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Squeezing and Antibunching in Three-mode Atom-Molecule Bose-Einstein Condensates

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ABSTRACT

We report the presence of the nonclassical properties namely squeezing and antibunching in three mode Bose-Einstein Condensate (BEC) system where the atomic mode is coupled with two molecular modes. Here photo associative stimulated Raman adiabatic passage (STIRAP) prepares the molecular modes in conjugation with Bose stimulation. Both squeezing and antibunching are found for atomic mode whereas the molecular modes remain coherent. The coupled mode squeezing is observed if one of the mode is necessarily atomicandthe coupled mode antibunching is present only for atomic-excited molecular mode. No nonclassicalities are found in excited molecular-stable molecular mode. The model Hamiltonian is solved analytically by a well-established approximation technique and the solutions are plotted with rescaled time. These solutions are well supported by numerical simulations. The criteria used here to examine nonclassicalities are practically realizable.

Keywords: Squeezing, Antibunching, Nonclassicality, BEC, STIRAP, Sen-Mandal approach, MBEC.

1. Introduction

The squeezing and antibunching are the nonclassical states required for the implementation of various other quantum mechanical states in numerous useful purposes such as dense coding [1], quantum cryptography [2,3] or quantum teleportation [4]. Antibunching is a quantum mechanical characteristic that is essential in realizing single photon sources [5]. In the recent past, BEC based systems are reported to make notable contributions in the actualization of quantum computing devices [6-9] e.g., the optical fiber coupled cavity consisting of two component BECs can transfer quantum mechanical states [9].

The single photon state is said to be the most nonclassical of all the quantum states of light [10]. But the nonclassicalitycan also involve a large number of photons. For the criterion of nonclassicality, one may think of Glauber-Sudarshan quasi-probabilty distribution or the P function [11]. For all the nonclassical states, P function is negative or more singular than δfunction. For practical applications, several other constructions of nonclassicality criterion such as negativity of Wigner function, Q parameters etc. have come into existence. We have resorted to some experimentally realizable nonclassial criteria relating to squeezing, intermodal squeezing, antibunching and intermodal antibunching.

The quantum states of BEC have marked both the theoretical and experimental significance [12-14]. Molecular Bose-Einstein Condensate (MBEC) can be produced using coherent photo association from an atomic BEC [15,16]. In this report, we investigate squeezing and antibunching (single mode as well as coupled mode) in a system of three mode atom-molecule BECs prepared by STIRAP having interactions between different modes in which the stable molecular mode has maximal population density in comparison with excited molecular mode [17,18].

The paper is organized as follows. In sec. 2, the analytical solutions of the equations of motion constructed from the Hamiltonian of the system of interest are presented. Sec. 3 gives the account of the presence of squeezing and sec. 4 deals with the occurrence of antibunching. Finally, it is concluded in sec 5.

2. The model Hamiltonian and the analytical solutions

The Hamiltonian of the model system [17] is as follows

$$H = \delta b^\dagger b - \frac{\omega}{2}\left(a^{\dagger 2} b + a^2 b^\dagger\right) - \frac{\epsilon}{2}\left(b^\dagger c + b c^\dagger\right). \quad (1)$$

where a, b and c are the bosonic annihilation operators for atomic mode, excited molecular mode and stable molecular mode with corresponding eigenstates$|1\rangle$, $|2\rangle$ and $|3\rangle$ respectively.The stable and excited molecular states differ by energy δ whereas ω and ϵ are the interactions between atomic and excited MBEC and between excited and stable MBEC respectively. The commutation relations among the field operators are given by

$$\begin{aligned} &[a, a^\dagger] = 1, [a, b] = 0 \\ &[b, b^\dagger] = 1, [b, c] = 0 \\ &[c, c^\dagger] = 1, [c, a] = 0 \end{aligned} \quad (2)$$

The stable molecular mode is maximally populated and for two photon resonance, atomic and stable molecular energy levels are exactly same as shown in the following schematic [17],

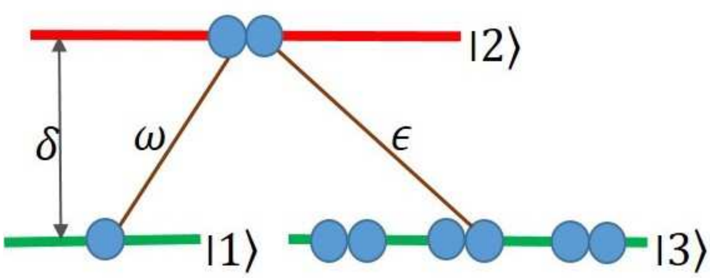


Figure 1: The schematic of three mode atom-molecule BEC

The Heisenberg's equations of motions for the field operators are given by the following set of equations (eqn. (3)). Here $\hbar = 1$ is taken throughout the solution.

$$\begin{aligned}
\dot{a}(t) &= i\omega a^\dagger(t)b(t),\\
\dot{b}(t) &= -i\delta b(t) + i\frac{\omega}{2}a^2(t) + i\frac{\epsilon}{2}c(t),\\
\dot{c}(t) &= i\frac{\epsilon}{2}b(t).
\end{aligned} \tag{3}$$

The coupled, nonlinear set of equations (3) does not have an exact analytical solution. Thus, an approximate technique called Sen-Mandal approach [19] has been adopted. These solutions are also featured in some of the authors' earlier work [20] and are given by the following set of equations considering terms upto $O(\epsilon^2)$ and $O(\omega^2)$,

$$\begin{aligned}
a(t) &= f_1 a(0) + f_2 a^\dagger(0)b(0) + f_3 a^\dagger(0)a^2(0) + f_4 a(0)b^\dagger(0)\mathrm{b}(0)\\
&\quad + f_5 a^\dagger(0)c(0),\\
b(t) &= g_1 b(0) + g_2 a^2(0) + g_3\mathrm{c}(0) + g_4 b(0) + g_5 a^\dagger(0)a(0)\mathrm{b}(0),\\
c(t) &= h_1 c(0) + h_2 b(0) + h_3 a^2(0) + h_4 c(0).
\end{aligned} \tag{4}$$

The parametric functions $f_i (i = 1,2,3,4,5)$, $g_i (i = 1,2,3,4,5)$ and $h_i (i = 1,2,3,4)$are,

$$\begin{aligned}
f_1 &= h_1 = 1,\\
f_2 &= 2g_2 = \frac{\omega}{\delta}G(t),\\
f_3 &= -\frac{f_4}{2} = -\frac{\omega^2}{2\delta^2}[G(t) - i\delta t],\\
f_5 &= 2h_3 = -\frac{\omega\epsilon}{2\delta^2}[G(t) - i\delta t],\\
g_1 &= e^{-i\delta t},\\
g_3 &= h_2 = \frac{\epsilon}{2\delta}G(t),\\
g_4 &= -\frac{2\omega^2 + \epsilon^2}{4\delta^2}g_1[G^*(t) + i\delta t],\\
g_5 &= -\frac{\omega^2}{\delta^2}g_1[G^*(t) + i\delta t],\\
h_4 &= -\frac{\epsilon^2}{4\delta^2}[G(t) - i\delta t].
\end{aligned} \tag{5}$$

The present set of solutions satisfies the following commutation relations which are called equal time commutation relations,

$$\left[b(t), b^\dagger(t)\right] = 1 \qquad \left[a(t), a^\dagger(t)\right] = 1 \qquad \left[c(t), c^\dagger(t)\right] = 1 \tag{6}$$

The conservation law of total particle number is also satisfied,

$$a^{\dagger}(t)a(t) + 2b^{\dagger}(t)b(t) + 2c^{\dagger}(t)c(t) = \text{constant}. \tag{7}$$

3. Squeezing

The stable molecular state is considered to have maximal population initially as evident with STIRAP. All the three modes are in coherent states at the beginning and the composite state is given by,

$$|\psi(0)\rangle = |\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle. \tag{8}$$

where $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ are eigen states of a,$b$and $c$with the eigenvalues α, β and γ respectively. Consequently, following eigenvalue equations are valid,

$$a(0)|\psi(0)\rangle = \alpha|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle,$$
$$b(0)|\psi(0)\rangle = \beta|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle, \tag{9}$$
$$c(0)|\psi(0)\rangle = \gamma|\alpha\rangle \otimes |\beta\rangle \otimes |\gamma\rangle.$$

To study the squeezing effect in various modes, the quadrature operators are defined as follows,

$$X_j = \frac{1}{2}[\, j(t) + j^{\dagger}(t)]$$
$$Y_j = -\frac{i}{2}[\, j(t) - j^{\dagger}(t)] \tag{10}$$

where$j = a, b$ or c.

The possibility of squeezing in any mode i is there if any one of the following inequalities is satisfied,

$$(\Delta X_j)^2 < \frac{1}{4}, (\Delta Y_j)^2 < \frac{1}{4} \tag{11}$$

And for the coupled mode squeezing, the quadrature operators are given by,

$$X_{jk} = \frac{1}{2\sqrt{2}}[\, j(t) + j^{\dagger}(t) + k(t) + k^{\dagger}(t)]$$
$$Y_{jk} = \frac{1}{2\sqrt{2}i}\left[\, j(t) - j^{\dagger}(t) + k(t) - k^{\dagger}(t)\right] \tag{12}$$

Whereas occurrence of any one of the following inequalities validates the coupled mode squeezing,

$$(\Delta X_{jk})^2 < \frac{1}{4},\ (\Delta Y_{jk})^2 < \frac{1}{4} \tag{13}$$

where $k = a, b$ or c and $j \neq k$.

Using (4), (5), (9) and (10), the fluctuations in the quadrature of mode a is given by,

$$\begin{pmatrix}(\Delta X_a)^2\\(\Delta Y_a)^2\end{pmatrix} = \frac{1}{4}[1 + |f_2|^2|\beta|^2 \pm (f_1 f_2 \beta + f_1 f_3 \alpha^2 + f_1 f_5 \gamma + c.c)] \tag{14}$$

where $c.c$ stands for complex conjugate and the upper and lower signs correspond to $(\Delta X_a)^2$ and $(\Delta Y_a)^2$ respectively. In case of the other modes, it is found that

$$\begin{pmatrix}(\Delta X_b)^2\\(\Delta Y_b)^2\end{pmatrix} = \frac{1}{4} \tag{15}$$

and

$$\begin{pmatrix}(\Delta X_c)^2\\(\Delta Y_c)^2\end{pmatrix} = \frac{1}{4} \tag{16}$$

Using (4), (5), (9) and (12) we obtain the variances for the coupled modes as follows,

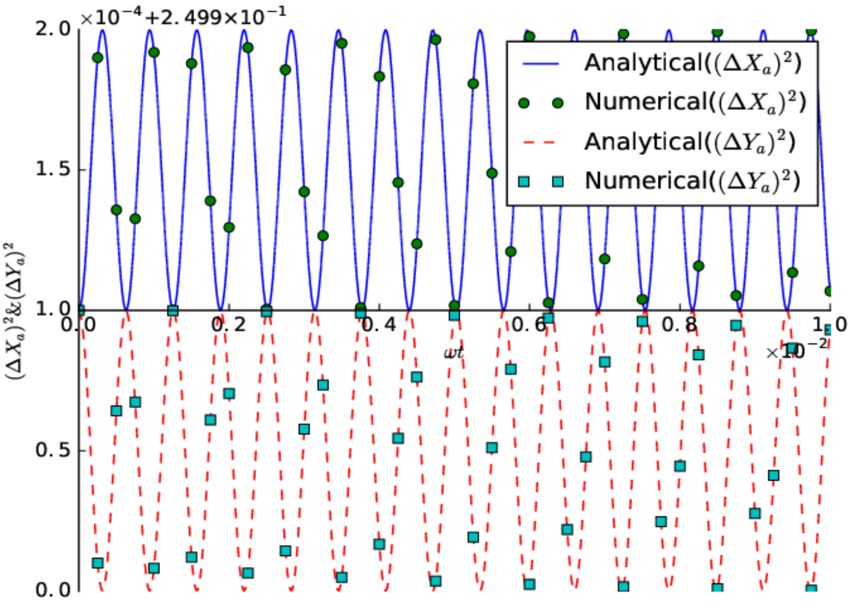
$$\begin{pmatrix}(\Delta X_{ab})^2\\(\Delta Y_{ab})^2\end{pmatrix} = \frac{1}{4}[1 + \frac{1}{2}|f_2|^2|\beta|^2 + \frac{1}{2}\{(f_1 g_5^* + f_4 g_1^*)\alpha\beta^* + (f_1 g_5 + g_1 f_4 + 2g_2 f_2)\alpha\beta \pm (f_1 f_2 \beta + f_1 f_3 \alpha^2 + f_1 f_5 \gamma) + c.c\}] \tag{17}$$

$$\begin{pmatrix}(\Delta X_{ac})^2\\(\Delta Y_{ac})^2\end{pmatrix} = \frac{1}{4}[1 + \frac{1}{2}|f_2|^2|\beta|^2 \pm \{\frac{1}{2}(f_1 f_2 \beta + f_1 f_3 \alpha^2 + f_1 f_5 \gamma) + c.c.\}] \tag{18}$$

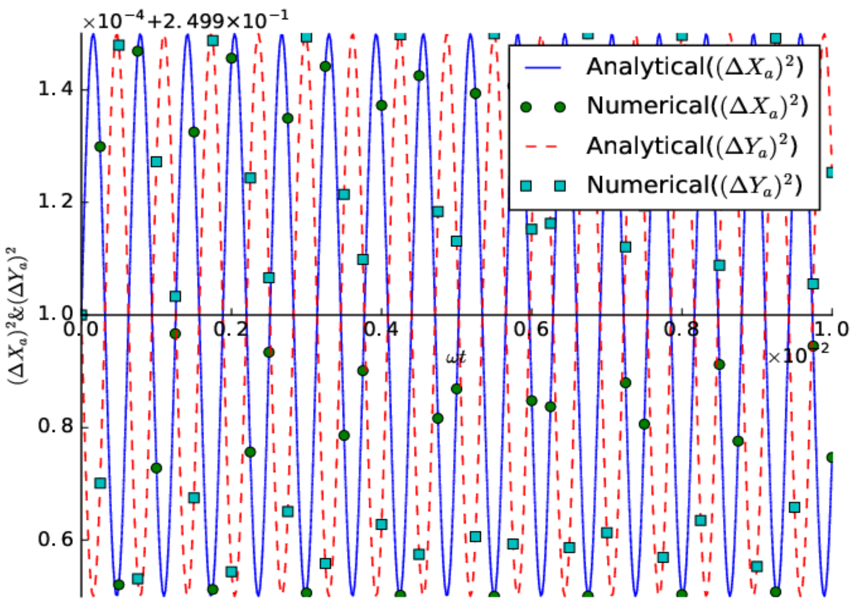
and

$$\begin{pmatrix}(\Delta X_{bc})^2\\(\Delta Y_{bc})^2\end{pmatrix} = \frac{1}{4} \tag{19}$$

In order to trace the signature of squeezing, the expressions of the equations (14), (17) and (18) are plotted with the dimensionless time ωt which are shown in figure (2) and (3). All the figures clearly depict the presence of squeezing. The analytical treatments are well supported by the numerical simulations. It can be noted that the amount of squeezing can be variedby controlling coupling constants (not shown in figure). The modes b and c remain coherent all through the time evolution.

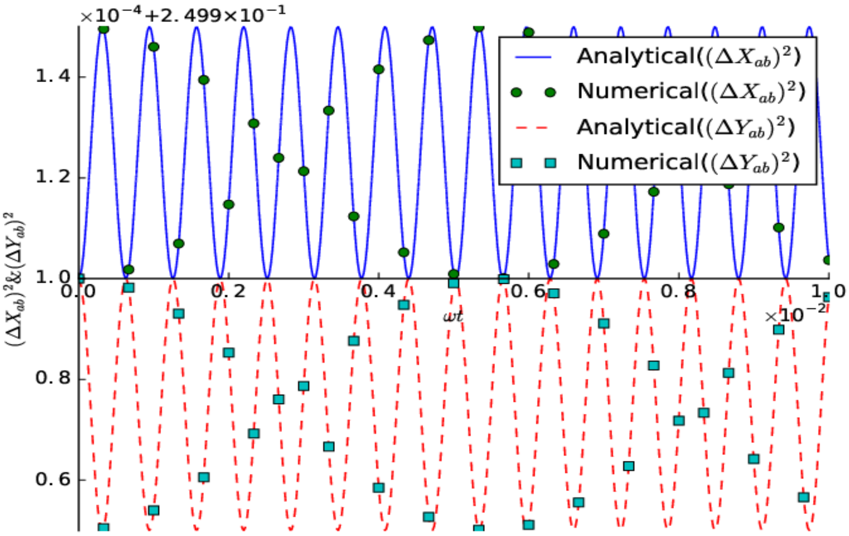


(a)

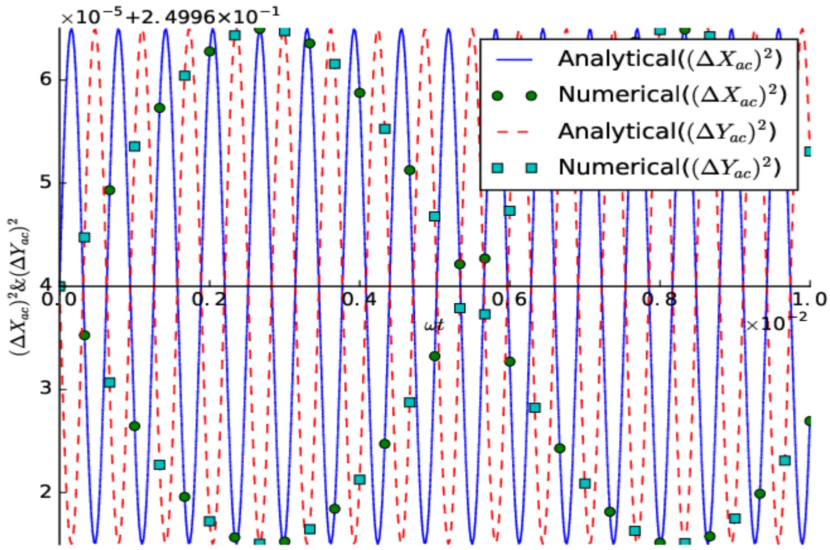


(b)

Figure 2: Plot of quadrature squeezing in mode a where in (a) β is taken as 1.0 and (b) $\beta = -i$. The other parameters are taken as,$\delta = 5.0$, $\omega = \epsilon = 5.0 \times 10^{-4}$, $\alpha = 1.0$ and $\gamma = 1.0$.



(a)



(b)

Figure 3: Plot of quadrature squeezing for (a) the coupled mode $a - b$ and (b) the coupled mode $a - c$. The rest of the parameters are taken as $\delta = 5.0$, $\omega = \epsilon = 5.0 \times 10^{-4}$, $\alpha = 1.0$ and $\gamma = 1.0$. Here for (a) $\beta = 1.0$ and for (b) $\beta = -i$.

4. Antibunching

The quantum statistical properties for mode $a$can be studied by calculating the second order correlation function for zero-time delay

$$g^{(2)}(0) = \frac{<a^{\dagger}(t)a^{\dagger}(t)a(t)a(t)>}{<a^{\dagger}(t)a(t)><a^{\dagger}(t)a(t)>} \quad (20)$$

The particle number distribution for mode a is sub-PoissonianIf $0 < g^{(2)}(0) < 1$and the mode is associated with the nonclassical phenomenon called antibunching [10]. The eqn. (21) can be written in the form

$$g^{(2)}(0) - 1 = \frac{(\Delta N)^2 - <N>}{<N>^2} \quad (21)$$

It is evident that numerator $D = (\Delta N)^2 - < N >$ in the right hand side of eqn. (21) determines the quantum statistical properties. Specifically speaking, the conditions $D < 0$, $D = 0$ and $D > 0$ give the sub-Poissonian, Poissonian and super-Poissonian statistics respectively. Using eqn. (4), the analytical expression for D_a is

$$D_a = |f_2|^2 \left(|\beta|^2 + 6|\alpha|^2|\beta|^2 - \frac{1}{2}|\alpha|^4\right) + (f_1^* f_2 \alpha^{*^2}\beta + f_1^* f_5 \alpha^{*^2}\gamma + c.c.). \quad (22)$$

Similarly, for the other modes

$$D_b = D_c = 0 \quad (23)$$

For intermodal antibunching, the following expression can be used

$$g^{(2)}(0) = \frac{<a^{\dagger}(t)b^{\dagger}(t)b(t)a(t)>}{<a^{\dagger}(t)a(t)><b^{\dagger}(t)b(t)>} \quad (24)$$

Correspondingly

$$D_{ab} =< a^{\dagger}(t)b^{\dagger}(t)b(t)a(t) > - < a^{\dagger}(t)a(t) >< b^{\dagger}(t)b(t) > . \quad (25)$$

Using equations (4) and (25), the following expression for the quantum statistics is obtained

$$D_{ab} = -(4|g_2|^2 - |g_3|^2)|\alpha|^2|\beta|^2. \quad (26)$$

We plot equations (22) and (26) with the scaled time ωt (figure 4) and from these plots the presence of antibunching is ascertained. No antibunching is found from any other combination of modes.

5. Conclusion

We consider three mode atom-molecule BEC prepared by Bose Stimulated Raman Adiabatic Passage. The Heisenberg's equations of motion derived from the model Hamiltonian is solved analytically using Sen-Mandal Technique. Using the solutions, we study two of the nonclassical properties of various modes namely, squeezing and antibunching. The plots of the solutions suggest the existence of both squeezing and antibunching in atomic mode as well as the coupled modes involving atoms and the analytical solutions are well supported by numerical ones. Particularly for antibunching to be observed in atomic mode, complex eigenvalue for the bosonic field operator corresponding to excited molecular mode is taken. The stable and the excited molecular mode states remain coherent all through the time evolution. There is further scope of research if the system is made coupled with external cavity.

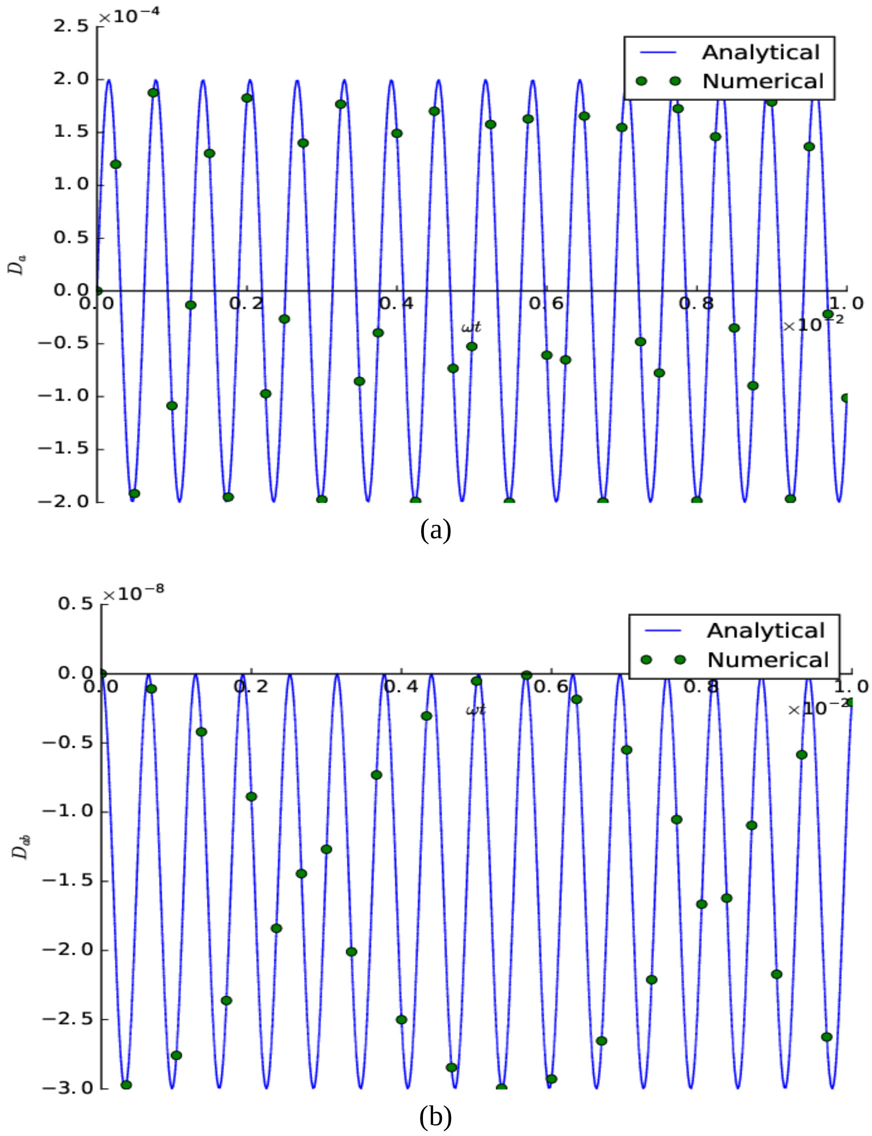


Figure 4: Plot for antibunching for (a) mode a and (b) coupled mode $a - b$. The parameters are taken as $\delta = 5.0$, $\omega = \epsilon = 5.0 \times 10^{-4}$, $\alpha = 1.0$ and $\gamma = 1.0$. Here for (a) $\beta = -i$ and for (b) $\beta = 1.0$.

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