An EOQ model of an item with imprecise seasonal time via genetic algorithm

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Abstract: Here, an inventory model of an item during its seasonal time is considered where duration of the season of the item is imprecise in nature in non-stochastic sense, i.e., fuzzy in nature. Demand of the item is price dependent and unit cost of the item is time dependent. Unit cost is a decreasing function at the beginning of the season and an increasing function at the end of the season and is constant during the remaining part of the season. The model is formulated to maximise the average proceeds out of the system from the planning horizon which is fuzzy in nature. As optimisation of fuzzy objective is not well defined, optimistic/pessimistic return of the objective function (using possibility/necessity measure of fuzzy event) is optimised. A fuzzy simulation process is proposed to evaluate this optimistic/pessimistic return. A genetic algorithm (GA) is developed based on entropy theory where region of search space gradually decreases to a small neighbourhood of the optima. This is named as region reducing genetic algorithm (RRGA) and is used to solve the model. The model is illustrated with some numerical examples and some sensitivity analyses have been done. In a particular case when planning horizon is crisp the model is solved via RRGA.

Keywords: fuzzy planning horizon; seasonal product; region reducing genetic algorithm; RRGA; fuzzy simulation; EOQ.

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1 Introduction

For items like paddy, wheat, pulses, potato, onion, etc., it is normally observed that price of the item decreases with time at the beginning of the production season due to availability in the market and reaches to a minimum value. Price of the item remains constant at this minimum value during the major part of the season due sufficient availability of the item in the market and towards the end of the season due to scarcity, cost again increases gradually and reaches its off season value. This price remains stable during the remaining part of the year. A considerable number of research work has been done for seasonal products by several researchers (Zhou et al., 2004; Chen and Chang, 2007; Panda et al., 2008; Banerjee and Sharma, 2010a, 2010b). In most of these research works it is assumed that price of the item decreases with time or demand increases with time. But the above mentioned real life phenomenon of a seasonal product is overlooked by the researchers. The another shortcoming of these research work is the assumption of the duration of the season of such products as finite crisp value. Although the duration of the season for an item is finite it varies from year to year due to environmental changes.

So it is worthwhile to assume this duration as a fuzzy parameter. The aim of this research work is twofold:

- Firstly to model price of a seasonal product as a function $f_1(t)$ of time which decreases monotonically for a duration H_1 at the beginning of the season and reaches a minimum value $f_1(H_1)$. The price remains at this value $f_1(H_1)$ during a period H_2. Then it again follows an increasing function $f_2(t)$ and after a period H_3 it reaches the off season value, i.e., $f_1(0) = f_2\ (H_1 + H_2 + H_3)$.
- Secondly to model the season length $(H_1 + H_2 + H_3)$ as an imprecise parameter.

Here, inventory model for a seasonal product is developed whose demand depends upon the unit cost of the product. Unit cost of the product is time dependent. During the beginning of the period as availability of the item gradually increases, unit cost decreases monotonically with time and reaches a constant value when availability of the item becomes stable. Unit cost remains constant until the items availability again decreases towards the end of the season. Then as availability decreases, unit cost gradually increases and reaches its value as it was at the beginning of the season and then the season ends. Here linear increasing and decreasing rate of unit cost function is considered. It is assumed that time horizon of the season is fuzzy in nature. In fact three parts in which unit cost function can be divided are considered as fuzzy number. The model is formulated to maximise optimistic/pessimistic return of the profit from the system for the season (Maiti and Maiti, 2006; Maiti, 2008, 2011). Optimistic/pessimistic return of the profit function is obtained using possibility/necessity measure on fuzzy event and a simulation approach is proposed to find this optimistic/pessimistic return of the profit (Liu and Iwamura, 1998a, 1998b). A genetic algorithm (GA) is developed based on entropy theory where region of search space is gradually decreases to a small neighbourhood of the optima (Bessaou and Siarry, 2001). This is named as region reducing genetic algorithm (RRGA) and is used to solve the model when planning horizon is crisp. When this algorithm is used to solve the fuzzy model using fuzzy simulation process to evaluate the optimistic/pessimistic return of the profit, the algorithm is named as fuzzy simulation-based region reducing genetic algorithm (FSRRGA). The model is illustrated with some numerical examples and some sensitivity analyses have been presented.

2 Literature review

Items considered in inventory control problems are broadly classified into – seasonal and normal products. The products whose demands are normally stable throughout the year are treated as normal product. Demand of these items are mainly constant (Ruiz-Torres and Santiago, 2007; Mahata and Goswami, 2010), stock-dependent (Jain et al., 2008; Guchhait et al., 2010) or price dependent (Maiti and Maiti, 2006; Pal et al., 2009). The products which are mostly found at a particular period of time in the year are termed as seasonal product. Fruits like mango, orange, etc., vegetables like cabbage, cauliflower, etc., sea fish like hilsa, winter garments, X-mass cake are some examples of seasonal items.

These items are found in the market for a particular period in each year and demand of these items increases at the beginning of the season, in the mid of the season it becomes steady and towards the end of the season it decreases and becomes asymptotic. A considerable number of research paper has been published in this direction. Chen and Chang (2007) developed a seasonal demand inventory model with variable lead time and resource constraints. Panda et al. (2008) developed an inventory model for perishable seasonal products with ramp-type time dependent demand. Banerjee and Sharma (2010b) studied optimal procurement and pricing policies for inventory models with price and time dependent seasonal demand. They (Banerjee and Sharma, 2010a) developed another inventory model for seasonal demand with option to change the market. Tripathi (2012) developed an EOQ model for deteriorating items with linear time dependent demand rate under permissible delay in payments. So demand of seasonal product is normally time dependent.

On the other hand there are some items whose stable demand exists throughout the year, but at the time of their grown due to sufficient availability price decreases and so demand increases (e.g., rice, wheat, pulses, etc.). The time of grown of these items are treated as season of these items. For these types of products it is normally observed that due to production of the items in the field by the farmers price of these items decrease with time at the beginning of the season and reach to a minimum value. Price of the item remains constant at this minimum value during the major part of the season and towards the end of the season it again increases gradually and reaches its off season value. As unit price influences the demand, demand increases with time at the beginning of the season and reaches to its maximum value. Demand of the item remains constant at this maximum value during the major part of the season and towards the end of the season it again decreases gradually and reaches to its off season level. Inventory practitioners overlooked the natural phenomenon of these types of items. In this research work an attempt has been made to developed an inventory model of this type of items.

After the introduction of fuzzy set theory by Zadeh (1965), it has been applied to different fields of optimisation including inventory control problems. In the last two decades extensive research work has been done on inventory control problems in fuzzy environment (Lee et al., 1991; Lam and Wong, 1996; Roy and Maiti, 2000; Mandal and Maiti, 2002; Kao and Hsu, 2002; Bera et al., 2012). These problems considered different inventory parameters as fuzzy numbers which render fuzzy objective function. As optimisation in fuzzy environment is not well defined some of these researcher transform the fuzzy parameters as equivalent crisp number or crisp interval and then the objective function is transformed to an equivalent crisp number/interval (Maiti and Maiti, 2007; Bera et al., 2012). Some of the researchers (Mandal and Maiti, 2002) set the fuzzy objective as fuzzy goal whose membership function as a linear/non-linear fuzzy number and try to optimise this membership function using Bellman-Zadeh's principle (Bellman and Zadeh, 1970). Maiti and Maiti (2006) propose a technique where instead of objective function pessimistic return of the fuzzy objective is optimised. They use necessity measure on fuzzy event to determine this pessimistic return and proposes fuzzy simulation process to find this return function. Recently Maiti (2008, 2011) proposes a technique where possibility/necessity measure of objective function (fuzzy profit) on fuzzy goal is optimised to find optimal decision. In this research work, approach followed by Maiti and Maiti (2006) is used to find optimal decision for the decision maker (DM) for fuzzy inventory model.

3 Possibility/necessity in fuzzy environment

Let $\Re$ represents the set of real numbers and $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively. Then taking degree of uncertainty as the semantics of fuzzy number, according to Liu and Iwamura (1998a, 1998b):

$$\text{Pos}\left(\tilde{A} \star \tilde{B}\right) = \sup\left\{\min\left(\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(y)\right),\ x,\ y \in \Re,\ x \star y\right\} \tag{1}$$

where the abbreviation Pos represent possibility and $\star$ is any one of the relations >, <, =, ≤, ≥. Analogously if $\tilde{B}$ is a crisp number, say b, then

$$\text{Pos}\left(\tilde{A} \star b\right) = \sup\left\{\mu_{\tilde{A}}(x),\ x \in \Re,\ x \star b\right\} \tag{2}$$

On the other hand necessity measure of an event $\tilde{A} \star \tilde{B}$ is a dual of possibility measure. The grade of necessity of an event is the grade of impossibility of the opposite event and is defined as:

$$\text{Nes}\left(\tilde{A} \star \tilde{B}\right) = 1 - \text{Pos}\left(\overline{\tilde{A} \star \tilde{B}}\right) \tag{3}$$

where the abbreviation Nes represents necessity measure and $\overline{\tilde{A} \star \tilde{B}}$ represents complement of the event $\tilde{A} \star \tilde{B}$.

If $\tilde{A}, \tilde{B} \in \Re$ and $\tilde{C} = f(\tilde{A}, \tilde{B})$ where $f : \Re \times \Re \rightarrow \Re$ be a binary operation then membership function μC˜ of C˜ can be obtained using *fuzzy extension principle* (Zadeh, 1965, 1973) as

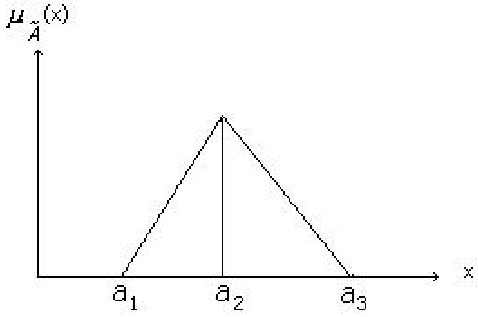
$$\mu_{\tilde{C}}(z) = \sup\left\{\min\left(\mu_{\tilde{A}}(x),\ \mu_{\tilde{B}}(y)\right),\ x,\ y \in \Re,\ \text{and}\ z = f(x, y), \forall z \in \Re\right\} \tag{4}$$

3.1 Triangular fuzzy number

A triangular fuzzy number (TFN) $\tilde{A}$ is specified by the triplet (a_1, a_2, a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x) : X \rightarrow [0,\ 1]$ as follows (cf., Figure 1):

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if}\ \ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if}\ \ a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 Triangular fuzzy number



4 Optimisation of fuzzy objective using possibility/necessity measure

A general single-objective unconstrained mathematical programming problem is of the following form:

$$\begin{array}{ll} \max & f(x,\ \xi) \\ \text{subject to} & x \in X \end{array} \tag{5}$$

where x is a decision vector, ξ is a vector of crisp parameters, $f(x, \xi)$ is the return function, X is the search space. In the above problem when ξ is a fuzzy vector $\tilde{\xi}$, then return function $f(x,\tilde{\xi})$ becomes imprecise in nature. In that case the statement maximise $f(x,\tilde{\xi})$ is not well defined. In that situation one can maximise the optimistic (pessimistic) return, z, corresponding to the objective function using possibility (necessity) measure of the fuzzy event $\{\tilde{\xi} \mid f(x,\tilde{\xi}) \geq z\}$ as suggested by Liu and Iwamura (1998a, 1998b), Maiti and Maiti (2006). So when ξ is a fuzzy vector one can convert the above problem (5) to the following equivalent possibility/necessity constrained programming problem (analogous to the chance constrained programming problem).

$$\begin{array}{ll} \max & z \\ \text{subject to} & \text{pos}\big/\text{nes}\left\{\tilde{\xi}\,\middle|\,f\left(x,\tilde{\xi}\right) \geq z\right\} \geq \beta \\ & x \in X \end{array} \tag{6}$$

where β is the predetermined confidence level for fuzzy objective, pos{.} (nes{.}) denotes the possibility (necessity) of the event in {.}. Here the objective value z should be the maximum that the objective function $f(x,\tilde{\xi})$ achieves with at least possibility (necessity) β, in optimistic (pessimistic) sense.

4.1 Fuzzy simulation

The basic technique to deal problem (6) is to convert the possibility/necessity constraint to its deterministic equivalent. However, the procedure is usually very hard and successful in some particular cases (Maiti and Maiti, 2006). Liu and Iwamura (1998a, 1998b) proposed fuzzy simulation process to determine optimum value of z for the problem (6) under possibility measure of the event $\{\tilde{\xi} \mid f(x,\tilde{\xi}) \geq z\}$. Following Liu and Iwamura (1998b) two algorithms are developed to determine z in (6) and are presented below.

Algorithm 1 Algorithm to determine z, for problem (6) under possibility measure of the event $\{\tilde{\xi} \mid f(x,\tilde{\xi}) \geq z\}$

1	Set $z = -\infty$.
2	Generate ξ_0 uniformly from the β cut set of fuzzy vector $\tilde{\xi}$.
3	If $z < f(x, \xi_0)$ then set $z = f(x, \xi_0)$.
4	Repeat Steps 2 and 3, N times, where N is a sufficiently large positive integer.
5	Return z.
6	End algorithm.

We know that $nes\{\tilde{\xi} \mid f(x,\tilde{\xi}) \geq z\} \geq \beta \Rightarrow pos\{\tilde{\xi} \mid f(x,\tilde{\xi}) < z\} < 1-\beta$. Now roughly find a point ξ_0 from fuzzy vector ξ, which approximately minimises f. Let this value be z_0 and ε be a positive number. Set $z = z_0 - \varepsilon$ and if $pos\{\tilde{\xi} \mid f(x,\tilde{\xi}) < z\} < 1-\beta$ then increase z with ε. Again check $pos\{\tilde{\xi} \mid f(x,\tilde{\xi}) < z\} < 1-\beta$ and it continues until $pos\{\tilde{\xi} \mid f(x,\tilde{\xi}) < z\} \geq 1-\beta$. At this stage decrease value of ε and again try to improve z.

When ε becomes sufficiently small then we stop and final value of z is taken as value of z. Using this criterion, Algorithm 2 is developed.

Algorithm 2 Algorithm to determine z, for problem (6) under necessity measure of the event $\{\tilde{\xi} \mid f(x,\tilde{\xi}) \geq z\}$

1	Set $z = z_0 - \varepsilon$, F $= z_0 - \varepsilon$, $F_0 = z_0 - \varepsilon$.
2	Generate ε_0 uniformly from the $1 - \beta$ cut set of fuzzy vector ξ.
3	If $f(x,\tilde{\xi}_0) < z$.
4	then go to Step 10.
5	End If
6	Repeat Step 2 to Step 5 N times.
7	Set $F = z$.
8	Set $z = z + \varepsilon$.
9	Go to Step 2.
10	If($z = F$)//In this case optimum value of $z < z_0 - \varepsilon$
11	Set $z = z_0 - \varepsilon$, $F = F - \varepsilon$, $F_0 = F_0 - \varepsilon$.
12	Go to Step 2
13	End If
14	If ($\varepsilon < tol$)
15	go to Step 20
16	End If
17	$\varepsilon = \varepsilon/N$
18	$z = F + \varepsilon$
19	Go to Step 2.
20	Output F.

5 Fuzzy simulation-based region reducing genetic algorithm

Gas are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation, etc.) and have been developed by Holland, his colleagues and students at the University of Michigan (Goldberg, 1989). Because of its generality and other advantages over conventional optimisation methods it has been successfully applied to different decision making problems (Zegordi et al., 2010; Simon et al., 2011; Das et al., 2012).

Generally a GA starts with a single population (Goldberg, 1989; Michalewicz, 1992), randomly generated in the search space. One of the difficulties of Gas is that they often converge too quickly and tend to make quickly, uniform the population of the chromosomes. Consequently they are easily trapped into local optima of the objective function. This difficulty is mainly due to the premature loss of diversity of the population during the search. To overcome this difficulties, Bessaou and Siarry (2001) propose a GA where initially more than one population of solutions are generated. Genetic operations are done on every population a finite number of times to find a promising zone of optimum solution. Finally a population of solutions is generated in this zone and genetic operations are performed on this population a finite number of times to get a final solution. Again the convergence towards the global optima of a GA, operating with a constant probability of crossover p_c, is ensured if the probability of mutation $p_m(k)$ follows a given decreasing law, in function of the generation number k (Davis and Principe, 1991). Following Bessaou and Siarry (2001) a GA is developed using the entropy generated from information theory, where promising zone is gradually reduces to a small neighbourhood of the optimal solution. In the algorithm any possibility constraint on objective function is checked via fuzzy simulation technique. This algorithm is named as FSRRGA and is used to solve our models. The algorithm is given below:

Algorithm 3 FSRRGA algorithm

1 Initialise probability of crossover p_c and probability of mutation p_m.

2 Set iteration counter T = 0.

3 Generate M sub-populations of solutions, each of order N (i.e., each sub-population contains N solutions), from search space of optimisation problem under consideration, such that the diversity among the solutions of each population is maintained. Diversity is maintained using the entropy originating from information theory [cf., § 2.1-(b)]. Solutions for each of the population are generated randomly from the search space in such a way that the constraints of the problem are satisfied. Possibilistic constraints are checked using the algorithms of Section 4.1. Let $P_1, P_2, \ldots P_M$ be these populations.

4 Evaluate fitness of each solution of every populations.

5 Repeat

   A Do for each sub-populations P_i.

      a Select N solutions from P_i^1, for mating pool using Roulette-wheel selection process (Michalewicz, 1992) (These N solutions may not be distinct.

      Solution with higher fitness value may be selected more than once). Let this set be P1.

      B Select solutions from P1, for crossover and mutation depending on p_c and p_m respectively.

      C Make crossover on selected solutions for crossover.

D Make mutation on selected solutions for mutation.

E Evaluate fitness of the child solutions.

F Replace the parent solutions with the child solutions.

G Replace P_i with P_i^1.

B End Do

C Reduce probability of mutation p_m.

6 Until number of generations < Maxgen1, where Maxgen1 represents the maximum number of generations to be made on initial populations.

7 Select optimum solutions from each sub-populations and S^* be the best among these solutions.

8 Select a neighbourhood V(T) of S^*

9 Repeat

a Generate a population of solutions of size N in V(T). Let it be P.

b Evaluate fitness of each solutions.

c Initialise probability of mutation p_m.

d Repeat

1 Select N solutions from P for mating pool using Roulette-wheel selection process. Let this set be P^1.

2 Select solutions from P^1 for crossover and mutation depending on p_c and p_m respectively.

3 Make crossover on selected solutions for crossover.

4 Make mutation on selected solutions for mutation.

5 Evaluate fitness of the child solutions.

6 Replace the parent solutions with the child solutions.

7 Replace P with P^1.

8 Reduce probability of mutation p_m.

e Until number of generations < Maxgen2, where Maxgen2 represents the maximum number of generations to be made on this population.

f Update S^* by the best solution found.

g Reduce the neighbourhood V(T).

h Increment T by 1.

10 Until T < Maxgen3, where Maxgen3 represents the maximum number of times for which the search space to be reduced.

5.1 FSRRGA procedures for the proposed model

a *Representation:* A '*K*-dimensional real vector' $X_{li} = (x_{li1}, x_{li2}, ... x_{liK})$ is used to represent i^{th} solution in l^{th} population, where $x_{li1}, x_{li2}, ... x_{liK}$ represent different decision variables of the problem such that constraints of the problem are satisfied.

b *Initialisation:* At this step *M* sub-populations, each of size *N* are randomly generated in the search space in such a way that diversity among the solutions of each of the populations is maintained and the constraints of the problem are satisfied. Possibility constraints are checked using the algorithms of Section 4.1. Let $X_{l1}, X_{l2}, ... X_{lN}$, are the solutions of l^{th} population P_l, $l = 1, 2, ...M$. Diversity can be maintained using the entropy originating from information theory. Entropy of j^{th} variable for the l^{th} population P_l can be obtained by the formula:

$$E_j\left(P_l\right) = \sum_{i=1}^{N} \sum_{k=i+1}^{N} -p_{ik} log\left(p_{ik}\right),$$

where p_{ik} represents the probability that the value of j^{th} variable of i^{th} solution (x_{lij}) is different from the one of the j^{th} variable of the kth solution (x_{lkj}) and is determined by the formula:

$$p_{ik} = 1 - \frac{\left|X_{lij} - X_{lkj}\right|}{U_j - L_j},$$

where $[L_j, U_j]$ is the variation domain of the j^{th} variable. The average entropy $E(P_l)$ of the l^{th} subpopulation P_l is taken as the average of the entropies of the different variables for the population, i.e.,

$$E\left(P_l\right) = \frac{1}{K} \sum_{j=1}^{K} E_j\left(P_l\right).$$

It is clear that if P_l is made-up of same solutions, then $E(P_l)$ vanishes and more varied the solutions, higher the value of $E(P_l)$ and the better is its quality. So to maintain diversity, every time a new solution is randomly generated for P_l from the search space, the entropy between this one and the previously generated individuals for P_l is calculated. If this value is higher than a fixed threshold E_0, fixed from the beginning, the current chromosome is accepted. This process is repeated until N solutions are generated. Following the same procedure all the sub-populations P_l, $l = 1, 2, ..M$ are generated. This solution sets are taken as initial sub-populations.

c *Fitness value:* Value of the objective function due to the solution X_{ij} (j^{th} solution in i^{th} population), is taken as fitness of X_{ij}. Let it be $f(X_{ij})$. Objective function is calculated using Algorithm 2 of Section 4.1.

d *Selection process for mating pool:* The following steps are followed for this purpose:

1 For each population Pi, find total fitness of the population $F_i = \sum_{j=1}^{N} f(X_{ij})$.

2 Calculate the probability of selection pr_{ij} of each solution X_{ij} by the formula $pr_{ij} = f(X_{ij})/F_i$.

3 Calculate the cumulative probability qr_{ij} for each solution X_{ij} by the formula

$$qr_{ij} = \sum_{k=0}^{j} pr_{ik}.$$

4 Generate a random number 'r' from the range [0, 1].

5 If $r < qr_{i1}$ then select X_{i1} otherwise select $X_{ij} (2 \le j \le N)$ where $qr_{ij-1} \le r < qr_{ij}$.

6 Repeat Step 4 and 5 N times to select N solutions for mating pool. Clearly one solution may be selected more than once.

7 Selected solution set is denoted by P_i^1 in the proposed FSRRGA algorithm.

e *Crossover:*

1 Selection for crossover: For each solution of P_i^1 generate a random number r from the range [0, 1]. If $r < p_c$ then the solution is taken for crossover, where p_c is the probability of crossover.

2 Crossover process: Crossover taken place on the selected solutions. For each pair of coupled solutions Y_1, Y_2 a random number c is generated from the range [0, 1] and Y_1, Y_2 are replaced by their offspring's Y_{11} and Y_{21} respectively where $Y_{11} = cY_1 + (1 - c)Y_2$, $Y_{21} = cY_2 + (1 - c)Y_1$.

f *Mutation:*

1 Selection for mutation: For each solution of P_i^1 generate a random number r from the range [0, 1]. If r< pm then the solution is taken for mutation, where pm is the probability of mutation.

2 Mutation process: To mutate a solution $X_{li} = (x_{li1}, x_{li2}, ... x_{liK})$ select a random integer r in the range [1, *K*]. Then replace x_{ijr} by randomly generated value within the boundary of r^{th} component of X_{ij}.

g *Reduction process of* p_m*:* Let $p_m(0)$ is the initial value of p_m. $p_m(T)$ is calculated by the formula $p_m(T) = p_m(0)\exp(-T/\alpha)$, where α is calculated so that the final value of p_m is small enough (10^{-3} in our case). So $\alpha = \text{Maxgen1}/\log\left[\frac{p_m(0)}{10^{-3}}\right]$ for initial populations P_i, $i = 1, 2, .. M$ and $\alpha = \text{Maxgen2}/\log\left[\frac{p_m(0)}{10^{-3}}\right]$ for the population P(T) in the promising zone.

h *Reduction process of neighbourhood:* V(0) is the initial neighbourhood of S^*. V(T) is calculated by the formula $V(T) = V(0)\exp(-T/\alpha)$, where α is calculated so that the final neighbourhood is small enough(10^{-2} in our case). So

$$\alpha = \text{Maxgen2}/\log\left[\frac{V(0)}{10^{-2}}\right].$$

6 Assumptions and notations for the proposed models

The following notations and assumptions are used in developing the models.

1 Inventory system involves only one item.

2 Time horizon (*H*) is finite and $H = H_1 + H_2 + H_3$.

3 Unit cost, i.e., purchase price, $p(t)$ is a function of time t and is of the form

$$p(t) = \begin{cases} a - bt & \text{for } 0 \le t \le H_1 \\ a - bH_1 & \text{for } H_1 \le t \le H_1 + H_2 \\ A + Bt & \text{for } H_1 + H_2 \le t \le H_1 + H_2 + H_3 \end{cases}$$

where $A = a - \frac{bH_1(H_1 + H_2 + H_3)}{H_3}$ and $B = \frac{bH_1}{H_3}$.

4 Selling price $s(t)$ is mark-up m of $p(t)$ and m takes the values m_1, m_2 and m_3 during $(0, H_1)$, $(H_1, H_1 + H_2)$ and $(H_1 + H_2, H_1 + H_2 + H_3)$, i.e., $s(t) = m[= m_1, m_2, m_3]p(t)$.

5 Demand is a function of selling price $s(t)$ and is of the form

6 The lead time is zero.

7 T_i is the total time that elapses up to and including the i^{th} cycle $(i = 1, 2, ..., n_1 + n_2 + n_3)$ where $n_1 + n_2 + n_3$ denotes the total number of replenishment to be made during the interval $(0, H_1 + H_2 + H_3)$ and $T_0 = 0$.

8 n_1 is the number of replenishment to be made during $(0, H_1)$ at $t = T_0, T_1 ...., T_{n1-1}$. So, there are n_1 cycles in this duration. As purchase cost decreases during this session, demand increases, so successive cycle length must decrease. Here, α is the rate of reduction of successive cycle length and t_1 is the first cycle length. So, i^{th} cycle length $t_i = t_1 - (i - 1)\alpha$.= it1 − α.

$$T_i = \sum_{j=1}^{i} t_j = it_1 - \alpha\frac{i(i-1)}{2}$$

for $i = 1, 2, ..., n_1$. Clearly, $T_{n1} = H_1$.

Thus, $n_1t_1 - \alpha\frac{n_1(n_1-1)}{2} = H_1$

$$\Rightarrow \alpha = \frac{2(n_1t_1 - H_1)}{n_1(n_1-1)} \tag{7}$$

Here, t_1 and α are decision variables.

9 n_2 is the number of replenishment to be made during $(H_1, H_1 + H_2)$. Since purchase cost is constant, demand is also constant during this interval. So, all the subcycle length in this interval is assumed as constant. Replenishment are done at $t = T_{n_1}, T_{n_1+1}, \dots, T_{n_1+n_2-1}$ where $T_{n_1+j} = T_{n_1} + (j-1)\frac{H_2}{n_2}$ for $j = 1, 2, ..., n_2$.

10 n_3 is the number of replenishment to be made during $(H_1 + H_2, H_1 + H_2 + H_3)$. During this interval, purchase cost increases, as a result demand decreases. So, duration of order gradually increases. Here, β be the rate of increase of cycle length. Let $t_1^{/}$ be the initial cycle length and i^{th} cycle length $t_i^{/} = t_1^{/} + (i-1)\beta$. Thus, $t_{n3}^{/} = t_1^{/} + (n_3 - 1)\beta$. Orders are made at $t = T_{n_1+n_2}, T_{n_1+n_2+1}, \dots, T_{n_1+n_2+n_3-1}$ where

$$T_{n_1+n_2+i} = T_{n_1+n_2} + \sum_{j=1}^{i} t_j^{/} = (H_1 + H_2) + it_1^{/} + \beta\frac{i(i-1)}{2}$$

Clearly,

$$T_{n_1+n_2+n_3} = H_1 + H_2 + H_3.$$

$$\Rightarrow H_1 + H_2 + n_3 t_1^{/} + \beta \frac{n_3\left(n_3-1\right)}{2} = H_1 + H_2 + H_3.$$

$$\Rightarrow \beta = \frac{2\left(H_3 - n_3 t_1^{/}\right)}{n_3\left(n_3-1\right)} \tag{8}$$

11 c_h is the holding cost per unit/unit time.

12 c_o is the ordering cost.

13 $Q(T_i)$ is the order quantity at $t = T_i$.

14 $q(t)$ is the inventory level at time t.

15 Shortages are not allowed.

16 Z_1 and Z_2 are the total profit for Scenario 1 and 2 respectively.

A wavy bar (˜) is used with these symbols to represent corresponding fuzzy numbers when required.

Figure 2 Pictorial representation of $p(t)$

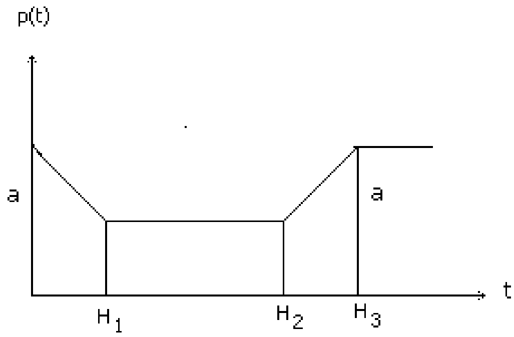
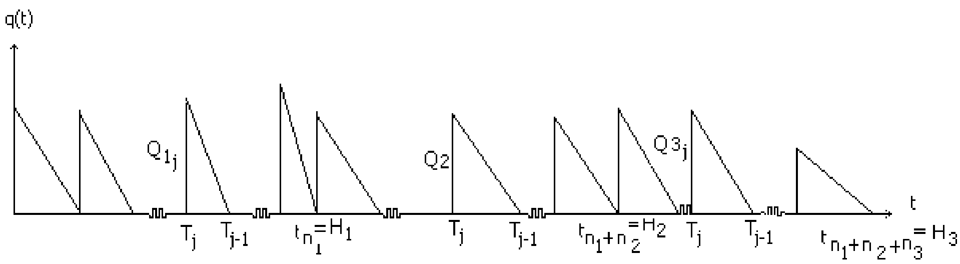


Figure 3 Inventory situation of the model



7 Model development and analysis

In the development of the model, it is assumed that at the beginning of every j^{th} cycle $[T_{j-1}, T_j]$, an amount $Q1_j$ units of the item is ordered. As lead time is negligible, replenishment of the item occurs as soon as order is made. Item is sold during the cycle and inventory level reaches zero at time $t = T_j$. Then order for next cycle made. Depending upon the selling price of seasonal product, two scenarios may arise.

Scenario 1 When selling price is a mark-up of present purchase cost.

Scenario 2 When selling price is a mark-up initial purchase cost for each cycle.

7.1 Formulation of Scenario 1 in crisp environment

This part is formulated in three phases.

7.1.1 Formulation for first phase (i.e., $0 \le t \le H_1$)

Duration of $j^{\text{th}}(1 \le j \le n_1)$ cycle is $[T_{j-1}, T_j]$, where $T_j = jt_1 - \alpha j(j-1)/2$ and at the beginning of the cycle inventory level is $Q1_j$. So instantaneous state $q(t)$ of the item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} = -\frac{D_1}{(a-bt)^{\gamma}} \tag{9}$$

Solving the above differential equation using the initial condition at $t = T_j$, $q(t) = 0$ we get

$$q(t) = \frac{D_1}{b(1-\gamma)}\left[(a-bt)^{1-\gamma} - \left(a-bT_j\right)^{1-\gamma}\right] \tag{10}$$

when

$$t = T_{j-1}, Q1_j = q\left(T_{j-1}\right) = \frac{D_1}{b(1-\gamma)}\left[\left(a-bT_{j-1}\right)^{1-\gamma} - \left(a-bT_j\right)^{1-\gamma}\right] \tag{11}$$

So, holding cost for $j^{\text{th}}(1 \le j \le n_1)$ cycle, $H1_j$, is given by

$$H1_j = c_h \int_{T_{j-1}}^{T_j} q(t)dt$$

$$= \frac{c_h D_1}{b(1-\gamma)}\left[\frac{\left(a-bT_{j-1}\right)^{2-\gamma} - \left(a-bT_j\right)^{2-\gamma}}{b(2-\gamma)} - \left(a-bT_j\right)^{1-\gamma}\left(T_j - T_{j-1}\right)\right]$$

Thus, total holding cost during $(0, H_1)$, $HOC1$, is given by

$$HOC1 = \sum_{j=1}^{n_1} H1_j \tag{12}$$

Total purchase cost during $(0, H_1)$, $PC1$, is given by

$$\begin{aligned} PC1 &= \sum_{j=1}^{n_1}\left[Q\left(T_{j-1}\right)p\left(T_{j-1}\right)\right] \\ &= \sum_{j=1}^{n_1}\frac{D_1}{b(1-\gamma)}\left[\left(a-bT_{j-1}\right)^{1-\gamma}-\left(a-bT_j\right)^{1-\gamma}\right]\left(a-bT_{j-1}\right) \end{aligned} \tag{13}$$

Total ordering cost during $(0, H_1)$, $OC1$, is given by

$$OC1 = \sum_{j=1}^{n_1}\left[c_{o1}+c_{o2}Q\left(T_{j-1}\right)\right] \tag{14}$$

Selling price for $j^{\text{th}}(1 \le j \le n_1)$ cycle, $SP1_j$, is given by

$$\begin{aligned} SP1_j &= \int_{T_{j-1}}^{T_j} m_1(a-bt)\frac{D_1}{(a-bt)^{\gamma}}dt \\ &= \frac{mD_1}{b(2-\gamma)}\left[\left(a-bT_{j-1}\right)^{2-\gamma}-\left(a-bT_j\right)^{2-\gamma}\right] \end{aligned} \tag{15}$$

Thus, total selling price during $(0, H_1)$, $SP1$, is given by

$$SP1 = \sum_{j=1}^{n_1} SP1_j \tag{16}$$

7.1.2 *Formulation for second phase (i.e., $H_1 \le t \le H_1 + H_2$)*

In the second phase ($H_1 \le t \le H_1 + H_2$), demand of customer is taken as constant, i.e., $D(t) = \dfrac{D_1}{(a-bH_1)^{\gamma}}$. Duration of $j^{\text{th}}(n_1 \le j \le n_1 + n_2)$ cycle is $[T_{j-1}, T_j]$, where $t_j = \dfrac{H_2}{n_2}$ and at the beginning of the cycle, inventory level is $Q2$ which is constant in each of cycle during $H_1 \le t \le H_1 + H_2$. Thus, order quantity, $Q2$, is given by

$$Q2 = \frac{D_1H_2}{(a-bH_1)^{\gamma}} \tag{17}$$

So, total holding, ordering and purchase costs, $HOC2$, $OC2$ and $PC2$ are respectively given by

$$HOC2 = \frac{c_hH_2Q_2}{2} \tag{18}$$

$$OC2 = n_2\left(c_{o1}+c_{o2}Q2\right) \tag{19}$$

$$PC2 = n_2Q2\left(a-bH_1\right) \tag{20}$$

Total selling price $SP2$, is given by

$$SP2 = Q2\left[m_2\left(a - bH_1\right)\right] \tag{21}$$

7.1.3 Formulation for third phase (i.e., $H_1 + H_2 \le t \le H_1 + H_2 + H_3$)

In this phase, duration of $j^{\text{th}}(n_1 + n_2 \le j \le n_1 + n_2 + n_3)$ cycle is $[T_{j-1}, T_j]$, where $T_j = H_1 + H_2 + (j - n_1 - n_2)t_1^{/} + (j - n_1 - n_2)(j - n_1 - n_2 - 1)\beta / 2$ and at the beginning of the cycle inventory level is $Q3_j$. So instantaneous state $q(t)$ of the item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} = -\frac{D_1}{(A - Bt)^{\gamma}} \tag{22}$$

Solving the above differential equation using the initial condition at $t = T_j$, $q(t) = 0$ we get

$$q(t) = -\frac{D_1}{B(1-\gamma)}\left[\left(A + BT_j\right)^{1-\gamma} - \left(A + Bt\right)^{1-\gamma}\right] \tag{23}$$

When

$$t = T_{j-1}, Q3_j = q\left(T_{j-1}\right) = \frac{D_1}{B(1-\gamma)}\left[\left(A + BT_j\right)^{1-\gamma} - \left(A + BT_{j-1}\right)^{1-\gamma}\right] \tag{24}$$

So, holding cost for $j^{\text{th}}(n_1 + n_2 \le j \le n_1 + n_2 + n_3)$ cycle, $H3_j$, is given by

$$\begin{aligned} H3_j &= c_h \int_{T_{j-1}}^{T_j} q(t)dt \\ &= \frac{c_h D_1}{B(1-\gamma)}\left[\left(A - BT_j\right)^{1-\gamma}\left(T_j - T_{j-1}\right) - \frac{\left(A + BT_j\right)^{2-\gamma} - \left(A + BT_{j-1}\right)^{2-\gamma}}{B(2-\gamma)}\right] \end{aligned}$$

Thus, total holding cost during ($H_1 + H_2$, $H_1 + H_2 + H_3$), $HOC3$, is given by

$$HOC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} H3_j \tag{25}$$

Total purchase cost during ($H_1 + H_2$, $H_1 + H_2 + H_3$), $PC3$, is given by

$$\begin{aligned} PC1 &= \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3}\left[Q3_j\, p\left(T_{j-1}\right)\right] \\ &= \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \frac{D_1}{b(1-\gamma)}\left[\left(A + BT_j\right)^{1-\gamma} - \left(A + BT_{j-1}\right)^{1-\gamma}\right]\left(A + BT_{j-1}\right) \end{aligned} \tag{26}$$

Total ordering cost during ($H_1 + H_2$, $H_1 + H_2 + H_3$), $OC3$, is given by

$$OC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[c_{o1} + c_{o2} Q3_j \right] \tag{27}$$

Selling price for j^{th}($n_1 + n_2 \le j \le n_1 + n_2 + n_3$) cycle, $SP3_j$, is given by

$$\begin{aligned} SP3_j &= \int_{T_{j-1}}^{T_j} m_3 (A + Bt) \frac{D_1}{(A+Bt)^{\gamma}} dt \\ &= \frac{mD_1}{B(2-\gamma)} \left[\left(A + BT_j \right)^{2-\gamma} - \left(A + BT_{j-1} \right)^{2-\gamma} \right] \end{aligned} \tag{28}$$

Thus, total selling price during ($H_1 + H_2$, $H_1 + H_2 + H_3$), $SP3$, is given by

$$SP3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} SP3_j \tag{29}$$

Thus, total profit under the Scenario 1 over the planning horizon $H_1 + H_2 + H_3$, Z_1, is given by

$$\begin{aligned} Z_1 = &(SP1 + SP2 + SP3) - (PC1 + PC2 + PC3) \\ &- (HOC1 + HOC2 + HOC3) - (OC1 + OC2 + OC3) \end{aligned} \tag{30}$$

7.1.4 Mathematical model of Scenario 1 in crisp environment

Now the problem reduces to determination of the decision variables t_1, $t_1^{/}$, m_1, m_2, m_3, n_1, n_2 and n_3 so as to

$$\text{Maximise} \quad Z_1 \tag{31}$$

7.1.5 Mathematical model of Scenario 1 in fuzzy environment

Generally for seasonal items the duration of above mentioned phases change due to various environmental effects. So, in general H_1, H_2 and H_3 are imprecise in nature. Thus, we take H_1, H_2 and H_3 as fuzzy numbers, i.e., $\tilde{H}_1$, $\tilde{H}_2$ and $\tilde{H}_3$ respectively. According to this assumption, α and β becomes fuzzy numbers $\tilde{\alpha}$ and $\tilde{\beta}$ respectively and then the profit Z_1 becomes fuzzy number $\tilde{Z}_1$, whose membership function is a function of the decision variables $t_1, t_1^{/}, m_1, m_2, m_3, n_1, n_2$ and n_3. In this case, since optimisation of a fuzzy number is not well defined one can optimise the optimistic (pessimistic) return corresponding to the fuzzy objective $\tilde{Z}_1$, with some degree of possibility (necessity) α_1 (α_2), as described in §3. Accordingly, in optimistic sense the problem reduces to

$$\begin{aligned} &\max \quad z \\ &\text{subject to,} \quad pos\{ Z_1 \ \ge \ z \} \ge \alpha_1 \end{aligned} \tag{32}$$

and in pessimistic sense the problem reduces to

$$\begin{aligned} &\max \quad z \\ &\text{subject to,} \quad nes\{Z_1 \geq z\} \geq \alpha_2 \\ &\text{i.e.,} \quad pos\{Z_1 \leq z\} < 1-\alpha_2 \end{aligned} \tag{33}$$

7.2 Formulation of Scenario 1 in crisp environment

This part is formulated in three phases.

7.2.1 Formulation for first phase (i.e., $0 \leq t \leq H_1$)

Duration of $j^{th}(1 \leq j \leq n_1)$ cycle is $[T_{j-1}, T_j]$, where $T_j = jt_1 - \alpha j(j-1)/2$ and at the beginning of the cycle inventory level is $Q1_j$. So instantaneous state $q(t)$ of the item during $T_{j-1} \leq t \leq T_j$ is given by

$$\frac{dq(t)}{dt} = -\frac{D_1}{\left(a - bT_{j-1}\right)^{\gamma}} \tag{34}$$

Solving the above differential equation using the initial condition at $t = T_j$, $q(t) = 0$ we get

$$q(t) = \frac{D_1}{\left(a - bT_{j-1}\right)^{\gamma}}\left[T_j - t\right] \tag{35}$$

When

$$t = T_{j-1}, Q1_j = q\left(T_{j-1}\right) = \frac{D_1}{\left(a - bT_{j-1}\right)}\left[T_j - T_{j-1}\right] \tag{36}$$

So, holding cost for $j^{th}(1 \leq j \leq n_1)$ cycle, $H1_j$, is given by

$$\begin{aligned} H1_j &= c_h \int_{T_{j-1}}^{T_j} q(t)dt \\ &= \frac{c_h D_1 \left(T_j - T_{j-1}\right)^2}{2\left(a - bT_{j-1}\right)^{\gamma}} \end{aligned}$$

Thus, total holding cost during $(0, H_1)$, $HOC1$, is given by

$$HOC1 = \sum_{j=1}^{n_1} H1_j \tag{37}$$

Total purchase cost during $(0, H_1)$, $PC1$, is given by

$$\begin{aligned} PC1 &= \sum_{j=1}^{n_1}\left[Q1_j\, p\left(T_{j-1}\right)\right] \\ &= \sum_{j=1}^{n_1}\frac{D_1}{\left(a-bT_{j-1}\right)^{\gamma}-1}\left(T_j-T_{j-1}\right) \end{aligned} \tag{38}$$

Total ordering cost during $(0, H_1)$, $OC1$, is given by

$$OC1 = \sum_{j=1}^{n_1}\left[c_{o1}+c_{o2}Q1_j\right] \tag{39}$$

Selling price for $j^{\text{th}}(1 \le j \le n_1)$ cycle, $SP1_j$, is given by

$$\begin{aligned} SP1_j &= \int_{T_{j-1}}^{T_j} m_1(a-bT_{j-1})\frac{D_1}{(a-bT_{j-1})^{\gamma}}dt \\ &= \frac{mD_1}{\left(a-bT_{j-1}\right)^{\gamma-1}}\left[T_j-T_{j-1}\right] \end{aligned} \tag{40}$$

Thus, total selling price during $(0, H_1)$, $SP1$, is given by

$$SP1 = \sum_{j=1}^{n_1} SP1_j \tag{41}$$

7.2.2 Formulation for second phase (i.e., $H_1 \le t \le H_1 + H_2$)

In the second phase ($H_1 \le t \le H_1 + H_2$), demand of customer is taken as constant, i.e., $D(t) = \dfrac{D_1}{(a-bH_1)^{\gamma}}$. So, selling price and all costs are same, as done in Section 7.1.2.

7.2.3 Formulation for third phase (i.e., $H_1 + H_2 \le t \le H_1 + H_2 + H_3$)

In this phase, duration $j^{\text{th}}(n_1 + n_2 \le j \le n_1 + n_2 + n_3)$ cycle is $[T_{j-1}, T_j]$, where $T_j = H_1 + H_2 + (j-n_1-n_2)t_1^{/} + (j-n_1-n_2)(j-n_1-n_2-1)\beta/2$ and at the beginning of the cycle inventory level is $Q3_j$. So instantaneous state $q(t)$ of the item during $T_{j-1} \le t \le T_j$ is given by

$$\frac{dq(t)}{dt} = -\frac{D_1}{(A-BT_{j-1})^{\gamma}} \tag{42}$$

Solving the above differential equation using the initial condition at $t = T_j$, $q(t) = 0$ we get

$$q(t) = -\frac{D_1}{\left(A+BT_{j-1}\right)^{\gamma}}\left[T_j - t\right] \tag{43}$$

When

$$t = T_{j-1}, Q3_j = q\left(T_{j-1}\right) = \frac{D_1}{\left(A + BT_{j-1}\right)^{\gamma}}\left[T_j - T_{j-1}\right] \tag{44}$$

So, holding cost for $j^{th}(n_1 + n_2 \leq j \leq n_1 + n_2 + n_3)$ cycle, $H3_j$, is given by

$$\begin{aligned} H3_j &= c_h \int_{T_{j-1}}^{T_j} q(t)dt \\ &= \frac{c_h D_1}{2\left(A - BT_{j-1}\right)^{\gamma}}\left[\left(T_j - T_{j-1}\right)^2\right] \end{aligned}$$

Thus, total holding cost during $(H_1 + H_2, H_1 + H_2 + H_3)$, $HOC3$, is given by

$$HOC3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} H3_j \tag{45}$$

Total purchase cost during $(H_1 + H_2, H_1 + H_2 + H_3)$, $PC3$, is given by

$$\begin{aligned} PC1 &= \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} \left[Q3_j\, p\left(T_{j-1}\right)\right] \\ &= \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} Q3_j \left(A + BT_{j-1}\right) \end{aligned} \tag{46}$$

Total ordering cost during $(H_1 + H_2, H_1 + H_2 + H_3)$, $OC3$, is given by

$$OC3 = \sum_{j=1}^{n_1}\left[c_{o1} + c_{o2} Q3_j\right] \tag{47}$$

Selling price for $j^{th}(n_1 + n_2 \leq j \leq n_1 + n_2 + n_3)$ cycle, $SP3_j$, is given by

$$\begin{aligned} SP3_j &= \int_{T_{j-1}}^{T_j} m_3\left(A + BT_{j-1}\right)\frac{D_1}{\left(A + BT_{j-1}\right)^{\gamma}} dt \\ &= mD_1\left(A + BT_{j-1}\right)^{1-\gamma}\left[T_j - T_{j-1}\right] \end{aligned} \tag{48}$$

Thus, total selling price during $(H_1 + H_2, H_1 + H_2 + H_3)$, $SP3$, is given by

$$SP3 = \sum_{j=n_1+n_2+1}^{n_1+n_2+n_3} SP3_j \tag{49}$$

Thus, total profit under the Scenario 2 over the planning horizon $H_1 + H_2 + H_3$, Z_2, is given by

$$Z_2 = (SP1 + SP2 + SP3) - (PC1 + PC2 + PC3) - (HOC1 + HOC2 + HOC3) - (OC1 + OC2 + OC3) \tag{50}$$

7.2.4 Mathematical model of Scenario 2 in crisp environment

Similar to Scenario 1, the problem reduces to determine the decision variables t_1, $t_1^{/}$, m_1, m_2, m_3, n_1, n_2 and $n_3$so as to

$$\text{Maximise} \quad Z_2 \tag{51}$$

7.2.5 Mathematical model of Scenario 2 in fuzzy environment

As in Scenario 1, when phase intervals H_1, H_2 and H_3 are imprecise in nature, i.e., $\tilde{H}_1$, $\tilde{H}_2$ and $\tilde{H}_3$ respectively, the profit Z_2 reduces to the fuzzy number $\tilde{Z}_2$, whose membership function is a function of the decision variables t_1, $t_1^{/}$, m_1, m_2, m_3, n_1, n_2 and n_3. Proceeding the same way, in this case also, in optimistic sense the problem reduces to

$$\begin{aligned} &\max && z \\ &\text{subject to,} && pos\{Z_2 \ \geq \ z\} \geq \alpha_1 \end{aligned} \tag{52}$$

and in pessimistic sense the problem reduces to

$$\begin{aligned} &\max && z \\ &\text{subject to,} && nes\{Z_2 \geq \ z\} \geq \alpha_2 \\ &\text{i.e.,} && pos\{Z_2 \leq z\} < 1 - \alpha_2 \end{aligned} \tag{53}$$

8 Numerical experiments

8.1 Results obtained for crisp environment

To illustrate the model following hypothetical set of data is used. This dataset is taken for items like rice, potato, wheat, onion, etc., whose demand exists in the market throughout the year. When new crops come in the market, then its price gradually decreases during some weeks (say H_1) and reaches a lowest level. This minimum price prevails for few weeks (say H_2). Then again it gradually increases during few weeks (say H_3) and reaches its normal value. This normal price prevails remaining part of the year. For an item like potato values of H_1, H_2 and H_3 are about 5 weeks, 15 weeks, 7 weeks in the state of West Bengal, India. Normal price of the item through out the year is about \$3 for a 10 kg bag. Lowest price of it in the season is about \$2 for a 10 kg bag. Keeping this real life situation in mind the following dataset is fixed to illustrate the modes in crisp environment. In the dataset 10 kg of the item is considered as one unit item, one week is considered as unit time and costs are represented in \$.

$$a = 3, b = 0.2, H_1 = 5, H_2 = 15, H_3 = 7,$$
$$D_0 = 3{,}000, \ \gamma = 4.5, \ ch = 0.5, \ co_1 = 2, \ co_2 = 0.1.$$

For the above parametric values, results are obtained via RRGA and presented in Table 1. From Table 1, it is found that profit for Scenario 1 gives better than that of Scenario 2 which normally occurs.

Table 1 Results obtained for the Scenario 1 and 2 via RRGA

Scenario	n_1	n_2	n_3	t_1	$t_1^{/}$	m_1	m_2	m_3	*Profit($)*
Scenario 1	7	9	7	0.97	0.63	1.47	1.62	1.43	217.17
Scenario 2	6	9	7	1.14	0.66	1.44	1.62	1.47	216.98

For the above parametric values, results are obtained for different values of γ and presented in Table 2. It is observed that as γ increases profit decreases due to decrease of demand which agrees with reality.

Table 2 Results obtained for Scenario 1 and 2 for different γ

Scenario	γ	n_1	n_2	n_3	t_1	$t_1^{/}$	m_1	m_2	m_3	*Profit($)*
Scenario 1	4.6	7	9	6	0.97	0.74	1.46	1.61	1.43	187.79
	4.7	6	9	6	1.13	0.73	1.48	1.60	1.42	162.15
	4.8	5	9	5	1.35	0.89	1.50	1.59	1.43	139.42
	4.9	5	9	5	1.35	0.88	1.50	1.58	1.43	119.68
	5.0	5	9	5	1.36	0.87	1.49	1.57	1.42	102.22
Scenario 2	4.6	6	9	7	1.15	0.65	1.43	1.61	1.46	187.63
	4.7	6	9	6	1.15	0.77	1.42	1.60	1.48	161.87
	4.8	5	9	6	1.38	0.76	1.44	1.59	1.47	139.26
	4.9	5	9	6	1.39	0.76	1.43	1.58	1.46	119.34
	5.0	5	9	5	1.39	0.93	1.42	1.57	1.49	101.89

8.2 Results obtained for fuzzy environment

To illustrate the proposed inventory models, following input data are considered. In this case also hypothetical dataset is used and source of this data has discussed for crisp model. For crisp model it was considered that unit price of the item decreases during a period H_1 = 5 weeks, but in reality it is about five weeks which is fuzzy in nature. Due to this reason here H_1 is considered as a TFN (4.5, 5, 5.5). Following same argument other fuzzy parameters are fixed and the dataset is presented below. In the dataset fuzzy numbers are considered as TFN types.

$$a = 3,\ b = 0.2,\ \tilde{H}_1 = (4.5,\ 5,\ 5.5),\ \tilde{H}_2 = (14,\ 15,\ 16),\ \tilde{H}_3 = (6.5,\ 7,\ 7.5),$$
$$D_0 = 3{,}000,\ \gamma = 4.5,\ \alpha_1 = 0.9,\ \alpha_2 = 0.1\ c_h = 0.5,\ co_1 = 2,\ co_2 = 0.1.$$

For the above parametric values, results are obtained via FSRRGA for both the Scenarios 1 and 2 in optimistic and pessimistic sense and presented in Tables 3 and 4 respectively.

Here, results are presented via FSRRGA in optimistic and pessimistic. It is observed from Tables 3 and 4 that profit for Scenario 1 is better than Scenario 2. But for optimistic

DM more risk is involved in the decision. On the other hand for pessimistic DM minimum profit is ensured.

Table 3 Results obtained via FSRRGA in optimistic sense

Scenario	n_1	n_2	n_3	t_1	t_1'	m_1	m_2	m_3	*Profit($)*
Scenario 1	7	9	7	0.98	0.63	1.48	1.62	1.43	221.42
Scenario 2	6	9	7	1.15	0.66	1.44	1.62	1.47	221.18

Table 4 Results obtained via FSRRGA in pessimistic sense

Scenario	n_1	n_2	n_3	t_1	t_1'	m_1	m_2	m_3	*Profit($)*
Scenario 1	6	9	6	1.11	0.75	1.50	1.61	1.44	212.99
Scenario 2	6	9	7	1.13	0.66	1.44	1.61	1.47	212.85

Table 5 Sensitivity analysis for the Scenario 1 and 2 in optimistic sense

Scenario	α_1	n_1	n_2	n_3	t_1	t_1'	m_1	m_2	m_3	*Profit($)*
Scenario 1	0.92	7	9	7	0.98	0.63	1.48	1.62	1.43	220.56
	0.94	7	9	7	0.97	0.63	1.47	1.62	1.43	219.71
	0.96	7	9	7	0.97	0.63	1.47	1.62	1.43	218.86
	0.98	7	9	7	0.97	0.63	1.47	1.62	1.43	218.01
	1.0	7	9	7	0.97	0.63	1.47	1.62	1.43	217.17
Scenario 2	0.92	6	9	7	1.15	0.66	1.44	1.62	1.47	220.33
	0.94	6	9	7	1.15	0.66	1.44	1.62	1.47	219.49
	0.96	6	9	7	1.15	0.66	1.44	1.62	1.47	218.65
	0.98	6	9	7	1.14	0.66	1.44	1.62	1.47	217.82
	1.0	6	9	7	1.14	0.66	1.44	1.62	1.47	216.98

Table 6 Sensitivity analysis for the Scenario 1 and 2 in pessimistic sense

Scenario	α_2	n_1	n_2	n_3	t_1	t_1'	m_1	m_2	m_3	*Profit($)*
Scenario 1	0.12	6	9	6	1.10	0.75	1.49	1.61	1.44	212.18
	0.14	6	9	6	1.10	0.75	1.49	1.61	1.44	211.37
	0.16	6	9	6	1.10	0.75	1.49	1.61	1.44	210.57
	0.18	6	9	6	1.09	0.75	1.49	1.61	1.44	209.76
	0.20	6	9	6	1.09	0.75	1.49	1.61	1.44	208.96
Scenario 2	0.12	6	9	7	1.12	0.66	1.44	1.61	1.47	212.03
	0.14	6	9	7	1.12	0.66	1.44	1.61	1.47	211.22
	0.16	6	9	7	1.12	0.66	1.44	1.61	1.47	210.40
	0.18	6	9	7	1.11	0.66	1.44	1.61	1.47	209.59
	0.20	6	9	7	1.11	0.65	1.44	1.61	1.47	208.78

From Tables 5 and 6, it is observed that as the degree of acceptability (α_1) for optimistic sense increases, the profit decreases and the increase of degree of acceptability (α_2) for pessimistic sense brings down, the profit also decreases. All these observations agree with reality.

9 Practical implications

The present models have the following practical usages:

- It is applicable for the inventory control of seasonal goods like paddy, wheat, pulses, etc., whose demand exists throughout the year and their price found stable about half of the year. But at the beginning of the production season their price gradually decreases to a stable lowest value for a period. This lowest price persist for a period and then again gradually increases to normal price of the year. The model is developed for these types of items during their seasonal period.
- Determination of possibility/necessity of a fuzzy event is possible analytically only if membership function of the related fuzzy numbers are found. But most of the real life problems it is not possible to find analytical form of membership function of objectives. As a result possibility/necessity of any event related to objective can not be obtained analytically and so analytical solution cannot be found in those cases. In these cases fuzzy simulation can be used to find possibility/necessity measure of a fuzzy event and GA (or any other heuristic) can be used to find optimal decision. So present methodology is applicable in these real life situations.
- The methodology used for the formulation and determination of solution is quiet general and can be useable on any inventory control/supply chain/optimisation problem in fuzzy environment.

10 Conclusions

Here, a real-life inventory model for a seasonal product is developed whose demand depends upon the unit cost of the product in fuzzy environment. Unit cost of the product is time dependent. Unique contribution of the paper is three fold:

- The model is developed for such items like food grains, pulses, potato, onion, etc., whose stable demand exists in the market throughout the year but it fluctuates for a part of the year when they are produced in the field. Here modelling is done for such products during their season of grown. For the best of authors knowledge none have considered this type of inventory model.
- Here for the first time unit cost of an item is modelled following real life situation, which gradually decreases with time during grown of the item in the field, then it retains the lowest value for a period and again gradually increases with time to normal price of the year. Though it is found for above mentioned items in every year, inventory practitioners overlooked this real life phenomenon.
- It is assumed that time horizon of the season is fuzzy in nature. For the first time season of an item is considered as a combination of three imprecise intervals. In fact

three parts in which unit cost function can be divided are considered as fuzzy numbers, which agree with reality.

The model is formulated to maximise optimistic/pessimistic return of the profit from the system for the season. Optimistic/pessimistic return of the profit function is obtained using possibility/necessity measure on fuzzy event and a simulation approach is proposed to find this optimistic/pessimistic return of the profit. A GA is developed based on entropy theory where region of search space is gradually decreases to a small neighbourhood of the optima. This is named as RRGA and is used to solve the model when planning horizon is crisp. When this algorithm is used to solve the fuzzy model using fuzzy simulation process to evaluate the optimistic/pessimistic return of the profit, the algorithm is named as FSRRGA. For a DM following managerial implications can be made

- If the DM allows some risk for his/her concern then he/she will follow optimistic approach.
- If the DM thinks that any risk may effect very bad for his/her concern then he/she will follow pessimistic approach.
- He/she may follow the model during the season of the item. As demand and price of the item is approximately constant during other part of the year, so normal inventory decision is applicable for that period.

At length, though model is formulated in fuzzy environment, here demand is not considered as imprecise in nature, though it is appropriate for any product. In fact consideration of fuzzy demand leads to fuzzy differential equation for formulation of the model. Using proposed solution approach one can not considered imprecise demand, which is the major limitation of the approach. So further research work can be done incorporating fuzzy demand in imprecise planning horizon. Though the model is presented in crisp and fuzzy environment, it can also be formulated in stochastic, fuzzy-stochastic environment.

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