# 2022

## APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

## [P.G.]

## (M.Sc. Second Semester End Examination-2022)

### PAPER-MTM 202

***Full Marks: 50*** ***Time: 02 Hrs***

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

*Illustrate the answers wherever necessary*

**[Numerical Analysis]**

**Attempt Question No. 1 and any four from rest:**

1. Attempt any **four** questions: **4 × 2 = 8**

   a) A function f(x) is defined by

   $$f(x)=\begin{cases} 1+x, \forall x \in [0,3] \\ (x-3)^2+x+1, \forall x \in [3,4] \end{cases}$$. Prove that f(x) is a cubic spline.

   b) Prove that closed type Newton's quadrature formula is more accurate than open type.

   c) What type of stability in Milne's method? Justify your answers.

   d) Find the weights $w_1, w_2, w_3$ so that the relation

$$\int_{-1}^{1} f(x)dx = w_1 f\left(-\sqrt{0.6}\right) + w_2 f(0) + w_3 f\left(\sqrt{0.6}\right)$$

Is exact for the functions $f(x) = 1, x, x^2$

e) What are the advantages to approximate a function using orthogonal polynomials?

f) Discuss the merits and demerits of finite difference method to solve an ordinary differential equation.

2. Suppose a table of values $(x_i, y_i), i = 0,1,2,...,n,$ is given. Describe natural cubic spline method to fit this set of data. **8**

3. a) Describe a least square method for approximating a function by Chebyshev polynomials in [-1, 1].

   b) Solve the system of equation by Crout's decomposition method.

$$x + 2y = 3$$
$$x - 2y + 3z = 5$$
$$-3y + 2z = 1$$

**4 + 4**

4. Describe 3-point. Gauss-Legendre quadrature formula. Use this formula to find the value of

$$\int_0^2 \left(x^5 + 2x^2 + 3x\right)dx$$

**4 + 4**

5. Derive fixed point iteration method to solve a system of non-linear equations $f(x, y) = 0$ and $g(x, y) = 0$. Find its condition of convergence and also order of convergence using maximum norms of vector. **3 + 2 + 3**



6. a) Find the largest eigen value and corresponding eoigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ by power method.

   b) Solve the boundary problem $\frac{d^2y}{dx^2} = y$ with $y(0) = 0$ and $y(2) = 3.627$ and take $h = k = 0.5$ **4 + 4**

7. Describe the Crank-Nicolson implicit method to solve the following equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

Subject to the boundary conditions $u(0,t) = f_1(t), u(1,t) = f_2(t)$ and initial condition $u(x,0) = g(x)$. **8**

**[Internal Asssment-10]**