# 2022

# Mathematics

## [HONOURS]

## (CBCS)

## (B.Sc. Third End Semester Examinations-2022)

## Paper: MTM-GE-301

**[*Analytical Geometry, Algebra & Vector Algebra*]**

***Full Marks: 60*** ***Time: 03 Hrs***

*The figures in the right hand margin indicate marks*
*Candidates are required to give their answers in their own words as far as practicable*
*Illustrate the answers wherever necessary*

### Group – A
### (Analytical Geometry)

1. **Answer any three questions:** **3x2=6**

   a) Find the equation of curve $9x^2+4y^2+18x-16y-11$ referred to parallel axes through(-1,2).

   b) Determine the type of the conic which is represented by $4x^2-4xy+y^2-8x-6y+5=0$

   c) Find the equation of the common tangent of the circle $x^2+y^2=4ax$ and the parabola $y^2=4ax$.

d) Find the equation of the bisectors of the angle between the pair of lines $3x^2+8xy+4y^2=0$.

e) Find the points on the conic $\frac{15}{r}=1-4\cos\theta$ whose radius vector is 5.

**2. Answer any one question** **1x5=5**

a) Prove that the pair of straight lines joining the origin to the other two points of intersection of the curves

b) $ax^2+2hxy+by^2+2gx=0$ and $a'x^2+2h'xy+b'y^2+2g'x=0$ will be at right angles if $g(a+b)=g(a'+b')$.

c) Show that the semi latus rectum of a conic is a harmonic mean between the segments of any focal chord.

**3. Answer any one question** **1x10=10**

a) i) Reduce the following equation to the canonical form and determine the nature of the conic represented by $x^2-5xy+y^2+8x-20y+15=0$

ii) Find the value of $\chi$ for which the two lines $3x^2-8xy+\chi^2=0$ are perpendicular to one another.

(6+1)+3

b) i) Show that the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and the straight line $lx+my=1$ is right angled if $(a+b)(al^2+2hlm+bm^2)=0$



ii) Show that the locus of points such that two of the three normal drawn from them to the parabola $y^2=4ax$ coincide is $27ay^2=4(x-2a)^3$

## Group – B
## (Algebra)

**1. Answer any five questions:** **5x2=10**

a) Define power set Find the power set of the set $A=\{a,b\}$

b) Solve by crammer's rule $2x-y=3$

$6x-3y=1$

c) If $a^2$=e for all $a\in G$ prove that G is a commutative group.

d) Find $\chi$ such that $A=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & \chi\end{bmatrix}$ is an orthogonal matrix

e) Prove that set to odd integers does not form a group under addition.

f) Find whether or not the relation R in the set A={1,2,3} are reflexive, symmetric where $R=\{(1,2),(2,2)\}$

g) Show that the product of all the values of $\left(1+i\sqrt{3}\right)^{3/4}$ *is* 8.

h) Show that every skew symmetric determine of 2nd order is a perfect square.

**2. Answer any two question** **2x5=10**

a) Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta' = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

Where $A_1, B_1, C_1$ ...... are the respective cofactors of $a_1, b_1, c_1$ .....in $\Delta$. Prove that $\Delta^1 = \Delta^2$

b) If R be a relation in the set of integer Z defined by the open sentence 'x-y is divisible by $6'$ , that is

$R = \{(x, y) : x \in \mathbb{Z}, y \in \mathbb{Z}; x - y$ is divisible by 6$\}$

Then prove that R is an equivalence relation.

c) Show that $H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$ is a sub group of the multiplicative group of all real non singular matrices of order 2

**3. Answer any one question** **10x1=10**

a) i) Express the matrix $A = \begin{pmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{pmatrix}$ is the sum of two matrices of which one is symmetrical and the other is skew-symmetrical.



ii) In a class of 25 students, 12 have taken mathematics, 8 have taken mathematics but not biology. Find the number of students who have taken mathematics and biology and those who have taken biology but not mathematics. 5+5

b) i) Show that

$$\begin{vmatrix} 0 & (a-b)^2 & (a-c)^2 \\ (b-a)^2 & 0 & (b-c)^2 \\ (c-a)^2 & (c-b)^2 & 0 \end{vmatrix} = 2(b-c)^2(c-a)^2(a-b)^2$$

ii) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{pmatrix}$

iii) Show that the set of all roots of the equation $x^4 = 1$ is group under usual multiplication. 4+3+3

## Group – C
## (Vector Algebra)

**1. Answer any two questions:** **2x2=4**

a) Show that the vectors $\vec{a} = (1,2,3)$, $\vec{b} = (2,-1,4)$ and $\vec{c} = (-1,8,1)$ are linearly dependent.

b) Find the value of $\gamma$ such that the vectors $(2\vec{i} - \vec{j} + \vec{k})$, $(\vec{i} + 2\vec{j} + \gamma\vec{k})$ and $(3\vec{i} - 4\vec{j} + 5\vec{k})$ are coplanar.

c) Show that the angle between the vectors $(\vec{i} - 2\vec{j} - 2\vec{k})$ and $(2\vec{i} + 2\vec{j} - 2\vec{k})$ is $\cos^{-1}\frac{4}{9}$

**2. Answer any one question** **1x5=5**

a) If a vector $\vec{\alpha}$ be resolved into components parallel and perpendicular to another vector $\vec{\beta}$, then show that the components are $\frac{\vec{\beta}.\vec{\alpha}}{|\vec{\beta}|^2}\vec{\beta}$ and $\frac{\vec{\beta}\times(\vec{\alpha}\times\vec{\beta})}{|\vec{\beta}|^2}$ respectively.

b) i) If $\vec{a} = \left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$, $\vec{b} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ and $\vec{c} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ then find the value of $\vec{a}.(\vec{b}\times\vec{c})$

ii) Find the vector area of the triangle, the position vectors of those vertices are $(\hat{i} + \hat{j} + 2\hat{k})$, $(2\hat{i} + 2\hat{j} + 3\hat{k})$ and $(3\hat{i} - \hat{j} - \hat{k})$. 2+3