

2022
MATHEMATICS
[HONOURS]
(B.Sc. Sixth Semester End Examination-2022)
PAPER-MTMH DSE 601
[MECHANICS]

Full Marks: 60

Time: 03Hrs

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as
far as practicable
Illustrate the answers wherever necessary

- 1. Answer any ten questions: 2x10= 20**
- i) State Varignon's theorem of moments.
 - ii) Prove that if three coplanar forces acting on a rigid body are in equilibrium then they must either be concurrent or parallel to one another.
 - iii) Discuss: angle of friction, cone of friction.
 - iv) Mention the forces which may be omitted in the equation of virtual work.
 - v) Define: wrench, pitch
 - vi) State the theorem of pappus.
 - vii) Determine the moment of a force about an axis.

(2)

- viii) What is meant by momental ellipsoid at a point?
- ix) Find the moment of inertia of a solid right circular cone about its axis.
- x) Define D'Alembert's principle.
- xi) What is the principle of conservation of momentum?
- xii) The resultant of two forces P and $2P$, acting at a point, is perpendicular to P . Find the angle between the forces
- xiii) A uniform cubical box of edge a is placed on the top of a fixed sphere. Show that the least radius of the sphere for which the equilibrium will be stable is $\frac{a}{2}$
- xiv) Find the moment of inertia of the arc of a circle about the diameter bisecting the arc.
- xv) Find the length of the simple equivalent pendulums of a circular disc, axis being the tangent to it (Axis being horizontal).

2. Answer any four questions:

4x5 = 20

- i) A semicircular disc rests in a vertical plane with its curved edge on a horizontal plane and an equally rough vertical plane and coefficient of friction being μ . Find the greatest angle that the bounding diameter can make with the horizontal plane.
- ii) Drive the equation of the central axis when a system of non-coplanar forces acting on a rigid body at different points.

(3)

- iii) Find the position of the c.g. of the surface generated by revolution of a loop of the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line
- iv) Show that the momental ellipsoid at a point on the rim of a hemisphere is $2x^2 + 7(y^2 + z^2) - \frac{15}{4}zx = \text{Constant}$
- v) ABC is a triangular area and AD is perpendicular to BC and AE is a median O , is the middle point of DE . Show that BC is a principal axis of the triangle at O .
- vi) A uniform rod OA of length $2a$, free to turn about its end O , revolves with uniform angular velocity w about the vertical OZ through O , and is inclined at a constant angle α to OZ ; show that the value of α is either zero or $\cos\left(\frac{3g}{4aw^2}\right)$.

3. Answer any two questions:

2x10 = 20

- i) a) Prove the principle of virtual works for a system of coplanar forces acting at different points on a rigid body.
b) Find the moment of inertia of a rectangular lamina about a line through the centre and parallel to its length.
- ii) a) A square frame $ABCD$ of four equal joined rods hangs from A , the shape being maintained by a string joining mid-points of AB, BC . Find the ratio of the tension of the string to the reaction at C .

(4)

b) A force P acts along the axis of x and another force nP acts along a generators of the cylinder $x^2+y^2=a^2$.

Show that the central axis lies on the cylinder

$$n^2(nx-z)^2+(1+n^2)y^2=a^2n^4$$

vi) a) A uniform rod AB is freely movable on a rough inclined plane, whose inclination to the horizon is i and whose coefficient of friction is μ about a smooth pin fixed through the end A ; the bar is held in the horizontal position in the plane and allowed to fall from this position . If θ be the angle through which it falls from

rest show that $\frac{\sin \theta}{\theta} = \mu \cot i$.

b) Find the kinetic energy of a rigid body rotating about a fixed axis.
