2021

Applied Mathematics with Oceanology and Computer Programming

[P.G.]

(CBCS)

(M.Sc. First Semester EndExaminations-2021)

MTM – 101

(*REAL ANALYSIS*)

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

1. Answer any FOUR questions **4x2=8**

a) Show that every compact metric space is separable.

b) Find thevariation function for the function $f(x)=|x-1|$ on $[0,2]$

c) Show that any function from a discrete metric space into a metric space is continuous.

d) Let $S_1, S_2 : X \to \mathbb{R}^{\bullet}$ be two non-negative simple measurable functions on a measurable space *X*. Show that $\{x \in X : s_1(x) \geq s_2(x)\}$ is a measurable set in *X*.

e) If α is continuous and β is of bounded variation on $[a,b]$, show that $\int_a^b \alpha \, d\beta$ exists

f) Define Borel Set

g) Show that the set of all natural numbers is a null subset of $\mathbb{R}$

2. Answer any FOUR questions 4x8=32

a) i) Establish a necessary and sufficient condition for a function $f : [a,b] \to \mathbb{R}$ to be a function of bounded variation on $[a,b]$

ii) Show that the set of all functions of bounded variation on $[a,b]$ forms a vector space under usual addition and multiplication by scalars. 4+4

b) i) Show that every path connected metric space is connected.

ii) Show that the function $f(x) = x Sin \frac{\pi}{x} \; 0 < x \leq 1$

$= 0 \qquad x = 0$ 4+4

Is not of bounded variation through continuous.

c) i) State and prove Lebesgue's monotone Convergence theorem.

ii) Prove that a continuous image of a connected metric space is connected. 4+4

d) i) Suppose $f : X \to [0,\infty]$ is measurable and $\phi(E) = \int_E f \, d\mu$ for every measurable set E in X. Show that ϕ is a measure and $\int g \, d\phi = \int gf \, d\mu$ for every measurable function *g* on *X* with range in $[0,\infty]$

ii) If $f_n : X \to [0,\infty]$ is measurable for $n = 1,2,3,......$ and $f(x) = \sum_{n=1}^{\infty} f_n(x)$, $x \in X$ then show that $\int f \, d\mu = \sum_{n=1}^{\infty} \int f_n \, d\mu$ 4+4

e) i) Show that cantor set is a null set

ii) Evaluate the RS-integral

$$\int_1^4 \left(4x^4 - 3e^{6x} + 5x^3 - 4x + 3\right) d\left(2[x]+1\right)$$

f) i) Let $f(x) = \frac{1}{x^p}$ if $0 < x \leq 1$ and $f(0) = 0$. Find necessary and sufficient condition on p such that $f \in L^1[0,1]$. Compute $\int_0^1 f(x)\lambda(x)$ in that case.

ii) Evaluate the following :

$$\int_{-1}^{3} 2\cos x \, d\left(2x + [x]\right)$$ 5+3

[Internal Marks – 10]