

End Semester Examination, 2022**Semester - III****Physics****PAPER - CC-5T**

Full Marks : 40

Time : 2 Hours

The Figures in the right hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. **Answer any five questions :** **5x2=10**
- a) Check the singularity of the equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + l(l+1)y = 0$$
 at $x = +1$ and $x = -1$. 2
- b) Prove that, $P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)P_n(x)$ 2
- c) Write down the three dimensional wave equation in cylindrical co-ordinates. 2
- d) Show that, $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$ 2
- e) What is the value of Fourier Series at the point of discontinuity? 2
- f) What is Dirichlet's condition? 2
- g) Express $7x^3 + 2x - 3$ in terms of Legendre polynomials. 2

(Turn Over)

- h) If $f = \frac{5x^3y^4}{z^5}$ and errors in x, y, z be 0.0001, compute the maximum proportional error when $x = 0.001, y = 0.01$ and $z = 0.1$. 2

Group - B

Answer any four of the following questions :

4x5=20

2. What are Bessel's functions? Prove that the function $J_n(x)$ is the coefficient of Z^n in the expansion of $e^{\frac{1}{2}z(z-\frac{1}{z})}$. 2+3
3. Solve the differential equation by power series solution :- $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ about $x = 0$. 5
4. i) State Normal law of errors.
ii) By using the principle of least squares, find the equation of best fit straight line in the following data :-

| | | | | |
|-----------------|----|----|----|----|
| $x \rightarrow$ | 0 | 5 | 10 | 15 |
| $y \rightarrow$ | 12 | 15 | 17 | 22 |

1+3
5. Using the method of separation of variables, solve the spherical form of Laplace's equation. 5

6. Find half range cosine series of the function

$$f(x) = 1+x \quad 0 \leq x \leq 2$$

And hence prove that,

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8} \quad 3+2=5$$

7. Find the relation between Beta and Gamma functions. 5

Group - C

Answer any one of the following : 1x10=10

8. Deduce generating functions of Hermite Polynomials and show that :

$$\frac{1}{e} \text{Cosh} 2x = \sum_{n=0}^{\infty} \frac{1}{2^n n!} H_{2n}(x) \quad 6+4$$
9. Find out the permanent temperature within a solid sphere of radius unity when one half of the surface of the sphere is kept at constant temperature 0°C and the other half of the surface of the sphere is kept at 1°C . 10