2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. FirstSemester End Examinations-2021)

MTMH-C102

(Algebra)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group – A
[*CLASSICAL ALGEBRA*]

1. **Answer any FOUR questions:** **4x2=8**

   a) If $f(x)=x^4-3x^3+10x^2$ express $f(x+3)$ as polynomial in x.

   b) $x^3-3px-q$ has a factor of the form $(x-\alpha)^2$,show that $q^2+4p^3=0$

c) If a and b are positive real numbers and $a+b=4$ prove that

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2\geq\frac{25}{2}$$

d) Find fourth roots of (-1)

e) Find mod z and amp z (principal amplitude) when

$$z=1+\cos 2\theta+iSin2\theta,\ \frac{\pi}{2}<\theta<\pi$$

f) Find the quotient and reminder when x^6+x^3+1 is divided by $x+1$.

2. Answer any ONEquestions 5x1=5

a) If n be an odd positive integer prove that the equation $x^{2n}-1=0$ and $x^n-1=0$ have the same number of special roots.

b) Solve the equation $x^4-10x^3+35x^2-50x+24=0$ by Ferrari's method.

3. Answer any ONE question 10x1=10

a) i) If A and G be the arithmetic mean and geometric mean respectively of n positive real numbers $a_1, a_2.....a_n$ prove that if $K>0, (K+A)^n\geq(K+a_1)(K+a_2)...(K+a_n)\geq(K+G)^n$

ii) If Z_1, Z_2 are two non-zero complex numbers, Prove that

$$2|Z_1+Z_2|\geq(|Z_1|+|Z_2|)\left|\frac{Z_1}{|Z_1|}+\frac{Z_2}{|Z_2|}\right|$$ 5+5

b) i) If α,β,γ be the root of the equation $x^3+px^2+qx+r=0$ then find the equation whose roots are $\beta^2+\gamma^2-\alpha^2,\ \gamma^2+\alpha^2-\beta^2,\ \alpha^2+\beta^2-\gamma^2$.

ii) If $\alpha,\beta,\gamma,\delta$ are the roots of the equation $t^4+t^2+1=0$ and n is a positive integer, prove that

$$\alpha^{2n}+\beta^{2n}+\gamma^{2n}+\delta^{2n}=4\cos\frac{2n\pi}{3}$$

Group – B

[*ABSTRACT ALGEBRA - I*]

4. Answer any TWO questions 2x2=4

a) Find two integer u and v satisfying $54u+24v=30$

b) Let A be a set of n elements and B be a set of m elements. Show that if $n\leq m$ the total number of injective mappings from A to B is $\frac{m!}{(m-n)!}$

c) Show that the number of different reflective relations on a set of n elements is $2^{n^2}-n$

5. Answer any TWO questions 5x2=10

a) If a and b are integers, not both zero, then show that $\exists$ integers u and v such that $g(a,b)=au+bv$

b) Prove that $I^n-3^n-6^n+8^n$ is divisible by 10 for all $n\in$ □

c) Arelation β is defined on $\mathbb{Z}$ by "$x\beta y$" of and only if x^2-y^2 is divisible by 5 for $x,y\in\mathbb{Z}$. Prove that β is an equivalence relation on $\mathbb{Z}$. Show that there are three distinct equivalence classes.

Group – C
[*LINEAR ALGEBRA*]

6. Answer any FOUR question **4x2=8**

a) Prove that $adj\ A^{-1}=(adj\ A)^{-1}$ for a non-singular matrix A.

b) Let Vbe a vector space over a field$F$, then $-1\alpha=\alpha$

c) Is $S=\{(x,y,z)\in\mathbb{R}^3 : x-2y+z=0\}$ a subspace of $\mathbb{R}^3$.

d) Find the eigen value of the idempotent matrix.

e) Define linear mapping.

f) Define linear sum of two subspaces.

7. Answer any ONE question **5x1=5**

a) Define dimension of a vector space. Given $\{1,1+x,x+x^2\}$ as basis of $P_2(x)$ over $\mathbb{R}$ and the inner product defined as $(f,g)=\int_{-1}^{1}f(x)g(x)dx$ where $f,g\in P_2(x)$. Construct an orthonormal basis of $P_2(x)$ from given set.

b) Define rank of the matrix. Determine the conditions for the system of equations has only one solution, many solutions, no solution : $x+y+z=1, x+2y-z=b, 5x+7y+az=b^2$

8. Answer any ONE question **10x1=10**

a) i) Find the orthonormal basis of the row space of the matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ 2 & 2 & 1 \end{pmatrix}$$

ii) If S and T be two non-empty finite subsets of a vertex space V over a field F and each element of T is a linear combination of the Vectors of S. Then (T) $L(T)\subset L(S)$

5+5

b) i) Find a matrix $\underline{P}$ such that $P^{-1}AP$ is a diagonal matrix where $A=\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

ii) Show that the quadratic form $x^2+y^2+2xy+2yz$ is indefinite. 5+5

[The End]