A soft-computing approach to multi-item fuzzy EOQ model incorporating discount

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Abstract: Contractive mapping genetic algorithm (CMGA) with logic structures has been developed and implemented for multi-item EOQ models with all unit discount (AUD), incremental quantity discount (IQD) and a combination of these discounts having fuzzy objective goal and resources. Here, AUD or/and IQD with two price breaks on purchasing price are allowed. For storage, warehouse capacity is limited but imprecise in nature. Selling price is mark-up of the purchasing cost. Profit function is formulated for the system incorporating impreciseness in it and is maximised using CMGA. The impreciseness in storage space and profit goal has been represented by fuzzy linear membership functions. For the present model, CMGA has been developed in real code representation and has been successfully implemented to obtain the optimum order quantities for the fuzzy inventory model with price-breaks in order to achieve the maximum profit. Numerical examples are provided to illustrate the model and sensitivity analyses with respect to different demand have been performed.

Keywords: EOQ model; contractive mapping genetic algorithm; CMGA; ALU; incremental quantity discount; IQD; fuzzy inventory.

Reference to this paper should be made as follows: Bera, U.K. and Maiti, A.K. (2012) 'A soft-computing approach to multi-item fuzzy EOQ model incorporating discount', *Int. J. Information and Decision Sciences*, Vol. 4, No. 4, pp.313–328.

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1 Introduction

The development of EOQ model by Harris (1915), a lot of research works on inventory control system has been reported in the literature during last few decades. Goyal (1985) developed the EOQ model in permissible delay in payments and Cheng (2002) extended the EOQ model under cash discount and payment delay after modification of Goyal's model. Nowadays, in the third world countries, with the introduction of open market system and advent of multi-nationals, there is a stiff competition amongst the companies to win over the maximum possible market. They allure the customers by giving quantity discounts in different forms. In general, there are two types of discount, AUD and IQD. Under AUD, the unit price of 'all units' decreases as the order size increases according to the price schedule offered by the supplier. In AUD, the discounts are offered to every unit purchased whereas in IQD, discounts are offered only to the additional units ordered beyond a specified quantity over which the discount is given. In the literature of discounted inventory problems, Benton (1985) considered an inventory system with quantity discount for MRP lot-sizing, Majewicz and Swemson (1978) for dynamic lot-sizing, Goyal (1977) and Monahan (1984) for integrated decision-making by both supplier and buyer. Wee and Yu (1997) published a research paper for deteriorating items and allowed temporary price discounts.

But, all these investigations are related to a single item or multi-items. Algorithm for the solution of the inventory problems with multiple price breaks is quite complex. If it is a case of multi-item with multi-price breaks including constraints, the usual algorithm becomes almost intractable. For this reason, till now, none has attempted to develop the algorithm for solving multi-item inventory models with multi-price breaks and resource constraints.

But, GA, one of the recently developed evolutionary techniques for optimisation, generates the chromosomes (i.e., potential solutions) randomly i.e., assumes several random values within the feasible regions for the decision variables and evaluates the objective function for each set of decision variables. Normally, decision-making problems are formulated as unconstrained/constrained non-linear optimisation problem, which are solved by traditional direct and gradient-based optimisation method. Among the limitations of these methods, one is that the traditional non-linear optimisation methods very often stuck to the local optimum. To overcome some of these limitations, a soft computing method, GA is very popular. GAs have been applied in different areas like neural networks (c.f. Pal et al., 1997), scheduling (Davis, 1991), numerical optimisation (Michalewicz, 1992), pattern recognition (Gelsema, 1995), etc. There are a very few papers where GAs have been applied in the field of inventory control system (Sarkar and Newton, 2002). Disney et al. (2000) solved classical inventory control systems using genetic algorithm optimisation, Mondal and Maiti (2002) developed a multi-item fuzzy EOQ model using GA, Gaafar (2006) studied genetic algorithm to dynamic lot-sizing with batch ordering, Altiparmak et al. (2006) solved multi-objective supply chain networks via genetic algorithm, Smith (2003) applied genetic algorithm on multiple inventory problem. Recently, Maiti and Maiti (2005) did work on damageable items in imperfect production process via GA. Also Maiti and Maiti (2006a, 2006b) developed two-warehouse problem in fuzzy environment, Maiti et al. (2006a, 2006b) and Maiti and Maiti (2008) applied genetic algorithm respectively in two-warehouse inventory model with discount and with random planning horizon.

In the quantity discount inventory problems; the randomly generated order quantities in contractive mapping genetic algorithm (CMGA) can be easily tagged with the corresponding unit price using a logic structure for price-breaks.

Again, in a business, normally a retailer fixes a goal for profit. But, in reality, it is difficult to have a fixed profit goal. During the course of business, initially, the goal for profit may be not maximum. In order to get maximum possible profit, he/she (retailer/seller) may be forced to settle with lowest price to allure the customers for large amount of purchase.

Initially, a warehouse with a finite capacity may be selected to store the multi-items for business. But, later, to take the advantage of discount, less transport cost or some other economical concessions, he/she is compelled to augment some more additional storage space in the interest of business. Hence, in these two cases, both profit goal and storage area are fuzzy in nature. Kar et al. (2001) developed multi-item inventory model with imprecise goal and constraints. Recently some fuzzy inventory problems (Roy and Maiti, 1997; Mandal et al., 1998) have been reported in the literature.

In this paper, EOQ models for several items are considered. Here, AUD, IQD and a combination of AUD, IQD in the form two price breaks are allowed. The items are stored in a warehouse whose capacity is imprecise in nature. The cherished goal for profit is also uncertain in non-stochastic sense. Impreciseness of the storage area and profit goal is represented by fuzzy linear membership functions. A CMGM is developed in real code to solve the fuzzy EOQ model. For this first time, this algorithm with some logic structures connecting the price break rules has been used to solve the above inventory models and the optimum order quantities are determined in order to have maximum possible profit within a range. The whole process along with the algorithm has been illustrated through numerical examples and sensitivity analysis has been performed with respect to different demand.

2 Assumptions and notations

For i^{th} item, ($i = 1, 2, \ldots\ldots\ldots\ldots\ldots, n$)

$q_i(t)$ inventory level at time t

T_i time length for each cycle

C_{3i} set-up cost per cycle

Q_i maximum inventory level

D_i demand rate

p_{ij} purchasing prices per unit within different price-break blocks

Q_i^* optimum value of Q_i

$PF_i(Q_i)$ profit function

b_{ij} the price break points, j = 1, 2 (only two price-break points considered)

a_i storage space required by one unit

W total available warehouse space.

The inventory models are developed under the following assumptions:

1 Shortages are not allowed.

2 The replenishment is infinite.

3 The lead time is zero.

For i^{th} item

4 The selling price is fixed on the basis of purchasing cost. Let r_i fraction on the purchasing cost be made as profit and then the selling price is fixed as:

$$P_{ij} = p_{ij}\left(1+r_i\right), 0 < r_i < 1 \tag{1}$$

5 Demand D_i's are constant.

6 The holding cost c_{0i}'s are x_i percentages of the unit purchasing cost.

7 The purchasing price is offered as:

- For AUD scheme

$$\text{Unit purchasing price, } p_i = \begin{cases} p_{i1} & 0 \le Q_i \le b_{i1} \\ p_{i2} & b_{i1} \le Q_i \le b_{i2} \\ p_{i3} & b_{i2} \le Q_i \end{cases} \tag{2a}$$

- For IQD system

$$\begin{aligned} \text{Total purchase price for } Q_i &= p_{i1}Q_i \ \ for \ \ 0£ \ Q_i < b_{i1} \\ &= p_{i1}b_{i1} + p_{i2}\left(Q_i - b_{i1}\right) for \ b_{i1} \ £ \ Q < b_{i2} \\ &= p_{ij}b_{ij} + p_{i2}b_{i2} + p_{i3}\left(Q_i - b_{i2}\right) for \ \ b_{i2}£Q_i \end{aligned}$$

For both systems, $p_{i1} > p_{i2} > p_{i3}$ (2b)

- For AUD and IQD system

  In this case, unit price is given by (2a) for one item and by (2b) for another item.

3 Mathematical formulation

3.1 Crisp model

With the above notations and assumptions, the differential equation for the i^{th} item describing the EOQ model is:

$$\frac{dq_i}{dt} = -D_i \tag{3}$$

with the boundary conditions:

$$\begin{aligned} q_i(t) &= Q_i \qquad at \quad t = 0 \\ &= 0 \qquad at \quad t = T_i \end{aligned} \tag{4}$$

Here, $Q_i = D_i T_i$, Holding cost $= \frac{1}{2} C_{1i} Q_i T_i$ and Net revenue = $r_i\, p_i\, Q_i$.

Hence, the average profit is:

$$PF(Q_1, Q_2, \ldots\ldots\ldots, Q_n) = \sum_{i=1}^{n} r_i p_i D_i - \left(\frac{1}{2} C_{1i} Q_i + C_{3i} \frac{D_i}{Q_i} + p_i D_i \right) \tag{5}$$

and

$$\sum_{i=1}^{n} a_i Q_i \le W \tag{6}$$

Now, the problem is to find Q_i, i = 1, 2, … , n, so that PF is maximum with the price break structure (2a) or (2b) or combination of (2a) and (2b) subject to the constraint (6).

3.2 Fuzzy inventory model

In general, a fuzzy non-linear programming problem with fuzzy objective goal and imprecise resources is represented as:

$$\text{Find } X \text{ to } Max\, \tilde{f}_0(X) \text{ such that}$$
$$f_k(X) \le \tilde{b}_k, \quad k = 1, 2 \ldots\ldots\ldots, m$$

where X is a n-dimensional vector, b_k's are the constraints' goals, m, n are, respectively, the number of constraints and decision variables and the symbol '~' represents the fuzziness of the parameter.

Let b_0 is the target to be achieved by the objective function. In fuzzy set theory, the fuzzy objective and constraints are defined by their membership functions, which may be linear or non-linear. Here, we assume μ_0, μk(k = 1, 2, ... , m) to be the continuous linear membership functions for fuzzy objective and constraints respectively. These are represented as:

$$m_0\left(f_0(x)\right)=\begin{cases}1 & for \quad f_0(x)\ge b_0\\ 1-\dfrac{b_0-f_0(x)}{P_0} & for \quad b_0-P_0\le f_0(x)\le b_0\\ 0 & for \quad f_0(x)\le b_0-P_0\end{cases}$$

$$m_k\left(f_k\left(x\right)\right)=\begin{cases}1 & for \quad f_k(x)\ge b_k\\ 1-\dfrac{f_k(x)-b_k}{P_k} & for \quad b_k\le f_k(x)\le b_k+P_k\\ 0 & for \quad f_k(x)\ge b_k+P_k\end{cases}$$

where P_0 and P_k's are the maximally acceptable violations of the aspiration levels of b_0 (objective goal) and b_k's (resources).

The pictorial representations of these functions are given in Figures 1 and 2.

Figure 1 Membership function for objective function

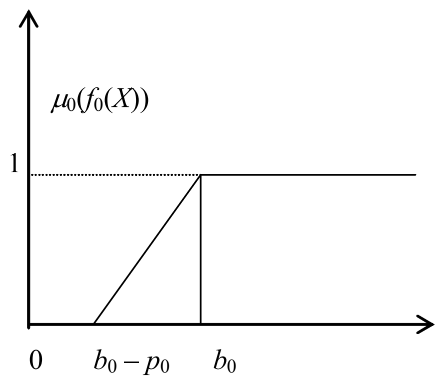
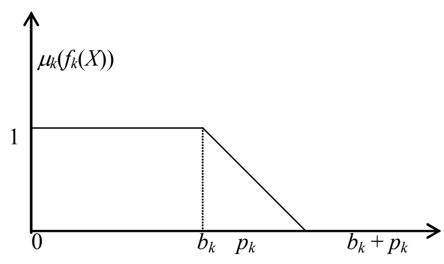


Figure 2 Membership function for resources



Using Bellman and Zadeh's (1970) max-min operator, the solution of the above problem can be obtained from:

$$\begin{aligned}
&Max\ \alpha \\
&subject\ to \\
&f_0(x) \leq m_0^{-1}(\alpha),\ f_k(x) \leq m_k^{-1}(\alpha), \\
&X \geq 0,\ \alpha \in [0,1],\ X = (x_1, x_2, \ldots\ldots, x_n)^T
\end{aligned}$$

In this case, $m_0^{-1}(\alpha) = b_0 - (1-\alpha)P_0$ *and* $m_k^{-1}(\alpha) = b_k + (1-\alpha)Pk$.

Now, it is a crisp deterministic problem and can be solved by CMGA.

Hence, fuzzy inventory model can be formulated as:

$$\begin{aligned}
&Max\ P\tilde{F}(Q_1, Q_2, \ldots\ldots, Q_n) \\
&subject\ to
\end{aligned}$$

$$\sum_1^n a_i Q_i \leq \tilde{W} \tag{7}$$

and its crisp version is:

$$Max\ \alpha \tag{8}$$

$$subject\ to$$

$$PF(Q_1, Q_2, \ldots\ldots, Q_n) \leq B - (1-\alpha)P_B \tag{9}$$

and

$$\sum_1^n a_i Q_i \leq W + (1-\alpha)P_W \tag{10}$$

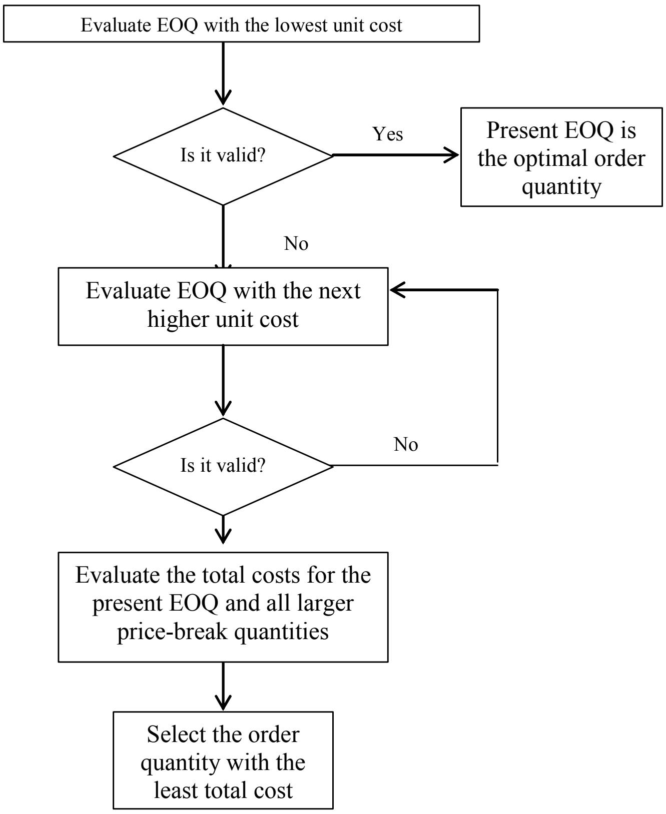
where B is profit goal, W the initially available storage space, P_B and P_W are the tolerance limits for profit goal and storage area respectively and PF is given by Monahan (1984). Here, profit goal may come down to $B - P_B$ and warehouse space may be augmented up to $W + P_W$ units

4 AUD logic structure for a single item

The conventional logic (cf. Tersine and Barman, 1991) to solve a single item inventory model with AUD is shown in Figure 3.

For the multi-items with resource constraints, the above logic structure is not applicable. For this reason, we propose CMGA with some logic structures for price-breaks to solve such multi-item inventory models with constraints.

Figure 3 Logic structure for AUD



5 Contractive mapping genetic algorithms

GA, initially introduced by Holland (1975) and later developed by Goldberg (1989), Davis (1991a) and Michalewicz (1992), is based upon the Darwin's theory – 'survival of the fittest'. It is a stochastic search process for optimisation. The algorithm mimics the selection process of nature and searches from a set of solutions. The main components of GA are:

1. chromosome representation
2. initial population
3. fitness evaluation
4. selection
5. crossover and mutation.

To solve a decision-making problem, the above genetic operations are performed sequentially and repeatedly.

In CMGA, movement from old population to new population takes place only when average fitness of new population is better than the old one. A CMGA in general form is given below:

```
Start
{
  L ← 0
    Initialise pc, pm //pc, pm are probabilities of crossover and mutation respectively //
      Initialise P(L) // P(L) is the population of potential solutions for iteration L //
         Evaluate P(L) // This function evaluates fitness of each member of P(L) //
         While (Not termination condition)
         Let this set be P/(L)
         Select solutions from P/(L) for crossover
         Made crossover on selected solutions to get population P1(L)
      Made mutation on selected solutions to get new population P(L + 1)
      Evaluate (P(L + 1))
    If average fitness of P(L + 1) > average fitness of P(L) then
    L(L + 1)
    }
End
```

6 Implementation of CMGA

Now, we shall develop a real coded CMGA for solving constrained maximisation problem. As mentioned in ($5), CMGA is developed as follows.

Parameters

The different parameters on which this GA is developed are population size (POPSIZE), probability of cross over (pc), probability of mutation (pm) and maximum number of generation (MAXGEN). In this case, $POPSIZE = 100$, $pc = 0.3$, $pm = 0.1$ and $MAXGEN = 2{,}000$.

Chromosome representation

A n-dimensional real vector $V = (Q_1, Q_2, ... Q_n)$, is used to represent a solution.

Initial population

Solution V_1, V_2, ……… , $V_{POPSIZE}$ are randomly generated by random number generator (using RND function) such that each chromosome satisfies the constraints (9) and (10).

Evaluation

To evaluate the value of the objective function $PF(V)$ due to the chromosome V = $(Q_1, Q_2, \ldots, Q_n)$, the corresponding unit purchase cost p_{ij} for Q_i is selected with the help of a logical statement using IF-THEN-ELSE (defined in $4). Following this, holding costs are also evaluated. The values of the objective function act as fitness values.

Selection

The selection scheme in a GA determines which solutions in the current population are to be selected for recombination. Many selection schemes have been proposed by researchers for various problems. For the present problem following Roulette wheel selection scheme is used.

Step 1 Find total fitness of the population $F = \sum_{K=1}^{POPSIZE} PF(V_K)$.

Step 2 Calculate the probability, P_K for the selection of each chromosome, V_k as = / F, (k = 1, 2, ... , *POPSIZE*).

Step 3 Calculate a cumulative probability Y_k for each chromosome as $Y_k = \sum_{j=1}^{k} P_j$.

Step 4 Select a single chromosome for the new population in the following ways:

- Step 4.1 Set a variable *COUNT* to 1.
- Step 4.2 Generate a random (float) number r within the range (0, 1).
- Step 4.3 If $r < Y_1$, then the first selected chromosomes is V_1 otherwise select the k^{th} chromosome $V_k(2 \le k \le POPSIZE)$ such that $Y_{k-1} < r \le Y_k$.
- Step 4.4 Compute $COUNT = COUNT + 1$.
- Step 4.5 If $COUNT < POPSIZE$, then go to Step 4.2.

Let, the selected solution set is denoted by $P(t)$.

Crossover

The exploration and exploitation of the solution space is made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates offspring solutions by recombining both parent solutions. After the selection of chromosomes for new population, the chromosomes for cross over are selected and then this operation is applied amongst them.

Selection for crossover

1 For each solution of $P(t)$, a random number r within [0, 1] is generated, If $r < p_e$, the chromosome (i.e., solution) is taken for cross over.

2 Crossover process: Crossover is done between the selected solutions. For each pair of coupled solutions V_1 and V_2, arithmetic cross over is performed as follows:

A random number c within [0, 1] is generated and parents V_1 and V_2 are replaced by their offspring's $V_1^{/}$ and $V_2^{/}$ where

$$V_1^{/} = c\ V1 + (1-c)V2$$

$$V_2^{/} = (1-c)V1 + c2$$

Mutation

Mutation operation is used to prevent the search process from converging to local optima rapidly. Unlike cross over, it is applied to a single chromosome V_k.

Selection for mutation

1 For each solution of $P(t)$, a random number r_1 within [0, 1] is generated. If $r_1 < P_m$ then the solution is taken for mutation.

2 Mutation process: To mutate a chromosome (i.e., solution) $V_k = (Q_{1k}, Q_{2k}, \ldots\ldots Q_{nk})$, select a random integer $r^{//}$ in the range $[1, n]$ and replace $Q_{r}^{//}\neg$ by randomly generated value within the boundaries of $r^{//\text{th}}$ component of V.

7 Logic structure for unit price and holding cost

7.1 AUD scheme

```
Q1 > = b12
   {
         calculate revenue, holding cost and then average profit with p = p13
   }
   else if (Q1 > = b11 and Q1 < b12)
   {
      calculate revenue, holding cost and the average profit with p = p12
      }
      else
      {
      calculate revenue, holding cost and then average profit with p = p11
}
```

The above structure is repeated for the second item, Q_2.

7.2 IQD scheme

For this system, the unit prices are calculated as:

$$p_{13} = p_1 * (1 - m_{12}) + p_1 * b_{12} * (m_{12} - m_{11}) / Q_{11} + p_1 * m_{11} * b_{11} / Q_1$$
$$p_{12} = p_1 * (1 - m_{11}) + p_1 * m_{11} * b_{11} / Q_1$$
$$p_{11} = p_{11}$$
$$p_{23} = p_2 * (1 - m_{22}) + p_2 * b_{22} * (m_{22} - m_{21}) / Q_2 + p_2 * m_{21} * b_{21} / Q_2$$
$$p_{22} = p_2 * (1 - m_{21}) + p_2 * m_{21} * b_{21} / Q_2$$
$$p_{21} = p_{21}$$

After the calculation of the unit prices, the same logic structure for AUD is used.

7.3 AUD and IQD scheme

Obviously, here AUD logic structure for one item, say Q_1, and IQD structure for other item, say Q_2 are used.

8 Numerical illustrations

For the illustration of the present model and proposed algorithm, inventory of two items (n = 2) with two price breaks are considered. The following values of inventory parameters are used to calculate the optimum order quantities for maximum profit.

Table 1 Input values for some inventory parameters

Item	*Set-up cost per set-up ($)*	*Demand*	*Holding cost × % of the unit cost*	*Mark up*	*Storage space for a unit (sq mt.)*
First	100	200	2%	1.3	2
Second	100	200	25%	1.2	3

Unit price structure for different discount system:

8.1 AUD system

Let,

For first item

$$p_1 = \begin{cases} \$10 & \text{for} \quad 0 < Q_1 < 500 \\ \$9.25 & \text{for} \quad 500 \le Q_1 < 750 \\ \$\ 8.75 & \text{for} \quad 750 \le Q_1 \end{cases}$$

For second item

$$p_2 = \begin{cases} \$10 & \text{for} \quad 0 < Q_2 < 50 \\ \$9 & \text{for} \quad 50 \le Q_2 < 100 \\ \$8 & \text{for} \quad 100 \le Q_2 \end{cases}$$

8.2 IQD system

For first item

$p_{11} = \$10 \quad for \quad (0 < Q_1 < 125)$

$p_{12} = \$(1 - m_{11})\, p_{11} \quad for \quad (125£\, Q_1 < 200)$

$p_{13} = \$(1 - m_{12})\, p_{11} \quad for \quad (200£\, Q_1)$

where $m_{11} = 0.075,\ m_{12} = 0.15$

For second item

$p_{21} = \$15 \quad for \quad (0 < Q_2 < 12)$

$p_{22} = \$(1 - m_{21})\, p_{21} \quad for \quad (15 \le Q_1 < 25)$

$p_{23} = \$(1 - m_{22})\, p_{21} \quad for \quad (25 \le Q_2)$

where $m_{21} = 0.088,\ m_{22} 0.14$

8.3 AUD and IQD system

Here, AUD scheme is allowed for the first item and IQD scheme for the second item.

For first item

$$p_1 = \begin{cases} \$10 & for \quad 0 < Q_1 < 125 \\ \$9.25 & for \quad 125 \le Q_1 < 200 \\ \$8.75 & for \quad 200 \le Q_1 \end{cases}$$

For second item

$p_{21} = \$10 \quad for \quad (0 < Q_2 < 15)$

$p_{22} = \$(1 - m_{21})\, p_{21} \quad for \quad (15 \le Q_1 < 25)$

$p_{23} = \$(1 - m_{22})\, p_{21} \quad for \quad (25 \le Q_2)$

where $m_{21} = 0.088, m_{22} = 0.14$

Imprecise storage space and profit are:

Table 2 Storage space and Profit for different discount system

Discount system	*Profit goal (\$ B)*	*Tolerance limit for profit goal (\$ P_B)*	*Initial storage area (W Sq. mt.)*	*Tolerance limit for storage area (P_W Sq. mt.)*
AUD	770	100	990	100
IQD	900	100	600	100
AUD and IQD	669	100	990	100

8.4 Optimal results

With the price structures in § 8.1, 8.2 and 8.3, the inventory models given by Mandal et al. (1998), Holland (1975) and Goldberg (1989) are solved using CMGA outlined in § 6 and logical structures in § 7.1, 7.2 and 7.3 and the corresponding optimum order quantities and maximum profits are presented in Table 3.

Table 3 Optimal Results for different discounted systems

Discount system	Q_1^*	Q_2^*	*PF*(in \$)*	α
AUD	447.44	49.99	768.05	0.980521
IQD	251.95	38.77	850.76	0.507583
AUD and IQD	442.98	55.18	668.9	0.999993

8.5 Sensitivity analysis and discussion

For the above mentioned model, sensitivity analysis with respect to demand is performed holding all other parameters at their previous values. Increasing D_i successively, the optimum order quantities and the average profit are calculated and presented in Table 4. From Table 4, it evident that the model is more sensitive with the change of demand. It also observed that with the increase of D_i, profit increases in AUD system. Same scenario have seen in IQD system also.

Table 4 Optimum profit when demand varies in AUD system

Demand (D_1, D_2)	Q_1^*	Q_2^*	*PF*(in \$)*
(205, 205)	449.25	52.99	770.45
(210, 210)	451.03	54.23	776.28
(215, 215)	452.56	55.92	783.61
(220, 220)	452.94	56.01	789.41
(225, 225)	453.12	56.78	794.64

9 Practical implication

With the introduction of open market system and advent of multi-nationals in the developing countries like India, Pakistan, Nepal, etc., inventory situation of these countries changes a lot. In these countries, multi-nationals bring their products and store at a place where financial benefits in terms of rent, taxes, etc., are less and form their store houses they do their business via different of dealers, sub-dealers and retailers. To capture the retail market, these multi-nationals offer different types quantity discount on their product to attract the retailers. As, a result, retailers try to purchase big lot size due price discount and to boost his/her customers by increasing stock. The present model is applicable in the real life situation.

10 Conclusions

This paper presents a realistic multi-item fuzzy inventory model with multiple price breaks and a resource constraint has been solved by CMGA. Here, randomly generated potential solutions of the problem have been logically connected with price breaks and it has been used to find the maximum profit after satisfying the available space constraint. The formulation and algorithm can be easily used for the inventory problems with large number of items (more than two) and several price breaks. The present methodology can be easily applied to the inventory problems with several constraints like limitation on invest of capital, space constraints, etc. Moreover, the present CMGA can be applied to other types of inventory models with variable demand, fixed time horizon, etc., along with discount formulated in probabilistic or fuzzy probabilistic environments.

Acknowledgements

The comments referees lead to improvement of the paper and greatly appreciated. The author would like to thank for their valuable suggestions.

References

Altiparmak, F., Gen, M., Lin, L. and Paksoy, T. (2006) 'A genetic algorithm approach for multi-objective optimization of supply chain networks', *Computers and Industrial Engineering*, Vol. 51, No. 2, pp.196–215.

Bellman, R.E. and Zadeh, L.A. (1970) 'Decision-making in a fuzzy environment', *Management Science*, Vol. 17, No. 4, pp.B141–B164.

Benton, E.C. (1985) 'Multiple price breaks and alternative purchase lot sizing procedures in material requirements planning system', *International Journal of Production Research*, Vol. 23, No. 5, pp.1025–1047.

Cheng, T.C. (2002) 'Extended EOQ model under cash discount and payment delay', *Information and Management Science*, Vol. 13, No. 3, pp.57–69.

Davis, L. (Ed.) (1991) *Hand Book of Genetic Algorithm*, Van Nostrand-Reinhold, New York.

Disney, S.M., Naim, M.M. and Towill, P.R. (2000) 'Genetic algorithm optimization of a classical inventory control systems', *International Journal of Production Economics*, Vol. 68, pp.259–278.

Gaafar, L. (2006) 'Applying genetic algorithm to dynamic lot-sizing with batch ordering', *Computers and Industrial Engineering*, Vol. 51, No. 3, pp.433–444.

Gelsema, E. (Ed.) (1995) 'Special issue of genetic algorithms', *Pattern Recognition Letters*, Vol. 16, No. 8, p.8.

Goldberg, D.E. (1989) *Genetic Algorithm in Search Optimization and Machine Learning*, Addison Wesley, Massachusetts.

Goyal, S.K. (1977) 'An integrated inventory model for a single supplier single customers problem', *International Journal of Production Research*, Vol. 15, No. 1, pp.107–111.

Goyal, S.K. (1985) 'Economic order quantity under conditions of permissible delay in payments', *Journal of Operations Research Society*, Vol. 36, No. 4, pp.335–338.

Harris, F. (1915) *Operations and Cost (Factory Management Series)*, A.W. Shaw Co., Chicago.

Holland, J.H. (1975) *A Adaptive in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor.

Kar, S., Roy, T.K. and Maiti, M. (2001) 'Multi-item inventory model with probabilistic price dependent demand and imprecise goal and constraints', *Yugolav Journal of Operations Research*, Vol. 11, No. 1, pp.93–103.

Maiti, A.K. and Maiti, M. (2008) 'Discounted multi-item inventory model via genetic algorithm in constraints bounded domains', *International Journal of Computer Mathematics*, Vol. 85, No. 9, pp.1341–1353.

Maiti, A.K., Bhunia, A.K. and Maiti, M. (2006a) 'An application of real-coded genetic algorithm (RCGA) for mixed integer non-linear programming in two-storage multi-item inventory model with discount policy', *Applied Mathematics and Computations*, Vol. 183, No. 2, pp.903–915.

Maiti, A.K., Maiti, M.K. and Maiti, M. (2006b) 'Two-storage inventory model with random planning horizon', *Applied Mathematics and Computations*, Vol. 183, No. 2, pp.1084–1097.

Maiti, M.K. and Maiti, M. (2005) 'Production policy for damageable items with variable cost function in an imperfect production process via genetic algorithm', *Mathematical and Computer Modeling*, Vol. 42, Nos. 9–10, pp.977–999.

Maiti, M.K. and Maiti, M. (2006a) 'Fuzzy inventory model with two-warehouses under possibility constraints', *Fuzzy Sets and System*, Vol. 157, No. 1, pp.52–73.

Maiti, M.K. and Maiti, M. (2006b) 'Two-storage inventory model with lot size dependent fuzzy lead-time under possibility constraints via genetic algorithm', *European Journal of Operations Research*, Vol. 179, No. 2, pp.352–371, available at http://www.sciencedirect.com.

Majewicz, D. and Swanson, I.A. (1978) *Inventory Ordering and Quantity Discounts with Time Varying Demand, a Programming Application, Production and Inventory Management*, Vol. 14, No. 1, pp.91–102.

Mandal, M., Roy, T.K. and Maiti, M. (1998) 'A fuzzy inventory model of deteriorating items stock dependent demand under limited storage space', *OPSEARCH*, Vol. 35, No. 4, pp.323–337.

Michalewicz, Z. (1992) *Genetic Algorithm + Data Structure = Evolution Programme*, Springer, Berlin.

Monahan, J.P. (1984) 'A quantity discount-pricing model to increase vendor profits', *Management Science*, Vol. 30, No. 6, pp.720–726.

Mondal, S. and Maiti, M. (2002) 'Multi-item fuzzy EOQ models using genetic algorithm', *Computers and Industrial Engineering*, Vol. 44, No. 1, pp.105–117.

Pal, S.K., De, S. and Ghosh, A. (1997) 'Designing Hopfield type networks using genetic algorithms and its caparisons with simulated annealing', *International Journal of Patterns Recognition and Artificial Intelligence*, Vol. 11, No. 3, pp.447–461.

Roy, T.K. and Maiti, M. (1997) 'A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity', *European Journal of Operational Research*, Vol. 99, No. 2, pp.425–432.

Sarkar, R. and Newton, C. (2002) 'A genetic algorithm for solving economic lot size scheduling problem', *Computers and Industrial Engineering*, Vol. 42, Nos. 2–4, pp.189–198.

Smith, J.C. (2003) 'A genetic algorithm approach to solving a multiple inventory problem', *International Journal of Engineering Theory*, Vol. 10, No. 1, pp.45–54.

Tersine, R.J. and Barman, S. (1991) 'Economic inventory/transport lot sizing with quantity and freight rate discounts', *Decision Science*, Vol. 22, No. 5, pp.1171–1179.

Wee, H.M. and Yu, J. (1997) 'A deteriorating inventory model with a temporary price discount', *International Journal of Production Economics*, Vol. 53, No. 1, pp.81–90.