2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Third End Semester Examinations-2021)

MTMH-C303

Full Marks: 60 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

[*REAL ANALYSIS - II*]

1. **Answer any TEN questions:** **10x2=20**

   a) Let *f, g* be defined on $A \subset \Re$ to $\Re$ and c be a limit point of A. If $\lim_{x \to c} f$ and $\lim_{x \to c} fg$ exists, does it follows that $\lim_{x \to c} g(x)$ exists.

   b) Show that $f(x) = |x|$ is continuous on $\Re$

   c) Suppose $f : \Re \to \Re$ and $g : \Re \to \Re$ are continuous on $\Re$ and that $f(x) = g(x)$ for all rational x. Prove that . $f(x) = g(x) \, \forall x \in \Re$

d) Prove that $f(x)=2\ln x+\sqrt{x}-2$ has root in the interval (1,2)

e) Show that $f(x)=x^3-3x^2-x+3$ has three zeroes in [-2, 4]

f) Use Mean value theorem prove that $|Sinx-Siny|\le|x-y|\forall x,y\in\Re$

g) Let *f*, *g* are differentiable on $\Re$ s.t. $f(0)=g(0)$ and $f'(x)\le g'(x)\forall x\ge 0$ Show that $f(x)\le g(x)$ for all $x\ge 0$

h) Find the supremum of $f(x)$ where $f(x)=\frac{x}{x^2+1}\forall x\in(-1,1)$

i) Examine if $\lim_{x\to 0} Cotx$ exists

j) Prove that $\log Sinx$ is continuous on $\left(0,\frac{\pi}{2}\right)$

k) Using sequential criterion for limit to show that $\lim_{x\to 0}\frac{1}{x}Sin\frac{1}{x}$ does not exists.

l) Let a function $f:\Re\to\Re$ is continuous on $\Re$ and $\mu\in\Re$. Prove that the set $Set\{x\in\Re: f(x)\ne\mu\}$ is an open set.

m) Verify Mean value theorem for the function $f(x)=4-(6-x)^{2/3}$ on [5, 7]

n) State Rolle's theorem for polynomial functions

o) Use Mean value theorem to prove that $0<\frac{1}{\log(1+x)}-\frac{1}{x}<1\forall x>0$

2. Answer any FOURquestions **5x4=20**

a) Let $C\in\Re$ and $f:(c,\alpha)\to\Re$ and $f(x)>0$ for all $x\in(c,\alpha)$ show that $\lim_{x\to c} f(x)=\alpha$ if and only it $\lim_{x\to c}\frac{1}{f(x)}=0$

b) Define $g:\Re\to\Re$ by $g(x)=2x$ is rational $=x+3$ if x is irrational.
Find all points at which *f*(*x*) is continuous

c) Let $g:\Re\to\Re$ satisfy the relation $f(x+y)=f(x).f(y)$ for all $x,y\in\Re$ Prove that if f is continuous at *x*=0 then g is continuous at every point in $\Re$. And if we have $g(a)=0$ for some $a\in\Re$ then $g(x)=0\forall x\in\Re$

d) Let $f:\Re\to\Re$ be continuous on $\Re$. A point $c\in\Re$ is said to be fixed point of *f* if $f(c)=c$ holds. Prove that the set of all fixed points of f is a closed set.

e) Let $f(x,y)=\frac{x^2y}{x^4+y^2}$. Discuss the existence of the limit of *f*(*x*,*y*) as $(x,y)\to(0,0)$

f) A function f is twice differentiable on [*a*, *b*] and $f(a)=f(b)=0$. If $f(c)>0$ for some $c\in(a,b)$ Prove that there exists a point ξ in (a, b) such that $f''(\xi)<0$

3. Answer any TWO question **10x2=20**

a) i) Let f, g defined on $A \subseteq \Re$ to $\Re$ and e be a limit point of A. Suppose that f is bounded on a $n.b.d$ of c and $\lim_{x \to c} g(x) = 0$ then prove that $\lim_{x \to c} fg = 0$

ii) Function f and g are defined on $\Re$ by $f(x) = x + 1$ and

$$g(x) = \begin{cases} 2, & if \ x \neq 1 \\ 0, & if \ x = 1 \end{cases}$$

I) Find $\lim_{x \to 1} g(f(x))$ and compare with the value of $g\left(\lim_{x \to 1} f(x)\right)$

II) Find $\lim_{x \to 1} f\left(g(x)\right)$ and compare with the value of $f\left(\lim_{x \to 1} g(x)\right)$

b) i) Let $f : D \to R$ when $D \subset R$ and closed and bounded interval and $\left[a,b\right] \subset D$ and $f(x)$ is continuous on D. If $f(a), f(b) < 0$ then prove that $f(x)$ =0 has a solution in (a,b)

ii) Let $f(x, y) = \dfrac{x^3 + y^3}{x - y}$ when $x \neq y, \ and = 0, x = y$ .show that both partial derivatives of $f(x, y)$ at (0, 0) exists and not continuous at (0,0).

c) i) Let $f : \left[a,b\right] \to R$ be such that it is differentiable on [a,b] and function has equal value at both end points. Prove that there is a point $\left(c, f(c)\right)$ on the curve $y = f(x)$ at which tangent is parallel to the x-axis and $c \in (a,b)$

iii) If f is differentiable on [0, 1]. Show that the equation $f(1) - f(0) = \dfrac{(e-1) f'(x)}{ex}$ has at least one solution in (0,1)

[The End]