2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Third Semester End Examinations-2021)

MTM-GE-301

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

[*Analytical Geometry, Algebra & Vector Algebra*]

1. **Answer any TEN questions:** **10x2=20**

   i) Show that the vectors $\left(9\hat{i}+\hat{j}-6\hat{k}\right)$ and $\left(4\hat{i}-6\hat{j}+5\hat{k}\right)$ are perpendicular to each other.

   ii) Determine the value of μ for which the vector $\mu\left(6\hat{i}+2\hat{j}-3\hat{k}\right)$ may be of unit length.

   iii) Transform to parallel axes through the point (2, -1) the *equation* $x^2-3y^2+4x+6y+1=0$

   iv) Find the modules and amplitude (Principle value) of the complex number -1 +*i*.

v) Express $\frac{3+5i}{7+3i}$ in the form $A+iB$

vi) Solve the equation $x^7=1$

vii) Determine the nature of the conic

$3x^2+4xy+3y^2+4x-4y-2=0$

viii) Define linearly independent and linearly dependent of a set of vectors.

ix) Find the vector product of two vectors $(3\hat{i}-2\hat{j}+\hat{k})$ and $(\hat{j}+4\hat{k})$

x) Define the terms minor and cofactor of a matrix

xi) Write any four properties of orthogonal matrices.

xii) Define union and intersection of sets.

xiii) Define sub-group with example.

xiv) Give an example of symmetric matrix and skew-symmetric matrix

xv) Express $\begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & 1 \\ 5 & -2 & 3 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.

2. Answer any FOURquestions **4x5=20**

i) Show that three vectors $\vec{\alpha}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-3\hat{j}-5\hat{k}$, $\vec{c}=3\hat{i}-4\hat{j}-4\hat{k}$ from the sides of a right angle triangle. 5

ii) a) Show that the following points are collinear $(\hat{i}-2\hat{j}+3\hat{k})$, $(2\hat{i}+3\hat{j}-4\hat{k})$, $(-7\hat{j}+10\hat{k})$

b) Find the angle between the vectors $\vec{a}=2\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+4\hat{k}$ 3+2

iii) State and prove De Movire's Theorem. 1+4

iv) Define Cartesian product of two sets. For three non-empty sets *A, B, C* prove that. $A\times(B\cap C)=(A\times B)\cap(A\times C)$

v) a) Find the value of the constant *d*, such that the vectors $(2\hat{i}-\hat{j}+\hat{k})$, $(\hat{i}+2\hat{j}-3\hat{k})$ and $(3\hat{i}+d\hat{j}+5\hat{k})$ are coplanar.

b) Show that $[\vec{\alpha}+\vec{\beta}.\vec{\beta}+\vec{\gamma}.\vec{\gamma}+\vec{\alpha}]=2[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]$, where $\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}$ are vectors. 2+3

vi) Show that the equation $5x^2+16xy+9y^2+22x+20y+9=0$ represents a pair of straight lines and find the angle between them. 5

3. Answer any TWO question **10x1=10**

1. a) Reduce the equations to the canonical forms and determine the nature of the conic represented by $3x^2+2xy+3y^2-16x+20=0$

   b) Show that distance between two points is an invariant under an orthogonal transformation 5+5.

2 a) If $\cos\alpha+\cos\beta+\cos\gamma=0=Sin\alpha+Sin\beta+Sin\gamma$ then prove that

i) $\cos\alpha+\cos\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$ and

$Sin3\alpha+Sin\beta+Sin3\gamma=3Sin(\alpha+\beta+\gamma)$

ii) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=Sin^2\alpha+Sin^2\beta+Sin^2\gamma=3/2$

b) Find the all roots of $x^5=-1$

3 a) Determine the matrices A and B, where

$A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$ and

$2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$

b) Prove that $\begin{vmatrix}(b+c)^2 & a^2 & a^2\\ b^2 & (a+c)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix}=2ab(a+b+c)^2$

c) If $A=\begin{bmatrix}-1 & 1 & -1\\ 3 & -3 & 3\\ 5 & -5 & 5\end{bmatrix}$

Then show that $A^k=A$ for some suitable value of k, find such K

d) Define Rank of a matrix. 3+3+3+1

4 If by a rotation of co-ordinate axes the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$ then show that

i) $a+b=a'+b'$

ii) $ab-h^2=a'b'-h'^2$ 5+5

[The End]