2022

Mathematics

[Honours]

(B.Sc. First Semester End Examination-2022)

PAPER-MTMH GE-101

(Numerical Methods and Differential Calculus-I)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group-A

[Numerical Methods]

1. Answer any seven questions: $7 \times 2 = 14$

a) Prove that $\Delta\nabla = \Delta\nabla$.

b) If $\pi = 3.142$ instead of $\frac{22}{7}$, calculate the absolute, relative and percentage error.

c) Find the loss of significant figures by subtracting 0.4329 from 0.4331.

d) Prove that $\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$, symbols have usual meaning.

e) Construct the difference table upto second order of $y = 3x^2 + 5$ for $x = 2, 4, 6, 8, 10$

f) Find $f(x)$ when its first difference is $x^3 + 4x^2 + 2x + 7$

g) Find the third degree polynomial which passes through the points (0, -1), (1, 1), (2, 1) and (3, -2)

h) What is mean by the degree of precision of a quadrature formula?

i) Why does the approximate value of $\int_0^1 x dx$ calculated by Trapezoidal rule becomes free inherent error?

j) If a number 0.000012 is approximated to 0.000009, find the number of significant digits for such approximation.

k) When Newton's Raphson method fails for computing the real root of the equation $f(x) = 0$.

2. Answer any two questions: $2 \times 5 = 10$

(a) Describe Euler's method in solving a differential equation. Comment on accuracy of Euler's method in solving a differential equation.

(b) Find the value of $\sqrt{2}$ correct upto four significant from the following table

x :	1.9	2.1	2.3	2.5	2.7
$\sqrt{x}$	1.3784	1.4491	1.5166	1.5811	1.6432

(c) Describe Gauss-seidal method for solving the system of linear equations.

(d) What is the lowest degree polynomial which takes the following values :

x:	0	1	2	3	4	5
f(x)	1	4	9	16	25	36

Hence find f(6) and f(0.5)

3. Answer any one question: $1 \times 10 = 10$

(a) (i) Describe the iteration method for computing a simple root of an equation $f(x)=0$.

Why iteration method is also called as fixed point iteration?

(ii) Calculate the values of $f(x)$ for x = 0.21 from the following table

x :	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$:	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

(b) (i) Compute y(0.8) by R-K method correct upto five decimal places from the equation. $\frac{dy}{dx} = xy,\ y(0) = 2,\ taking\ h = 0.2$.

(ii) Solve the system of equations by Gauss-Elimination method

$$x + 2y + z = 0$$
$$2x + 2y + 3z = 3$$
$$-x - 3y = 2$$

Group-B

[Differential Calculus-I]

4. Answer any three questions: $3\times 2=6$

(a) If In $=\int_0^{\pi/2} Sin^n x dx=\frac{n-1}{n}I_{n-2}$ then find $\int_0^{\pi/2} Sin^2 x dx$

(b) Find the nth derivative of $x^3\log x$.

(c) Determine the asymptotes of the curve

$$x^3+2x^2y-4xy^2-8y^3+8y-4x-1=0$$

(d) Find the curvature of the curve $y^2=4ax$ at $(a,2a)$ point

(e) Write down the properties of hyperbolic sine function and two properties of hyperbolic cosine function.

5. Answer any two questions: $2\times 5=10$

(a) If $y=\left(Sinh^{-1}x\right)^2$ prove that $\left(1+x^2\right)y_{n+2}+\left(2n+1\right)xy_{n+1}+n^2y_n=0$ using Leibnitz's rule.

(b) Find the envelope of the family of straight lines $x/a+y/b=1$ where the parameters a and b are related by $a^2+b^2=c^2$ c being constant.

(c) Find the area bounded by $y=6+4x-x^2$ and the chord joining (-2, -6) and (4, 6)

6. Answer any one question $1\times 10=10$

(a) (i) If $y^{1/m}+y^{-1/m}=2x$ prove that

$$\left(x^2-1\right)y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(n^2-m^2\right)y_n=0$$ 6

(ii) Evaluate $\underset{x\to 0}{Lt}\left(\frac{Sinx}{x}\right)^{1/x}$ 4

(b) (i) Find the area of the surface generated by revolving about the y-axis the part of the asteroid $x=a\cos^3\theta,\ y=a\sin^3\theta$ that lies in the first quadrant. 5

(ii) Find the point of inflexion, if any of the curve $y\left(a^2+x^2\right)=x^3$ 5
