2022

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth Semester End Examinations-2022)

MTMH-C501

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Real Analysis - III

1. Answer any TEN questions: **10x2=20**

a) Examine the Convergence of $\int_0^1 \frac{1}{(x+1)(x+2)\sqrt{x(1-x)}} dx$

b) Applying second mean value theorem show that $\left|\int_a^b \left(\frac{\sin x}{x}\right) dx\right| < \frac{4}{a}$ where $0 < a < b < \infty$.

c) Show that an enumerable subset of $\mathbb{R}$ is a set of measure zero.

d) Determine the radius of convergence of the power series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3i}+\frac{4^4x^4}{4!}+\ldots$

e) Prove that $\int_0^1 \log(1-x)dx=-1$ considering the following expansion $\log(1-x)=-1-\frac{x^2}{2}-\frac{x^3}{3}-\ldots\ldots$

f) Prove that if two power series $\sum_{n=0}^{\alpha} a_n\, x^n$ and $\sum_{n=0}^{\alpha} b_n\, x^n$ converge to *f*(*x*) then $a_n=b_n \forall n\in\mathbb{N}$

g) Examine the uniform convergence of the series $\sum_{n=0}^{\infty}\left(\frac{1}{nx+2}-\frac{1}{(n+1)x+2}\right)\forall x\in[0,1]$.

h) Prove that $\frac{\pi^2}{9}<\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{x}{\sin x}dx<\frac{2\pi^2}{9}$.

i) Let $f_n(x)=\frac{2x+n}{x+n}, x\in\mathbb{R}$ show that $\{f_n(x)\}$ converges uniformly [-b,b] but the convergence is not uniform in $a\leq x<\infty$.

j) Let $f_n(x)=\frac{n+\cos x}{2n+\sin^2}\forall x\in\mathbb{R}$ and $n\in\mathbb{N}$. Show that $\{fn(x)\}$ is convergent to $\mathbb{R}$

k) Let $f:[a,b]\to\mathbb{R}$ and $g:[a,b]\to\mathbb{R}$ are Riemann integrable than prove that $\max\{f,g\}$ is Riemann integral.

l) Examine the convergence of $\int_0^{\frac{\pi}{2}}\frac{1-\cos\frac{\pi}{2}}{x^m}dx$ for $m<m$

m) State Dirichlet's condition for convergence of Fouria series to a function f(x).

n) Using comparison test prove that $\int_1^{\infty}\frac{1}{x(1+x^2)}dx$ is convergent

o) Define uniform Convergent and pointwise Convergence of sequence of function $\{f_n(x)\}$ when $f_n:[a,b]\to\mathbb{R}\forall n\in\mathbb{N}$.

2. Answer any FOUR questions **5x4=20**

a) Let a function $f:[a,b]\to\mathbb{R}$ be integrable on $[a,b]$ $f(x)\geq 0\forall x\in[a,b]$. Show that if there exist a point c in $[a,b]$ such that f is continuous at c and $f(c)>0$ then $\int_a^b f>0$

b) State and prove the fundamental theorem of Integral calculus.

c) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ is convergent if and only if $m,n>0$

d) Prove that the series $\sum(-1)^n x^n(1-x)$ converges uniformly on $[0,1]$ but the series $\sum x^n(1-x)$ is not uniformly convergent on $[0,1$

e) Prove that $\int_0^1 \log(1-x)dx = -1$ considering the following expansion $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots$ for $-1 \le x < 1$

f) Let g be continuous on [0,1] and $f_n(x) = \frac{g(x)}{e^{nx}} \forall x \in [0,1]$ Prove that the sequence $\{f_n(x)\}$ is uniformly convergent on [0,1] if and only if $g(1)=0$ 5

3. Answer any TWO questions: **10x2=20**

a) i) Let $f_n : [a,b] \to \mathbb{R}$ and f_n is integrable on $[a,b] \forall n \in \mathbb{N}$. If $\{fn(x)\}$ converges to $F(x)$ on $[a,b]$ then prove that $F(x)$ is continuous on $[a,b]$ Where $F_n(x) = \int_a^x f(t)dt$. Is uniform convergence of the sequence $\{f_n(x)\}$ necessary for continuity of $F(x)$ on $[a,b]$? Converse of the theorem is true? If not then state theorem for partial converse of the theorem.

4+2+2+2

b) i) A power series can be integrated term by term on any closed and bounded interval contained with in the interval of convergence.

ii) What are the difference between Darboux sum and Riemann sum.

iii) Prove that $\int_0^1 \frac{dx}{(1-x^5)^{\frac{1}{5}}} = \frac{\pi}{5} \operatorname{cosec} \frac{\pi}{5}$, $n > 1$.

4+3+3

c) i) Let a function $f : [a,b] \to \mathbb{R}$ be Riemann integrable and $c \in (a,b)$. Then first prove that f is integrable on [a,c] and [c,b] and also prove that

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

ii) Show that Fourier series corresponding to x_2 on $-\pi \le x \le \pi$ is $\frac{\pi^2}{2} + 4\sum_{n=1}^{\alpha} (-1)^n \frac{\cos nk}{n^2}$ hence deduce the sum of $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ and $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$

5+(3+2)