M.Sc. First Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-103

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Ordinary Differential Equation and Special Functions]

Answer question no. 1 and any four from the rest

1. Answer any four questions: $2\times4=8$

a) Consider the second order homogeneous linear differential equation. $a_0(x)\dfrac{d^2y}{dx^2}+a_1(x)\dfrac{dy}{dx}+a_2(x)y=0$

When $a_0(x), a_1(x), a_2(x)$ are continuous function on a real interval $a\le x\le b$ and $a_0(x)\neq0$ for all $x\in[a,b]$. Let f_1 and f_2 are two solutions of the differential equation and has zero in $[a,b]$ then show that they are linearly dependent.

b) When a differential equation is said to be Fushian type and give an example.

c) Let $P_n(z)$ be the Legendre's polynomial of degree n. If

$$1+z^5=\sum_{n=0}^{5}C_nP_n(z)$$

Then find the value of C_5.

d) Find all the singularities of the following differential equation and then classify them:

$$z^2(z^2-1)^2\omega''-z(1-z)\omega+2\omega=0$$

e) Consider the linear linear system of differential equation

$$\frac{dx}{dt}=a_1x-b_1y$$

$$\frac{dy}{dx}=a_2x-b_2y$$

Where a_1,b_1,a_2,b_2 are real constants. Show that the system has two linearly independent solution of the form $x=Ae^{\lambda t}\ and\ y=Be^{\lambda t}\ if\ a_2b_1>0$

f) Derive the adjoint differential equation of the differential equation $x^2\dfrac{d^2y}{dx^2}+7x\dfrac{dy}{dx}+8y=0$

2. **Answer any four questions:** $4\times 8=32$

a) (i) Let $P_0(x),P_1(x),..P_n(x)$ are continuous on $[a,b]$ and $y_1(x),y_2(x),...y_n(x)$ are n solutions of the equation

$$P_0(x)\frac{d^ny}{dx^n}+P_1(x)\frac{d^{n-1}y}{dx^{n-1}}+...+P_n(x)y=0 \text{ where } P_0(x)\neq 0$$

Then prove that $W(y_1,y_2,...y_n)$ is identically zero or no where zero in $a\le x\le b$. If $P_0(x),P_1(x),..P_n(x)$ are all polynomial functions of degree n and has relative extremum at common point $x_0\in[a,b]$ then show that all solutions are linearly dependent.

(ii) Prove that $2J_n'(z)=J_{n-1}(z)-J_{nH}(z)$. (3+2)+3

b) (i) Prove that eigen functions of strum – Liuouville system $\dfrac{d}{dx}\left[p(x)\dfrac{dy}{dx}\right]+\left[\rho(x(x)-q(x)\right]y=0$ Where $\rho(x)>0\, and\, q(x)$ are both continuous function and λ is real value. $P(x)$ is continuously differentiable and positive, are orthogonal with boundary condition $y(a)=y(b)=0$.

(ii) Prove that $\sin(z\sin\theta)=2\sum_{n=1}^{\infty}J_{2n-1}(z)\sin(2n-1)\theta$ Hence find the series expansion of sin z terms Bessel functions.

c) (i) Solve by using green function the differential equation $\frac{d^2y}{dx^2}+k^2u=x(k\in\mathbb{N})$ subject to the boundary conditions $u(o)=\alpha, u'(1)=\beta$

(ii) Find all eigen values and eigen functions of the differential $\frac{d^2u}{dx^2}+\lambda u=0$ with $u^1(o)=u(\pi)=0, x\in[0,\pi]$ 5+3

d) (i) Determine whether the matrix $B=\begin{pmatrix} e^{4t} & 0 & 2e^{4t} \\ 2e^{4t} & 3e^{t} & 4e^{4t} \\ e^{4t} & e^{t} & 2e^{4t} \end{pmatrix}$ is a fundamental matrix system $\frac{dx}{dt}=AX$ where $X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $A=\begin{pmatrix} 1 & -3 & 9 \\ 0 & -5 & 18 \\ 0 & -3 & 10 \end{pmatrix}$ .If not justify and find another alternative fundamental matrix.

(ii) Prove that $\int_{1}^{1} P^2 m(z)\,dz=\frac{2}{2n+1}$ where $P_n(z)$ is the Legendre's polynomial. 4+4

e) (i) Solve the differential equation about z=0

$$8z^2\frac{d^2w}{dz^2}+2z\frac{dw}{dz}+w=0$$

(ii) Derive the integral representation of Confluent hyper geometric function of the equation

$$z\frac{d^2w}{dz^2}+(\gamma-z)\frac{dw}{dz}-\alpha w=0$$ 6+2

f) (i) Find the general solution of the equation $2z(1-z)w''(z)+w'(z)+4w(z)=0$ by the method of solution in series about z=0, and show that the equation has a solution which is polynomial in z.

(ii) Show that $nP_n(z)=zP_n(z)-P_{n-1}(z)$, where $P_n(z)$ denotes the Legendre's Polynomial of degree n. 4+4

[Internal assessment – 10]
