2021

Applied Mathematics with Oceanology and Computer Programming

[P.G.]

(CBCS)

(M.Sc. Third Semester End Examinations-2021)

MTM – 302

TRANSFORM AND INTEGRAL EQUATION

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

1. **Answer any FOUR questions.** **4x2=8**

a) Prove that $F\left\{e^{ibx} f(ax)\right\} = \frac{1}{a} F\left(\frac{k+b}{a}\right)$ F stands for Fourier transform.

b) Find the Fourier transform of $f(x)$ where $f(x) = xe^{-\alpha x^2}\ \alpha > 0$

c) Verify the final value theorem in connection with Laplace transform for the function $t^3 e^{-t}$

d) Define the inversion formula for Fourier cosine transform of the function $f(x)$. What happens if $f(x)$ is continuous ?

e) Transform the Initial value problem corresponding to the differential equation $u(x) = x + \int_0^x (x-t)\, u(t)dt$

f) Find the Laplace transform of $f(x) = [x]$, where $[x]$ represents the greatest integer less than or equal to x.

2. **Answer any FOUR questions.** **4x8=32**

a) **i)** Reduce the boundary value problem $\frac{d^2y}{dx^2} + \lambda xy = 1, 0 \le x \le l$ with boundary condition $y(0) = 0, y(e) = 1$ to an integral equation and find its Kernel.

ii) Show that if a function $f(x)$ defined on $(-\alpha, \alpha)$ and its Fourier transform $F(\xi)$ are both real, then $f(x)$ is even. Also show that if $f(x)$ is real and its Fourier transform $F(\xi)$ is purely imaginary.

5+3

b) **i)** Find the solution of the differential equation using Laplace transform $y'''(t) + y'(t) = Sin\,t$ with $y(0) = 0, y'(0) = -2, y''(0) = 0$

ii) Find the resolvent Kernel of the following integral equation and then solve it : $\varphi(t) = t^2 + \int_0^t \sin(t-y)\,\phi(y)dy$

4+4

c) i) If the Fourier transform of a function $f(x)$ and $x\,f(x)$ exist. Then the derivative of $F(k)$, the Fourier transform of $f(x)$ exists and is given by $\frac{d}{dx}[F(k)] = F[ix\,f(x)]$ hence show that $F[x\,f(x)] = -\frac{d}{dk}[kF(k)]$

ii) Find the solution of the integral equation

$$g(x) = x + \int_0^1 xt^2 g(t)dt$$ 6+2

d) i) If $f(t)$ is continuous and is of exponential order $0(e^{at})$ as $t \to \alpha$ and $f'(t)$ is piece wise continuous in any finite interval of *t*, then the Laplace trans form of $f'(t)$ exists for $\text{Re}(p) > a$ and is given by $L[f'(t)] = pf(p) - f(0)$ using this prove that $L[\phi(t)] = \frac{F(p)}{p}$ where $\phi(t) = \int_0^t f(\tau)d\tau$

ii) Solve the integral following equation

$y(x) = f(x) + \lambda \int_{-1}^{1} (xt + x^2t^2)\, y(t)dt$ and find the Eigen values. 3+1+4

e) i) Show that the integral equation $\phi(x)=1+\lambda\int_{0}^{2\pi} Sin(x+t)\phi(t)dt$ posses infinite number of solution.

ii) Find the solution of the differential equation using Laplace transform $c^2\dfrac{\partial^2 u}{\partial x^2}=\dfrac{\partial^2 u}{\partial t^2}$ $0\le x\le \ell,\ t>0$ Satisfying the following conditions

$$u(x,0)=0,\ \frac{\partial u(x,0)}{\partial t}=0, u(0,t)=a\ Sin\,\omega t,\ u(\ell,t)=0$$

f) i) State Parseval's identity of Fourier transform. Use generalization of Parseval's relation to show that

$$\int_{-\alpha}^{\alpha}\frac{dx}{\left(x^2+\alpha^2\right)\left(x^2+b^2\right)}=\frac{\pi}{ab\left(a+b\right)}, a,b>0$$

ii) Find $f(x)$ whose Laplace transform is $\dfrac{p}{\left(p2+4\right)^3}$ by using convolution theorem.

[Internal Marks – 10]