End Semester Examination, 2022

Semester - I

Physics

PAPER - CC-1T

Full Marks : 40

Time : 2 Hours

The Figures in the right hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. ***Answer any five questions :*** ***5x2=10***

a) Expand Sin x in powers of $(x - \pi/2)$ in Taylor series.

b) Prove that div curl $\vec{F} = 0$.

c) Using Green's theorem, show that area of plane region $A = \frac{1}{2}\oint(x\,dy - y\,dx)$.

d) If the thermodynamic variables pressure (P), volume (V) and Temperatue (T) are connected by the relation $f(P,V,T) = 0$, then prove that

$$\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T = -1$$

e) Evaluate $\int_0^4 t^3 \delta(t-5)\,dt$.

(Turn Over)

f) Find the solution of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-x}$ that satisfies the initial condition y = 0, $\frac{dy}{dx}=0$ at x = 0.

g) Let $f=x^2yz-4xyz^2$ be a scalar field. Find the directional derivative of f at P (1, 3, 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$.

h) Define Dirac delta function. Write down its properties.

Group - B

Answer any four questions : ***4x5=20***

2. a) If $\bar{V}=\bar{w}\times\bar{r}$. Prove $\bar{w}=\frac{1}{2}\bar{\nabla}\times\bar{v}$, where $\vec{w}$ is constant vector.

   b) Show that $\bar{\nabla}r^n=nr^{n-2}\bar{r}$. 3+2

3. Show that $\vec{A}=y^2\hat{i}+(2xy+z^2)\hat{j}+2yz\hat{k}$ is a conservative force field. Find the scalar potential and find work done in moving the object in the field from (1,1,1) to (2,1,3). 5

4. a) If ϕ is a continuously differentiable scalar function then $\oint\phi\, dr = \iint(\hat{n}\times\vec{\nabla}\phi)\, ds$.

   b) Prove that vectors $(\hat{i}-2\hat{j}+3\hat{k}), (-2\hat{i}+3\hat{j}-4\hat{k})$ and $(\hat{i}-3\hat{j}+5\hat{k})$ are coplanar. 3+2

5. Using method of variation of parameters solve

   $$\frac{d^2y}{dx^2}+4y=\tan x$$ 5

6. a) Find the order and degree of the following differential equation : $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/2}+xy=0$

   b) Solve $\frac{d^2y}{dx^2}+y=\sec^2 x$. 2+3

7. Let $\vec{F}$ be a conservative force field such that $\vec{F}=-\vec{\nabla}\phi$. Suppose a particle of constant mass m to move in this field. If A and B are any to points in space, prove that $\phi(A)+\frac{1}{2}mv_A^2=\phi(B)+\frac{1}{2}mv_B^2$

   Where v_A and v_B are the magnitude of velocities of the particles A and B respectively. 5

Group - C

Answer any one questions : ***1x10=10***

8. a) Write down the statement of Euler's Theorem.

   b) If f is homogeneous function of degree n in X and Y then prove that

   $$X^2\frac{\partial^2 f}{\partial X^2}+Y^2\frac{\partial^2 f}{\partial Y^2}+2XY\frac{\partial^2 f}{\partial x\times\partial Y}=n(n-1)f$$

   c) Verify Stoke's theoram for $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$, taken around the ractangle bounded by the lines $x=\pm a, y=0\ y=b$ 2+3+5

9. a) State and prove Green's theorem. 5

b) Prove $\iiint_V \vec{\nabla}\phi \, dV = \iint_s \phi \hat{n} \, dS$. 3

c) Evaluate $\iint_S \vec{F}.\hat{n} dS$, where

$\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$, where s is the surface of sphere with centre (3, −1, 2) and radius 3. 2