## Applied Mathematics with Oceanology and

## Computer Programming

[P.G.]
(CBCS)
(M.Sc. Third Semester End Examinations-2021)

$$
\text { MTM - } 304
$$

Full Marks: 50
Time: 02Hrs
The figures in the right hand margin indicate marks Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

## [DISCRETE MATHEMATICS]

1. Answer any FOUR questions.
a) What is inclusion-exclusion principle ?
b) Prove that the number of odd degree vertices in a graph is even.
c) What is tautology? Is $q \rightarrow(p v q)$ a tautology ?
d) Find the generating function of the sequence $\{1,-1,-1,1,-1,-1,1,-1,-1,1, \ldots \ldots \ldots$.
e) In a Boolean algebra $B$, prove that $\mathrm{x}=\mathrm{y}$ if $a+x=a+y$ and $a^{\prime}+x=a^{\prime}+y$
f) What is cardinal number? Find the cardinal number of , the set ofnatural numbers.
2. Answer any FOUR questions.
$4 \times 4=16$
a) Describe the time completing of maximum among $n$ number.
b) If $F_{n}$ is the nth Fibonacei number, prove that

$$
F_{n}=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n}-\left(\frac{1-\sqrt{5}}{2}\right)^{n}\right] . \quad n \quad \text { being }
$$

positive integer $\geq 1$
c) Prove that a simple graph with $n$ vertices and $K$ components can have at $\operatorname{most}(n-k)(n-k+1) / 2$ edges.
d) Express the DNF $F(x, y, z)=x y z+x y^{\prime} z+x y^{\prime} z^{\prime}$ in CNF.
e) Let $R$ be the relation on the set of people such that $x R y$ if $x$ and $y$ are people and $x$ is older than $y$ Show that $R$ is not a partial order relation.
f) Let S be the set of all positive integral divisors of 30 . Define the binary operations ( + ), (.), (/) on s by

$$
\begin{aligned}
& a+b=\ell . c . m \text { of } a \& b, \forall a, b \in S \\
& a . b=\text { ged of } a \& b, \forall a, b \in S
\end{aligned}
$$

And $\quad a^{\prime}=\frac{30}{a} \quad \forall a \in S$
Prove that $(S,+, ., /)$ is a Boolean algebra.

## 3. Answer any TWO questions.

$2 \times 8=16$
a) i) Using generating function, solve the recurrence relation $a_{n}=5 a_{n-1}-6 a_{n-2}, \forall_{n} \geq 2$ with $a_{0}=6$ and $a_{1}=30$
ii) Prove that number of $n$ vertices in a binary tree is always odd.
b) i) Show that the set of points in the closed interval [2, 4] and in the open interval $(1,2)$ are cardinally equivalent.
ii) Prove that the set $Q$ is enumerable.
$4+4$
c) i) Among the 1st 500 positive integers, determine the integers which are neither divisible 5,7 , nor 9 .
ii) Show that $(Z+, \leq)$ is a distributive lattice. $4+4$

