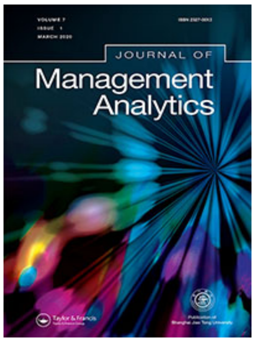

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Multi-item fuzzy inventory model for deteriorating items in multi-outlet under single management

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Multi-item inventory model with stock-dependent demand is developed in fuzzy environment. Items are deteriorated in constant rate and are sold from different outlets in the city under single management. Due to the impreciseness of different parameters, objectives as well as constraints are imprecise in nature. As optimization of fuzzy objectives as well as fuzzy constraints are not well defined, the model is formulated as a multi-objective chance constrained programming problem where optimistic/pessimistic return of the objectives with some degree of possibility/necessity are optimized and constraints are satisfied with some degree of necessity. The model is solved via Multi-Objective Genetic Algorithm (MOGA) when crisp equivalent of the problem is available. In other cases, fuzzy simulation process is proposed to check the constraints as well as to determine the optimistic/pessimistic return of the objectives. The model is illustrated with some numerical examples.

Keywords: possibility; necessity; fuzzy simulation; MOGA; inventory

1. Introduction

Most of the inventory determines optimal policies for single item, assuming that inventory policy for single item does not influence the cost of inventory as well as profit of the system. Instead of single item, many companies or enterprises or retailers are motivated to store several items in their shops for more profitable business affair. Another cause of their motivation is to attract the customers to purchase several items from one show/shop. Multi-item inventory first introduced by Federgruen, Groenevelt, and Tijma (1984) who found out that coordinated replenishments for multiple items can significantly reduce total inventory costs because placing orders for multiple items in one replenishment order would reduce set up costs. Multi-item classical inventory models under different resource constraints such as available floor space/shelf-space, capital investment, average number of inventory and so on are presented in the well-known books by Churchman, Ackoff, and Arnoff (1957), Silver and Peterson (1985) and others. Padmanabhan and Vrat (1990) developed a multi-item multi-objective inventory model of deteriorating items with stock-dependent demand by goal programming method. Recently, researchers have started to realize the complicated natures of the multi-item inventory systems. The demands for multiple inventory items might be correlated, affecting the optimal order policies for a

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multi-item inventory system (Liu and Yuan, 2000; Das, Roy, and Maiti, 2000; Bera and Maiti, 2012; Maiti and Maiti, 2008; Yadav, Singh, and Gautam, 2016; Garai, Chakraborty, and Roy, 2016, 2018a; Bera and Jana, 2017; Jana and Das, 2017; Chakraborty, Jana, and Roy, 2015; Tamjidzad and Mirmohammadi, 2018; Pakhira, Maiti, and Maiti, 2018; Shenoy and Mal, 2019; Pervin, Roy, and Weber, 2019, etc.). When one product is out of stock, the demand might be satisfied with other available products, requiring an inventory model for substitutable products (Yadavalli, Van Schoor, and Udayabaskaran, 2006).

At the same time, multi outlets are one of the most important aspects for a successful businessman/retailer, operating across various locations of city or different cities for inventory management. Now-a-days, it is observed that businessman/retailer possesses more than one outlet in a city or different cities to capture the market. So, to get more profit, the retailer takes big challenge to open multi-outlet with multi deteriorating items of homogeneous type items like fruits, vegetables, packaged products, etc. where fruits like apple, orange, mango, graphs, etc. are sold from first retail outlet, vegetables like tomato, cabbage, cauliflower, etc. are sold from second retail outlet and so on. A very few considered this type of phenomena in the inventory management. Kar, Bhunia, and Maiti (2001) solved two shop inventory model of deteriorating multi-items with constraints on space and investment. Das and Maiti (2003) developed inventory of a differential item from two shops under single management with shortages and variable demand. So, in this existing literature, inventory of multi deteriorating items with multi-retail outlets under a single management has been considered in fuzzy environment where inventory parameters are fuzzy. None has considered multi-item inventory models with different outlets under a single management where objectives with some degree of possibility/necessity are optimized and constraints are satisfied with some degree of necessity.

In many companies, convenience goods and products are offered to the consumers through the company controlled retail outlets. Examples of these products include packaged products, fast foods, fruits, vegetables, etc. where the respective outlets are situated in an important place like supermarkets, municipality markets, etc. In these important places, it is almost impossible to have big show-rooms/shops due the scarcity of space and high rent. They run the outlets with a limited storage space and limited investment. Though inventory models with space and investment constraints have been published by Kar et al. (2001), Maiti and Maiti (2006, 2007), Chou, Julian, and Hung (2009), Garai et al. (2016), El-Wakeel and Al-yazidi (2016), Tamjidzad and Mirmohammadi (2018) etc., a very few have considered multi-item inventory models with different outlets under a single management.

It has been recognized that one's ability to make precise statement concerning different parameters of an inventory model with increasing complexities of the environment are not defined. As a result, different inventory parameters, especially the purchase cost of an item fluctuates throughout the year. So, purchase cost is fuzzy in nature as well as selling price.

Normally, in inventory control systems, resource constraints are assumed deterministic. In real life, when different inventory parameters are imprecise then constraints also become imprecise. For example, at the beginning of a business, normally it is started with a fixed capital. But in course of business, to take some advantages like bulk transport, sudden increase of demand, price discount, etc., decision of acquiring more items force the investor to augment the previously fixed capital by some amount in some situations.

This augmented amount is clearly fuzzy in nature in the sense of degree of uncertainty (Dubois and Prade (1997)) and hence the total invested capital become imprecise in nature. When purchase costs and investment capital are fuzzy then the resource constraint becomes fuzzy in nature. As a fuzzy constraint represents a fuzzy event, it should be satisfied with some predefined necessity (Dubois and Prade, 1983), according to company's requirement. Like the chance constraint programming approach, proposed by Mohon (2000) in which minimum probability level for satisfying each of the constraint in stochastic environment, possibilistic constraints also may be defined as in Zadeh (1978), Dubois and Prade (1983), Liu and Iwamura (1998a, 1998b). When purchase costs are fuzzy, objective function (i.e. average profit) becomes fuzzy in nature. Since optimization of a fuzzy objective is not well defined, one can optimize the optimistic/pessimistic returns of the objectives with some degree of possibility/necessity according to requirement as proposed by Liu and Iwamura (1998a, 1998b), Maiti and Maiti (2006, 2007), Maiti, Maiti, and Maiti (2014), Garai et al. (2016, 2018b), etc.

In the present competitive market, the inventory/stock is decoratively displayed through electronic media to attract the customers and thus to boost the sale. Levin, Mclaughlin, Lamone, and Kottas (1972), Schary and Becker (1972), Wolfe (1968) and others established the impact of product availability for simulating demand. Mandal and Phaujder (1989), Datta and Pal (1988) and others considered linear form of stock-dependent demand, i.e. $D = c + \mathrm{d}q$, where D, q represent demand and stock levels respectively, c, d are two constants, so chosen to best fit the demand function, whereas Urban (1992), Giri, Pal, Goswami, and Chaudhuri (1996), Mandal and Maiti (2000), Maiti and Maiti (2006, 2007) and others took the demand of the form $D = \mathrm{d}q^{\beta}$, where β is a constant. So, extensive research works in inventory control problems with stock-dependent demand have been reported (Yang, 2014; Kumar and Kumar, 2016a, 2016b; Shukla, Tripathi, and Sang, 2017; Tripathi, Singh, and Aneja, 2018; Garai, Chakraborty, and Roy, 2019, etc.).

In real-world problems, deterioration is also a natural phenomenon. There are some physical goods which deteriorate with the progress of time during their normal storage. In this area, a lot of research papers have been published by several researchers, viz. Mandal and Phaujder (1989), Gupta and Agarwal (2000), Chang (2004), Chang, Ouyang, and Teng (2003), Balkhi (1998, 2004), Maragatham and Lakshmidevi (2014), Sharmila and Uthayakumar (2015), Muniappan, Uthayakumar, and Ganesh (2015), Saha and Chakraborti (2012), Chakraborty et al. (2015), Kumar and Kumar (2016a, 2016b), Pal, Sana, and Chaudhuri (2017), Rajan and Uthayakumar (2017), Pervin et al. (2019) and others.

The main contributions of this paper are as follows:

- Generally, inventory parameters may be considered precisely but due to practical situation inventory parameters like the purchase cost, investment amount, storage space are considered as fuzzy which are defuzzified using possibility/necessity measures for a given level of optimistic/pessimistic sense.
- Though a considerable number of research papers have been published with single shop/selling point, much attention has not been paid for the situation where more than one shops are run under a single management. So, the model is formulated with multi-item multi-outlets for deteriorating items under single management.
- Average profits from different outlets give different objectives. So the problem becomes multi-objective optimization problem.

- Due to fuzziness of the different parameters, the model is formulated as a multi-objective chance constrained programming problem where optimistic/pessimistic return of the objectives with some degree of possibility/necessity is optimized and constraints are satisfied with some degree of necessity.
- The models are solved by Multi-Objective Genetic Algorithm (MOGA) and Fuzzy-Simulation based Multi-Objective Genetic Algorithm (FSMOGA) and results are compared.
- Finally, the model is illustrated with some numerical examples and results are verified through sensitivity analyses.

2. Possibility/necessity in fuzzy environment

Any fuzzy number $\widetilde{a}$ of $\Re$ (where $\Re$ represents set of real numbers) with membership function

$\mu_{\tilde{a}}:\Re \rightarrow [0, 1]$ is called a fuzzy number. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to Zadeh (1978), Dubois and Prade (1983) and Liu and Iwamura (1998a, 1998b):

$$pos(\tilde{a}*\tilde{b}) = \sup\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), \ x, y \in \Re, \ x^* y\}, \tag{1}$$

where abbreviation *pos* represents possibility and * is any one of the relations $<, \ >, \ =, \ \leq, \ \geq$. Analogously, if $\tilde{b}$ is a crisp number, say, b, then

$$pos(\tilde{a}*b) = \sup\{\mu_{\tilde{a}}(x), \quad x \in \Re, \ x^* b\}. \tag{2}$$

The necessity measure of an event $\tilde{a}*\tilde{b}$ is a dual of the possibility measure. The grade of an event is the grade of impossibility of the opposite event and is defined as:

$$nes(\tilde{a}^*\tilde{b}) = 1 - pos(\overline{\tilde{a}^*\tilde{b}}), \tag{3}$$

where the abbreviation *nes* represents the necessity measure and $\overline{\tilde{a}^*\tilde{b}}$ represents the complement of the event $\tilde{a}*\tilde{b}$.

If $\tilde{a}$, $\tilde{b} \in \Re$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$ where $f:\Re \times \Re \rightarrow \Re$ is binary operation then, the extension principle by Zadeh (1978), the membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is given by

$$\mu_{\tilde{c}}(z) = \sup\{\min(\mu_{\tilde{a}}(x), \ \mu_{\tilde{b}}(y)), \ x, \ y \in \Re \text{ and } z = f(x, y), \ \forall z \in \Re\}. \tag{4}$$

2.1. *Triangular Fuzzy Number (TFN)*

A TFN $\tilde{a} = (a_1, a_2, a_3)$ (cf. Figure 1) has three parameters a_1, a_2, a_3 where $a_1<a_2<a_3$ and is characterized by the membership function $\mu_{\tilde{a}}(x)$, is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & \text{otherwise}. \end{cases} \tag{5}$$

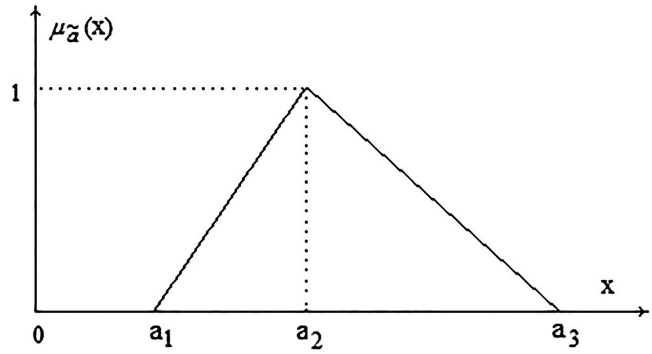

Figure 1. Membership function of a triangular fuzzy number.

2.2. *Parabolic Fuzzy Number (PFN)*

A PFN $\tilde{a} = (a_1, a_2, a_3)$ (cf. Figure 2) has three parameters a_1, a_2, a_3 where $a_1<a_2<a_3$ and is characterized by the membership function $\mu_{\tilde{a}}(x)$, is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \left(\dfrac{x - a_1}{a_2 - a_1}\right)^2, & a_1 \le x \le a_2, \\ 1 - \left(\dfrac{a_3 - x}{a_3 - a_2}\right)^2, & a_2 \le x \le a_3, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

2.3. *α cut of a fuzzy number*

α cut of a fuzzy number $\tilde{A}$ in $\Re$ with membership function $\mu_{\tilde{A}}$ denoted by A_α is defined as the crisp set $A_\alpha = \{x{:}\mu_{\tilde{A}}(x) \ge \alpha,\ x \in \Re\}$ where $\alpha \in [0, 1]$ A_α is a non-empty bounded closed interval contained in $\Re$ and it can be denoted by $A_\alpha = [A_L(\alpha),\ A_R(\alpha)]$.

Lemma 1: If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ are TFNs with $0 < a_1$ and $0 < b_1$ then

$$nes(\tilde{a} > \tilde{b}) \ge \alpha \textit{ iff } \frac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2} \le 1 - \alpha.$$

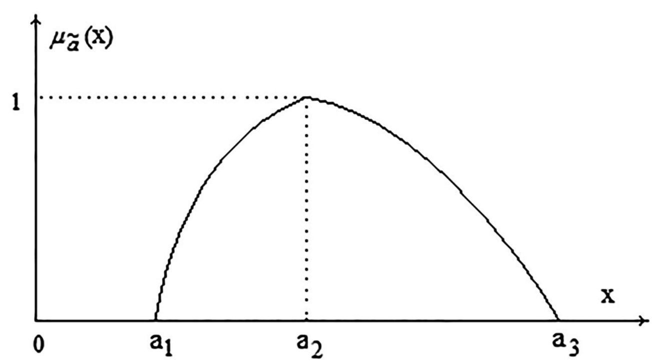

Figure 2. Membership function of a parabolic fuzzy number.

Proof: We have $nes(\tilde{a} > \tilde{b}) \geq \alpha \Rightarrow \{1 - pos(\tilde{a} \leq \tilde{b})\} \geq \alpha \Rightarrow pos(\tilde{a} \leq \tilde{b}) \leq 1 - \alpha$

So, from Figure 3, it is clear that $pos(\tilde{a} * \tilde{b}) = \delta = \dfrac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2}$ and hence the result follows.

Lemma 2: If $\tilde{a} = (a_1, a_2, a_3)$ be TFN with $0 < a_1$ and b is a crisp number, then

$$nes(\tilde{a} > b) \geq \alpha \text{ iff } \frac{b - a_1}{a_2 - a_1} \leq 1 - \alpha.$$

Proof: Proof follows from Lemma 1.

Lemma 3: If $\tilde{a} = (a_1, a_2, a_3)$ be TFN with $0 < a_1$ and b is a crisp number, then

$$pos(\tilde{a} > b) \geq \alpha \text{ iff } \frac{a_3 - b}{a_3 - a_2} \geq \alpha.$$

Proof: Proof follows from formula (1) and Figure 4.

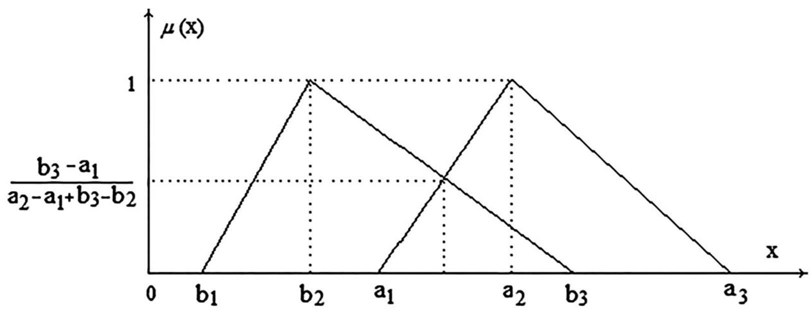

Figure 3. Comparison of two triangular fuzzy number.

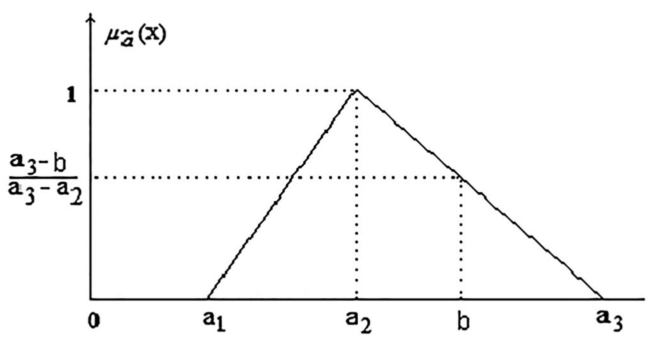

Figure 4. Comparison of a TFN with a crisp number.

3. Multi-objective optimization using possibility/necessity measure

A general multi-objective mathematical programming should have the following form:

$$\begin{aligned} &\text{Max}\, f_j(x,\ \xi),\ \ j = 1, 2,\ \ldots\ldots, m, \\ &\text{Subject to}\, g_i(x,\ \xi) \leq 0,\ \ \ i = 1,\ 2,\ \ldots.., n, \end{aligned} \tag{7}$$

where x is a decision vector, ξ is a vector of crisp parameters, $f_j(x,\ \xi)$ are return functions, $g_i(x,\ \xi)$ are constraint functions, $i = 1, 2, \ldots, n$. In the above problem, when ξ is a fuzzy vector $\tilde{\xi}$ (i.e. a vector of fuzzy numbers), then return functions and constraint functions $g_i(x,\ \tilde{\xi})$ are imprecise in nature and can be represented by two fuzzy numbers whose membership functions involve the decision variable x as a parameter and can be obtained when membership functions of the fuzzy numbers in $\tilde{\xi}$ are known (since f_j and g_i are functions of decision vector x and the fuzzy numbers in $\tilde{\xi}$). In that case the statements maximize $f_j(x,\ \tilde{\xi})$ as well as $g_i(x,\ \tilde{\xi}) \leq 0$ are not well defined. Since $g_i(x,\ \tilde{\xi})$ represents a fuzzy number whose membership function involves decision vector x and for a particular value of x, one can measure the necessity of $g_i(x,\ \tilde{\xi}) \leq 0$ using formula (3), so a value x_0 of the decision vector x is said to be feasible if necessity measure of the event $\{\tilde{\xi}{:}g_i(x_0,\ \tilde{\xi}) \leq 0\}$ exceeds some predefined level α_i in pessimistic sense, i.e. if $nes\{g_i(x_0,\ \tilde{\xi}) \leq 0\} \geq \alpha_i$ which is also written as $nes\{\tilde{\xi}{:}g_i(x_0,\ \tilde{\xi}) \leq 0\} \geq \alpha_i$. If analytical form of membership function of $g_i(x,\ \tilde{\xi})$ is available, then one can transform this constraint to an equivalent crisp constraint (cf. Lemma 1 of §2). Otherwise to check this necessity constraint, one can follow simulation process as proposed by Maiti and Maiti (2006).

Again since maximize $f_j(x,\ \tilde{\xi})$ are not well defined one can find maximum value of z_j such that $f_j(x,\ \tilde{\xi}) \geq z_j$. But $f_j(x,\ \tilde{\xi}) \geq z_j$ are also not well defined and so one can measure their possibility/necessity in optimistic/pessimistic sense and if this possibility/necessity measure exceeds some predefined level β_j, i.e. if $pos/nes\{f(x,\ \tilde{\xi}) \geq z\} \geq \beta$ (which are also written as $pos/nes\{\tilde{\xi}{:}f(x,\ \tilde{\xi}) \geq z\} \geq \beta$) then z_j taken as optimistic/pessimistic return of the fuzzy objective $f_j(x,\ \tilde{\xi})$ with degree of optimism/pessimism β_j. Since our aim is to maximize the objective functions, it is worthwhile to maximize the optimistic/pessimistic returns z_j and so one can find x for which z_j are maximum. When analytical form of membership functions of $f_j(x,\ \tilde{\xi})$ are available one can transform $pos/nes\{\tilde{\xi}{:}f(x,\ \tilde{\xi}) \geq z\} \geq \beta$ to equivalent crisp constraints, otherwise values of ξ are randomly generated from β_j cut set of fuzzy vector ξ and x is found from search space to maximize z_j as described in the next section (see Algorithm 2 and Algorithm 3 in §3.1).

So, when ξ is a fuzzy vector $\tilde{\xi}$ then one can convert the above problem (7) to the following chance constrained programming problems in optimistic and pessimistic sense respectively.

$$\begin{aligned} \max\quad & z_j,\ \ j = 1, 2,\ \ldots., m \\ \text{subject to}\quad & pos\{\tilde{\xi}{:}f_j(x,\ \tilde{\xi}) \geq z_j\} \geq \beta_j,\ j = 1, 2,\ \ldots., m \\ & pos\{\tilde{\xi}{:}g_i(x,\ \tilde{\xi}) \leq 0\} \geq \alpha_i,\ i = 1, 2,\ \ldots.., n \end{aligned} \tag{8}$$

$$\begin{aligned} \max \quad & z_j, \;\; j = 1, 2, \; \ldots., m \\ \text{subject to} \;\; & nes\{\tilde{\xi}{:}f_j(x, \tilde{\xi}) \geq z_j\} \geq \beta_j, \; j = 1, 2, \; \ldots., m \\ & pos\{\tilde{\xi}{:}g_i(x, \tilde{\xi}) \leq 0\} \geq \alpha_i, \;\; i = 1, 2, \; \ldots.., n, \end{aligned} \tag{9}$$

where α_i, $i = 1, 2, \ldots, n$, and β_j, $j = 1, 2, \ldots, m$ are predetermined confidence levels for fuzzy constraints and fuzzy objectives, respectively, $pos/nes\{.\}$ denotes the possibility/necessity of the event in $\{.\}$. So a point x is feasible if and only if the necessity measure of the set $\{\tilde{\xi}{:}g_i(x_0, \tilde{\xi}) \leq 0\}$ is at least α_i, $i = 1, 2, \ldots, n$. For each fixed feasible solution x, the objective value z_j should be the maximum that the objective function $f_j(x, \tilde{\xi})$ achieves with at least possibility/necessity β_j, $j = 1, 2, \ldots, m$.

3.1. *Fuzzy simulation*

The basic technique of chance constrained programming in a fuzzy environment is to convert the necessity constraints to their respective deterministic equivalents according to predetermined confidence level. However, the procedure is usually very hard and only successful for some special cases (cf. Lemma 1). Maiti and Maiti (2007) propose fuzzy simulation process to check feasibility of a solution x of the problems (8) and (9). The algorithm is presented below.

Algorithm 1: Algorithm to check $nes\{g_i(x, \tilde{\xi}) \leq 0\} \geq \alpha_i$, $i = 1, 2, \ldots, n$, for a particular value of decision vector x, for problem (8) and (9).

We know that $nes\{g_i(x_0, \tilde{\xi}) \leq 0\} \geq \alpha_i \Rightarrow pos\{g_i(x, \tilde{\xi}) > 0\} \leq 1 - \alpha_i$, $i = 1, 2, \ldots\ldots, n$. Using these criteria required algorithm is developed as below:

1. Set $i = 1$.
2. Generate ξ_0, uniformly from the $1 - \alpha_i$ cut set of fuzzy vectors $\tilde{\xi}$.
3. If $g_i(x, \xi_0) > 0$ go to step 7.
4. Repeat steps 2 to 3, N times.
5. Set $i = i + 1$, if $i \leq n$ go to step 2.
6. Return feasible.
7. Return infeasible.
8. End algorithm.

Again as stated earlier if analytical form of membership function of $f_j(x, \tilde{\xi})$ is available then only one can determine value of $z_j s$ in problems (8) and (9). However, in this case also, the procedure is usually very hard and only successful for some special cases (cf. Lemma 2, Lemma 3). To deal with the difficulties in evaluation of $z_j s$, following two simulation Algorithms are proposed for problems (8) and (9) respectively.

Algorithm 2: Algorithm to determine z_j, $j = 1, 2, \ldots, m$, for problem (8).
1. Do for $j = 1, 2, \ldots, m$.
2. Set $z_j = -\infty$ i.e. a large negative number.
3. Generate ξ_0 uniformly from β_j cut set of fuzzy vector $\tilde{\xi}$.
4. If $z_j < f_j(x, \xi_0)$ then $z_j = f_j(x, \xi_0)$.
5. Repeat steps 3 and 4, N times, where N is a sufficiently large positive integer.

6. End Do
7. Return z_j, $j = 1, 2, \ldots, m$.
8. End algorithm.

Algorithm 3: Algorithm to determine z_j, $j = 1, 2, \ldots, m$ for problem (9).

We know that $nes\{\tilde{\xi}: f_j(x, \tilde{\xi}) \geq z_j\} \geq \beta_j \Rightarrow pos\{\tilde{\xi}: f_j(x, \tilde{\xi}) < z_j\} < 1 - \beta_j$. Now roughly find a point ξ_0 from fuzzy vector $\tilde{\xi}$, which approximately minimizes f_j. Let this value be z_0 and ε be a positive number. Set $z_j = z_0 - \varepsilon$ and if $pos\{\tilde{\xi}: f_j(x, \tilde{\xi}) < z_j\} < 1 - \beta_j$ then increase z_j with ε. Again check $pos\{\tilde{\xi}: f_j(x, \tilde{\xi}) < z_j\} < 1 - \beta_j$ and it continues until $pos\{\tilde{\xi}: f_j(x, \tilde{\xi}) < z_j\} \geq 1 - \beta_j$. At this stage decrease value of ε and again try to improve z_j. When ε becomes sufficiently small then we stop and final value of z_j is taken. Using this criterion, required algorithm is developed as below:

1. Do for $j = 1, 2, \ldots, m$.
2. Initialize z_0 and ε.
3. Set $z_j = z_0 - \varepsilon$, $F_j = z_0 - \varepsilon$, $F_0 = z_0 - \varepsilon$
4. Generate ξ_0 uniformly from the $1 - \beta$ cut set of fuzzy vector $\tilde{\xi}$.
5. If $f_j(x, \tilde{\xi}_0) < z_j$
6. then go to step 12.
7. End If
8. Repeat step 4 to step 7 N times.
9. Set $F_j = z_j$.
10. Set $z_j = z_j + \varepsilon$.
11. Go to step 4.
12. If $(z_j = F_j)$ // In this case optimum value of $z_j < z_0 - \varepsilon$.
13. Set $z_j = z_j - \varepsilon$, $F_j = F_j - \varepsilon$, $F_0 = F_0 - \varepsilon$.
14. Go to step 4
15. End If
16. If $(\varepsilon < tol)$
17. go to step 22
18. End If
19. $\varepsilon = \varepsilon / N$
20. $z_j = F_j + \varepsilon$
21. Go to step 4
22. End Do
23. Output F_j, $j = 1, 2, \ldots, m$.

It is not possible to find an optimum solution of problem (8) or (9) using any traditional gradient based optimization technique or using any soft computing algorithm (MOGA, Multi Objective Simulated Annealing (MOSA), etc.) until the necessity constraints are converted to equivalent crisp constraints and analytical expressions of z_j are available. In almost all real-life problems, it is not possible to convert the necessity constraints to their crisp equivalents and it is very hard to get analytical expressions for z_js. In that case with the help of above algorithms any soft computing algorithm (MOGA, MOSA, etc.) can be used to solve the above problem (8) or (9). In this paper, MOGA is used for this purpose and since the above fuzzy simulation process is used to check the constraints in these situations as well as z_j are determined, the corresponding MOGA is called FSMOGA. In the next section, an MOGA is discussed to solve (8) and (9) with the help of above algorithms. This algorithm is named as FSMOGA.

4. Fuzzy simulation based multi-objective genetic algorithm (FSMOGA)

Genetic Algorithms are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation, etc.) and have been developed by Holland, his colleagues and students at the University of Michigan (c.f. Goldberg, 1989; Michalewicz, 1992, etc.). Because of its generality and other advantages over conventional optimization methods, it has been successfully applied to different decision making problems. There are several approaches using genetic algorithms to deal with the multi-objective optimization problems. The better known ones include the plain aggregation approach, the population-based non-pareto approach, the pareto-based approach and Niche induction approach by Deb (2001, 2002). Proposed multi-objective genetic algorithm has been developed following Deb, Pratap, Agarwal, and Meyarivan (2002) with the help of fuzzy simulation process to check the problem constraints and has the following two important components.

(a) Division of a population of solutions into subsets having non-dominated solutions: Consider a problem having M objectives and take a population P of feasible solutions of the problem of size N. We like to partition P into subsets F_1, F_2, … ., F_k, such that every subset contains non-dominated solutions, but every solution of F_i is not dominated by any solution of F_{i+1}, for $i = 1, 2, \ldots, k-1$. To do this for each solution, x, of P, calculate the following two entities.

(i) Number of solutions of P which dominate x, let it be η_x.
(ii) Set of solutions of P that is dominated by x. Let it be S_x.

The above two steps require O (MN^2) computations. Clearly F_1 contains every solution x having $\eta_x = 0$. Now for each solution $x \in F_1$, visit every member y of S_x and decrease η_y by 1. In doing so if for any member y, $\eta_y = 0$, then $y \in F_2$. In this way, F_2 is constructed. The above process is continued to every member of F_2 and thus F_3 is obtained. This process is continued until all subsets are identified.

For each solution x in the second or higher level of non-dominated subsets, η_x can be at most $N-1$. So each solution x will be visited at most $N-1$ times before η_x becomes zero. At this point, the solution is assigned a subset and will never be visited again. Since there is at most $N-1$ such solutions, the total complexity is O (N^2). So overall complexity of this component is O(MN^2).

(b) Determine distance of a solution from other solutions of a subset:

To determine distance of a solution from other solutions of a sub set following steps are followed:

(i) First sort the subset according to each objective function values in ascending order of magnitude.
(ii) For each objective function, the boundary solutions are assigned an infinite distance value (a large value).

(iii) All other intermediate solutions are assigned a distance value for the objective, equal to the absolute normalized difference in the objective values of two adjacent solutions.
(iv) This calculation is continued with other objective functions.
(v) The overall distance of a solution from others is calculated as the sum of individual distance values corresponding to each objective. Since M independent sorting of at most N solutions (in case the subset contains all the solutions of the population) are involved, the above algorithm has O(MNlogN) computational complexity.

Using the above two operations proposed multi-objective genetic algorithm takes the following form:

1. Set probability of crossover p_c and probability of mutation p_m.
2. Set iteration counter $T = 1$.
3. Generate initial population set of solution $P(T)$ of size N.
4. Select solution from $P(T)$ for crossover and mutation.
5. Made crossover and mutation on selected solution and get the child set $C(T)$.
6. Set $P_1 = P(T) \mathrm{U} C(T)$//Here U stands for union operation.
7. Divide P_1 into disjoint subsets having non-dominated solutions. Let these sets be F_1, F_2, … , F_k.
8. Select maximum integer n such that order of P_2 ($=F_1 \mathrm{U} F_2 \mathrm{U} \ldots \mathrm{U} F_n$) $\leq N$.
9. If O(P_2) < N sort solutions of F_{n+1} in descending order of their distance from other solutions of the subset. Then select first N –O(P_2) solutions from F_{n+1} and add with P_2, where O(P_2) represents order of P_2.
10. Set $T = T + 1$ and $P(T) = P_2$.
11. If termination condition does not hold go to step 4.
12. Output: $P(T)$
13. End algorithm

MOGAs that use non-dominated sorting and sharing are mainly criticized for their

- O(MN3) computational complexity
- non-elitism approach
- the need for specifying a sharing parameter to maintain diversity of solutions in the population.

In the above algorithm, these drawbacks are overcome. Since in the above algorithm computational complexity of step 7 is O(MN2), step 9 is O(MNlogN) and other steps are ≤ O(N), so overall time complexity of the algorithm is O(MN^2). Here the selection of new population after crossover and mutation on old population is done by creating a mating pool by combining the parent and offspring population and among them, best N solutions are taken as solutions of new population. By this way, elitism is introduced in the algorithm. When some solutions from a non-dominated set F_j (i.e. a subset of F_j) are selected for new population, those are accepted whose distance compared to others (which are not selected) are much i.e. isolated solutions are accepted. In this way taking some isolated solutions in the new population, diversity among the solutions is introduced in the algorithm. Different procedures of the above MOGA are discussed in the following section.

4.1. *Procedures of the proposed MOGA*

(a) **Representation**: A 'K dimensional real vector' $X = (x_1, x_2, \ldots . x_K)$ is used to represent a solution, where $x_1, x_2, \ldots, x_K$ represent different decision variables of the problem such that constraints of the problem are satisfied.

(b) **Initialization**: N such solutions $X_1, X_2, X_3, \ldots, X_N$ are randomly generated by random number generator from the search space such that each X_i satisfies the constraints of the problem. This solution set is taken as initial population $P(1)$. Also set $p_c = 0.3$, $p_m = 0.2$, $T = 1$.

(c) **Crossover**:

  (i) **Selection for crossover**: For each solution of $P(T)$ generate a random number r from the range [0, 1]. If $r < p_c$, then the solution is taken for crossover.

  (ii) **Crossover process**: Crossover taken place on the selected solutions. For each pair of coupled solutions Y_1, Y_2 a random number c is generated from the [0, 1] and offsprings Y_{11} and Y_{21} are calculated by $Y_{11} = cY_1 + (1-c)Y_2$, $Y_{21} = cY_2 + (1-c)Y_1$.

(d) **Mutation:**

  (i) **Selection for mutation:** For each solution of $P(T)$ generate a random number r from the range [0, 1]. If $r < p_m$ then the solution is taken for mutation.

  (ii) **Mutation process:** To mutate a solution $X = (x_1, x_2, x_3, \ldots, x_K)$ select a random integer r in the range [1, K]. Then replace x_r by randomly generated value within the boundary of rth component of X.

(e) **Division of $P(T)$ into disjoint subsets having non-dominated solutions:** Following the discussions of the previous section the following algorithm is developed for this purpose.

```
For every x ∈ P(T) do
        Set S_x=Φ, where Φ represents null set
        η_x = 0
        For every y ∈ P(T) do
            If x dominates y, then
                S_x = S_x U{y}
            Else if y dominates x then
                η_x = η_x + 1
            End if
        End for
    If η_x = 0 then
        F_1 = F_1 U{x}
    End If
  End For
  Set i = 1
  While F_i ≠ Φ do
        F_{i+1} = Φ
        For every x ∈ F_i do
    For every y ∈ S_x do
        η_y = η_y − 1
```

If $\eta_y = 0$ then
$F_{i+1} = F_{i+1}$ U$\{y\}$
End If
End For
End For
$i = i + 1$
End While
Output: $F_1, F_2, \ldots, F_{i-1}$.

(f) **Determine distance of a solution of subset *F* from other solutions**: Following algorithm is used for this purpose

Set n = number of solutions in F
For every $x \in F$ do
$x_{\text{distance}} = 0$
End For
For every objective m do
Sort F, in ascending order of magnitude of mth objective.
$F[1] = F[n] = M$, where M is a big quantity.
For $i = 2$ to $n{-}1$ do
$F[i]_{\text{distance}} = F[i]_{\text{distance}} + (F[i+1].\text{objm} - F[i-1].\text{objm})/(f_m^{\max} - f_m^{\min})$
End For
End For

In the algorithm $F[i]$ represents ith solution of F, $F[i]$.objm represent mth objective value of $F[i]$. $f_m^{\max}$ and $f_m^{\min}$ represent the maximum and minimum values of mth objective function.

5. Assumptions and notations for the proposed models

The following notations and assumptions are used in developing the models.

5.1. *Notations*

This model is developed for ith ($i = 1$,) outlet and jth item throughout the paper.

N	number of deteriorating items
M	number of outlets
W_i	storage area of ith outlet
λ	deterioration rate
I_{NV}	maximum investment amount
T_{ij}	cycle length
Q_{ij}	order quantity
Q_{0ij}	stock level above which stock has no effect on demand
$q_{ij}(t)$	inventory level at time t
D_{ij}	demand rate per unit time
$a_{ij},\ b_{ij}(>0)$	parameters of demand
A_{ij}	storage area per unit
T_{1ij}	time when inventory level reaches Q_{0ij}
c_{pij}	purchase cost per unit
c_{sij}	selling price per unit
c_{0ij}	ordering cost per cycle

c_{hij}	holding cost per unit
Z_{ij}	average profit.
F_i	average profit from *i*th outlet
P_i/N_i	optimistic/pessimistic return of the average profit F_i with degree of optimisim/ pessimism β_i
α_1, α_2	confidence levels for investment and space constraints respectively.

5.2. *Assumptions*

(i) The model is developed for M outlets, N deteriorating items and n_i items are sold from this outlet. So, $\sum_{i=1}^{M} n_i = N$.
(ii) Lead time is zero.
(iii) Shortages are not allowed.
(iv) Time horizon of the inventory system is infinite.
(v) The demand D_{ij} is linearly depend upon the stock level of the item and is of the form

$$D_{ij} = \begin{cases} a_{ij} + b_{ij}Q_{0ij}, & Q_{0ij} < q_{ij} \leq Q_{ij} \\ a_{ij} + b_{ij}q_{ij}, & q_{ij} \leq Q_{0ij} \end{cases}$$

(vi) Selling price c_{sij} is the mark-up of purchase cost, i.e. $c_{sij} = mc_{pij}$.
(vii) Ordering cost c_{0ij} linearly depends on order quantity and is of the form $c_{0ij} = c_{0ij1} + c_{0ij2}Q_{ij}$
(viii) The holding cost c_{hij} is multiple of purchase cost, i.e. $c_{hij} = h_{ij}c_{pij}$.

6. Model development and analysis

In the development of the model, it is assumed that the business man possesses M outlets and N items are sold from these outlets. For *j*th item in *i*th outlet a cycle starts with an inventory level Q_{ij}. Demand is stock dependent and when inventory level of the item reaches zero an order Q_{ij} for next cycle is made.

6.1. *Formulation for the* **j***th item in* **i***th outlet*

Depending upon the order quantity Q_{ij}, two cases may arise (i) Case I: $Q_{ij} > Q_{0ij}$; (ii) Case II: $Q_{ij} \leq Q_{0ij}$

6.1.1. *Case I ($Q_{ij} > Q_{0ij}$)*

The instantaneous state $q_{ij}(t)$ is given by the following differential equation:

$$\frac{\mathrm{d}q_{ij}(t)}{\mathrm{d}t} = \begin{cases} -(a_{ij} + b_{ij}Q_{0ij}) - \lambda q_{ij}(t), & Q_{0ij} < q_{ij}(t) \leq Q_{ij} \\ -(a_{ij} + b_{ij}q_{ij}(t)) - \lambda q_{ij}(t), & Q_{0ij} \geq q_{ij}(t) \geq 0 \end{cases} \tag{10}$$

with the boundary conditions $q_{ij}(0) = Q_{ij},\ q_{ij}(T_{1ij}) = Q_{0ij},\ q_{ij}(T_{ij}) = 0$.

Solving (10) we get

$$T_{1ij} = \frac{1}{\lambda}\log\left|\frac{a_{ij} + b_{ij}Q_{0ij} + \lambda Q_{ij}}{a_{ij} + b_{ij}Q_{0ij} + \lambda Q_{0ij}}\right|, \tag{11}$$

$$T_{ij} = T_{1ij} + \frac{1}{b_{ij} + \lambda} \log\left|\frac{a_{ij} + (b_{ij} + \lambda)Q_{0ij}}{a_{ij}}\right|, \tag{12}$$

$$q_{ij}(t) = \begin{cases} \dfrac{1}{\lambda}[(a_{ij} + b_{ij}Q_{0ij} + \lambda Q_{ij})e^{-\lambda t} - (a_{ij} + b_{ij}Q_{0ij})], & 0 < t \le T_{1ij} \\ \dfrac{1}{b_{ij} + \lambda}[\{a_{ij} + (b_{ij} + \lambda)Q_{0ij}\}e^{-(b_{ij}+\lambda)(t - T_{1ij})} - a_{ij}], & T_{1ij} \le t \le T_{ij}. \end{cases} \tag{13}$$

Sales revenue in $[0\,,\ T_{ij}]$ is $S_{pij} = c_{sij}S_{ij}$ *where* S_{ij} is given by

$$S_{ij} = \int_0^{T_{1ij}} (a_{ij} + b_{ij}\,Q_{0ij})\,\mathrm{d}t \;+ \int_{T_{1ij}}^{T_{ij}} (a_{ij} + b_{ij}\,q_{ij})\,\mathrm{d}t$$

$$= (a_{ij} + b_{ij}\,Q_{0ij})T_{1ij} + \frac{a_{ij}\lambda}{(b_{ij} + \lambda)^2}\log\left|\frac{a_{ij} + (b_{ij} + \lambda)Q_{0ij}}{a_{ij}}\right| + \frac{b_{ij}Q_{0ij}}{b_{ij} + \lambda}. \tag{14}$$

Holding cost in $[0, T_{ij}]$ is $c_{hij}H_{ij}$ where H_{ij} is given by

$$\begin{aligned} H_{ij} &= \int_{Q_{ij}}^{Q_{0ij}} \frac{-q_{ij}}{a_{ij} + b_{ij}Q_{0ij} + \lambda q_{ij}}\,\mathrm{d}q_{ij} \;+ \int_{Q_{0ij}}^{0} \frac{-q_{ij}}{a_{ij} + (b_{ij} + \lambda)q_{ij}}\,\mathrm{d}q_{ij} \\ &= -\frac{(a_{ij} + b_{ij}\,Q_{0ij})}{\lambda^2}\log\left|\frac{a_{ij} + b_{ij}Q_{0ij} + \lambda Q_{ij}}{a_{ij} + (b_{ij} + \lambda)Q_{0ij}}\right| + \frac{1}{\lambda}(Q_{ij} - Q_{0ij}) \\ &\quad + \frac{Q_{0ij}}{b_{ij} + \lambda} - \frac{a_{ij}}{(b_{ij} + \lambda)^2}\log\left|\frac{a_{ij} + (b_{ij} + \lambda)Q_{0ij}}{a_{ij}}\right|. \end{aligned} \tag{15}$$

6.1.2. *Case II* $(Q_{ij} \le Q_{0ij})$

The instantaneous state $q_{ij}(t)$ is given by the following differential equation:

$$\frac{\mathrm{d}q_{ij}(t)}{\mathrm{d}t} = -(a_{ij} + b_{ij}q_{ij}(t)) - \lambda q_{ij}(t) \tag{16}$$

with the boundary conditions $q_{ij}(0) = Q_{ij}, \quad q_{ij}(T_{ij}) = 0.$

Solving (16) we get

$$T_{ij} = \frac{1}{b_{ij} + \lambda}\log\left|\frac{a_{ij} + (b_{ij} + \lambda)Q_{ij}}{a_{ij}}\right| \tag{17}$$

$$q_{ij}(t) = \frac{1}{b_{ij} + \lambda}[\{a_{ij} + (b_{ij} + \lambda)Q_{ij}\}e^{-(b_{ij}+\lambda)t} - a_{ij}]. \tag{18}$$

Sales revenue in $[0, T_{ij}]$ is $S_{pij} = c_{sij} S_{ij}$ where S_{ij} is given by

$$S_{ij} = \int_0^{T_{ij}} (a_{ij} + b_{ij}\, q_{ij})\, \mathrm{d}t$$

$$= \frac{a_{ij}\lambda}{(b_{ij}+\lambda)^2} \log\left|\frac{a_{ij} + (b_{ij}+\lambda)Q_{ij}}{a_{ij}}\right| + \frac{b_{ij}Q_{ij}}{b_{ij}+\lambda}. \tag{19}$$

Holding cost in $[0, T_{ij}]$ is $c_{hij} H_{ij}$ where H_{ij} is given by

$$H_{ij} = \int_{Q_{ij}}^{0} \frac{-q_{ij}}{a_{ij} + (b_{ij}+\lambda)q_{ij}}\, \mathrm{d}q_{ij}$$

$$= \frac{Q_{ij}}{b_{ij}+\lambda} - \frac{a_{ij}}{(b_{ij}+\lambda)^2} \log\left|\frac{a_{ij} + (b_{ij}+\lambda)Q_{ij}}{a_{ij}}\right|. \tag{20}$$

Combining both the cases average profit from *j*th item in *i*th outlet Z_{ij} is given by

$$\begin{aligned} Z_{ij} &= [c_{sij}S_{ij} - c_{pij}Q_{ij} - c_{hij}H_{ij} - (c_{0ij1} + c_{0ij2}Q_{ij})]/T_{ij} \\ &= [\{mS_{ij} - Q_{ij} - h_{ij}H_{ij}\}c_{pij} - (c_{0ij1} + c_{0ij2}Q_{ij})]/T_{ij}. \end{aligned} \tag{21}$$

Average profit F_i from *i*th outlet is given by

$$F_i = \sum_{j=1}^{n_i} Z_{ij}. \tag{22}$$

6.2. *Crisp model in mathematical form*

From the above discussion, the problem reduces to the following multi-objective constrained optimization problem as

$$\text{Model 1:} \qquad \text{Maximize } F_i,\ i = 1, 2, \ldots\ldots, M \tag{23}$$

$$\text{Subject to } \sum_{i=1}^{M} \sum_{j=1}^{n_i} Q_{ij} c_{pij} \le I_{NV}$$

$$\sum_{j=1}^{n_i} Q_{ij} A_{ij} \le W_i,\ i = 1, 2, \ldots, M.$$

6.3. *Fuzzy models in mathematical form*

In the real world, purchase cost (c_{pij}), investment amount (I_{NV}), warehouse space (W_i) are normally imprecise, i.e. vaguely defined in some situations. So we take c_{pij}, I_{NV}, W_i are fuzzy numbers, i.e. as $\tilde{c}_{pij}$, $\tilde{I}_{NV}$, $\tilde{W}_i$ respectively. Then, due to this assumption, F_i

become fuzzy number $\tilde{F}_i$ and constraints in (23) also become imprecise in nature. Then as discussed in Section 3, the statement Maximize $\tilde{F}_i$ and constraints in (23) are not well defined. In this case following Section 3, one can maximize an optimistic (pessimistic) return value P_i (N_i) for the objective function $\tilde{F}_i$ with some degree of optimism (pessimism) β_i and the fuzzy constraints are satisfied with degree of pessimism α_1 and α_2 for investment and space constraints respectively. So in this case, the problem reduces to the following multi-objective chance constrained programming problems in optimistic and pessimistic senses respectively.

$$\text{Model 2:} \qquad \text{Maximize } P_i,\ i = 1, 2, \ldots\ldots., M \tag{24}$$

Subject to

$$pos\{\tilde{F}_i \geq P_i\} \geq \beta_i$$

$$nes\{\sum_{i=1}^{M}\sum_{j=1}^{n_i} Q_{ij}\tilde{c}_{pij} \leq \tilde{I}_{NV}\} \geq \alpha_1$$

$$nes\{\sum_{j=1}^{n_i} Q_{ij}A_{ij} \leq \tilde{W}_i\} \geq \alpha_2\ ,\ i = 1, 2, \ldots, M$$

$$\text{Model 3:} \qquad \text{Maximize } N_i,\ i = 1, 2, \ldots\ldots., M \tag{25}$$

Subject to

$$nes\{\tilde{F}_i \geq N_i\} \geq \beta_i$$

$$nes\{\sum_{i=1}^{M}\sum_{j=1}^{n_i} Q_{ij}\tilde{c}_{pij} \leq \tilde{I}_{NV}\} \geq \alpha_1$$

$$nes\{\sum_{j=1}^{n_i} Q_{ij}A_{ij} \leq \tilde{W}_i\} \geq \alpha_2,\ i = 1, 2, \ldots, M$$

If purchase cost $\tilde{c}_{pij}$, investment amount $\tilde{I}_{NV}$ and outlet capacity $\tilde{W}_i$ are TFNs with components $(c_{pij1}, c_{pij2}, c_{pij3})$, $(I_{NV1}, I_{NV2}, I_{NV3})$ and (W_{i1}, W_{i2}, W_{i3}) respectively then according to formula (4) $\tilde{Z}_{ij}$ becomes TFN with components $(Z_{ij1}, Z_{ij2}, Z_{ij3})$ where

$$Z_{ijk} = [\{mS_{ij} - Q_{ij} - h_{ij}H_{ij}\}c_{pijk} - (c_{0ij1} + c_{0ij2}Q_{ij})]/T_{ij},\ \ k = 1, 2, 3.$$

Then $\tilde{F}_i$ becomes TFN with components (F_{i1}, F_{i2}, F_{i3}) where $F_{ik} = \sum_{j=1}^{n_i} Z_{ijk}, k = 1, 2, 3.$

Also the quantity $\sum_{i=1}^{M}\sum_{j=1}^{n_i} Q_{ij}\,\tilde{c}_{pij}$ becomes TFN $(R_1,\ R_2, R_3)$ where $R_k = \sum_{i=1}^{M}\sum_{j=1}^{n_i} Q_{ij}\tilde{c}_{pijk}, k = 1, 2, 3.$ Then using the lemmas of Section 3, the problems

(24) and (25) reduce to following crisp multi-objective constrained optimization problems (26) and (27) respectively.

$$\text{Maximize}\, P_i,\, i = 1, 2, \ldots, M \tag{26}$$

Subject to

$$\frac{F_{i3} - P_i}{F_{i3} - F_{i2}} \geq \beta_i$$

$$\frac{R_3 - I_{NV1}}{I_{NV2} - I_{NV1} + R_3 - R_2} \leq 1 - \alpha_1$$

$$\frac{\sum_{j=1}^{n_i} Q_{ij}A_{ij} - W_{i1}}{W_{i2} - W_{i1}} \leq 1 - \alpha_2,\, i = 1, 2, \ldots, M \tag{27}$$

and Maximize $N_i,\, i = 1, 2, \ldots, M$

Subject to

$$\frac{N_i - F_{i1}}{F_{i2} - F_{i1}} \geq \beta_i$$

$$\frac{R_3 - I_{NV1}}{I_{NV2} - I_{NV1} + R_3 - R_2} \leq 1 - \alpha_1$$

$$\frac{\sum_{j=1}^{n_i} Q_{ij}A_{ij} - W_{i1}}{W_{i2} - W_{i1}} \leq 1 - \alpha_2,\, i = 1, 2, \ldots, M.$$

So when fuzzy parameters are TFN type then problems (24) and (25) can be transformed to equivalent crisp problems and can be solved via MOGA. But if the parameters are of PFN type then sum of two PFNs is not a PFN so in that case problems (24) and (25) cannot be transformed to equivalent crisp problems. In that case problems can be solved using FSMOGA with the help of simulation algorithms in Section 3.1.

7. Numerical illustration

7.1. *Crisp model*

To illustrate the crisp model (23), the following example 1 is given below.

Example 1: A businessman sells five items ($N = 5$) from two outlets ($M = 2$). From the first outlet, three types of fruits (i.e. apple, orange, mango) are sold and two types of vegetables (i.e. cauliflower, tomato) are sold from second outlet. For this problem, demand of these items is stock dependent which is shown in assumption (v), selling price is the mark-up ($m = 1.5$) purchase cost (i.e. c_{p11} =\$ 9.5, c_{p12} = \$10.5, c_{p13} = \$8.5, c_{p21} =\$ 9, c_{p22} =\$ 8), items deteriorated with the constant rate $\lambda = 0.02$, the storage space for two outlets respectively are $W_1 = 60$ sq. ft and $W_2 = 35$ sq ft. In this problem, holding cost (h_{ij}) has been decided according to the assumption (viii) and the ordering cost (c_{0ij}) is linearly dependent on order quantity (Q_{ij}) which is defined according to assumption (vii). Here, the businessman invested maximum amount (I_{NV}) \$1550 and all other parametric values has been presented in Table 1. Now, the problem is to find the optimum order quantity to get maximum profit.

Solution: In this problem, the input parameters are given in Table 1. This problem cannot be solved analytically. So, MOGA is applied to obtain the pareto optimal solution which has been presented in Table 2.

It is observed that pareto optimality of a solution does not imply total profit (F_1+F_2) from the system is maximum, which agrees with reality.

7.2. *Fuzzy model*

For illustration of the fuzzy models (24) and (25) following examples (Example 2 and Example 3) are used.

Example 2: Suppose the businessman starts business in an important place like supermarkets or municipality markets from where he runs two outlets for selling of fruits (i.e. apple, orange, mango, etc.) and vegetables (cabbage, tomato, cauliflower, etc.). Due to scarcity of space and high rent, it is impossible to have big showroom/shop. So, the businessman runs the outlets within the limited storage space and limited investment. But he store more at a time due to the customers demand which leads purchase cost (c_{pij}), investment amount (I_{NV}), warehouse space (W_i) becomes imprecise, i.e. vaguely defined in some situations. So we take c_{pij}, I_{NV}, W_i are fuzzy numbers, i.e. as $\tilde{c}_{pij}$, $\tilde{I}_{NV}$, $\tilde{W}_i$ respectively. Then, due to this assumption, F_i become fuzzy number $\tilde{F}_i$ and constraints in (23) also become imprecise in nature. Then as discussed in Section 3, the statement Maximize $\tilde{F}_i$ and constraints in (23) are not well

Table 1. Common input data for different examples.

Outlet (i)	Item (j)	a_{ij}	b_{ij}	h_{ij}	c_{0ij1}	c_{0ij2}	A_{ij}	Q_{0ij}
1	1	5	2.5	0.15	50	0.5	0.5	10
	2	10	2.2	0.15	50	0.5	0.45	10
	3	12	2.0	0.15	50	0.5	0.55	10
2	1	14	1.8	0.15	50	0.5	0.35	10
	2	8	2.1	0.15	50	0.5	0.45	10

Table 2. Results of Example 1 (Model 1) via MOGA.

Q_{11}	Q_{12}	Q_{13}	Q_{21}	Q_{22}	F_1	F_2	F_1+F_2
36.21	37.84	29.64	30.80	34.33	140.77	72.47	213.23
36.84	34.40	28.05	35.06	35.22	137.16	76.49	213.64
36.71	33.08	35.93	28.74	35.89	143.44	71.06	214.50
34.11	31.08	36.82	32.75	36.10	140.60	75.26	215.86
36.77	31.26	32.48	35.73	34.02	139.30	76.19	215.48

defined. In this case following Section 3, one can maximize an optimistic (pessimistic) return value P_i (N_i) for the objective function $\tilde{F}_i$ with some degree of optimism (pessimism) β_i and the fuzzy constraints are satisfied with degree of pessimism α_1 and α_2 for investment and space constraints respectively. So in this case the problem reduces to the following multi-objective chance constrained programming problems in optimistic and pessimistic senses respectively. Now the problem is to find the optimum profit in optimistic and pessimistic sense.

Here it is assumed that $\tilde{c}_{pij} = (c_{pij1}, c_{pij2}, c_{pij3})$, $\tilde{I}_{NV}$=(I_{NV1}, I_{NV2}, I_{NV3}), $\tilde{W}_i = (W_{i1}, W_{i2}, W_{i3})$ as TFNs with

$$c_{p111} = 9,\ c_{p112} = 9.5,\ c_{p113} = 10,\ c_{p121} = 9,\ c_{p122} = 10.5,\ c_{p123} = 11,\ c_{p131} = 8,$$
$$c_{p132} = 8.5,\ c_{p133} = 9.5,\ c_{p211} = 8.5,\ c_{p212} = 9,\ c_{p213} = 10.5,\ c_{p221} = 7.5,\ c_{p222} = 8,$$
$$c_{p223} = 9,\ I_{NV1} = \$1500,\ I_{NV2} = \$1550,\ I_{NV3} = \$1600,\ W_{11} = 50,\ W_{12} = 60,$$
$$W_{13} = 65,\ W_{21} = 30,\ W_{22} = 35,\ W_{23} = 40,\ \alpha_1 = 0.5,\ \alpha_2 = 0.5.$$

In this case problem (24), (25) can be transformed into corresponding crisp problem (26), (27) respectively. Problem (26) is solved for $\beta_1 = 0.9$, $\beta_2 = 0.9$ and (27) is solved for $\beta_1 = 0.1$, $\beta_2 = 0.1$ via MOGA and results are presented in Table 3 and Table 4 respectively.

In this case problem (24), (25) are also directly solved via FSMOGA and results are presented in Table 3 and Table 4 respectively. It is observed that results obtained by both the techniques are almost same.

Table 3. Results of Example 2 in model 2.

Method	Q_{11}	Q_{12}	Q_{13}	Q_{21}	Q_{22}	P_1	P_2
MOGA	27.84	32.27	29.97	36.96	31.98	132.47	78.08
	30.60	34.38	32.82	31.48	29.01	139.85	71.51
	34.14	33.50	31.21	34.08	25.26	140.59	69.10
	31.68	37.63	32.27	29.42	26.34	142.25	66.28
FSMOGA	35.91	26.89	32.35	35.51	29.06	135.50	74.66
	36.03	28.96	32.78	33.54	28.15	138.73	72.34
	30.60	34.38	32.82	31.48	29.01	139.85	71.51
	31.18	36.85	32.39	29.78	27.54	141.51	68.18

Table 4. Results of Example 2 in model 3.

Method	Q_{11}	Q_{12}	Q_{13}	Q_{21}	Q_{22}	N_1	N_2
MOGA	33.08	30.78	35.21	30.31	30.06	135.93	67.65
	31.77	31.32	33.67	35.08	27.17	134.31	68.61
	31.85	33.00	31.71	34.42	27.95	134.19	69.07
	28.46	31.94	31.35	35.12	32.77	129.59	73.92
FSMOGA	29.64	29.14	35.44	33.23	32.36	131.22	72.27
	30.88	30.61	32.53	36.59	28.78	131.86	71.31
	32.01	29.31	35.67	30.45	32.35	133.76	69.71
	31.67	30.30	35.21	31.37	30.95	134.29	69.52

Table 5. Results of Example 3 in model 2.

Method	Q_{11}	Q_{12}	Q_{13}	Q_{21}	Q_{22}	P_1	P_2
FSMOGA	31.23	32.65	31.74	31.31	29.10	142.68	77.21
	27.76	28.86	32.06	35.42	32.51	135.43	83.33
	26.51	28.03	32.44	36.39	33.11	133.08	84.31
	26.26	27.93	32.42	36.66	33.42	132.61	84.67

Table 6. Results of Example 3 in model 3.

Method	Q_{11}	Q_{12}	Q_{13}	Q_{21}	Q_{22}	N_1	N_2
FSMOGA	31.82	38.14	30.24	26.75	27.57	130.24	58.28
	31.27	32.43	30.81	29.35	32.55	126.63	66.38
	28.82	33.67	30.37	30.71	32.46	124.77	67.84
	26.61	32.23	30.03	32.67	34.87	120.54	71.23

Example 3: Here it is assumed that $\tilde{c}_{pij} = (c_{pij1}, c_{pij2}, c_{pij3})$ as PFNs with $c_{p111} = 9,\ c_{p112} = 9.5,\ c_{p113} = 10,\ c_{p121} = 9,\ c_{p122} = 10.5,\ c_{p123} = 11,\ c_{p131} = 8,$ $c_{p132} = 8.5,\ c_{p133} = 9.5, c_{p211} = 8.5, c_{p212} = 9, c_{p213} = 10.5, c_{p221} = 7.5, c_{p222} = 8,$ $c_{p223} = 9,\ \alpha_1 = 0.5,\ \ \alpha_2 = 0.5.$
All other parameters are same as Example 2.

In this case problem (24), (25) cannot be reduced to equivalent crisp problem and so are solved via FSMOGA only and results are presented in Table 5 and Table 6 respectively.

8. Practical implications

Let a businessman/retailer sell fruits (i.e. apple, orange, mango, etc.) and vegetables (cauliflower, cabbage, tomato, etc.) from two outlets situated in the place like supermarkets or municipality markets. To sell more items, he stocked more items in two outlets in order to get maximum profit. Here each item has different demand rate depending upon displayed stock level and the space used for two outlets are about 60 sq ft and 35 sq ft. which appears in an imprecise sense. The businessman/retailers decide how much order quantity be stocked at two outlets so that the profit will be maximum? For such real-life problem, present model has been implemented and solved by MOGA and FSMOGA. So, this model gives the managerial insight to the decision maker.

9. Conclusion and future research work

For the first time deteriorating multi-item inventory model is developed with multi-outlet facilities under a single management. The model is formulated as a multi-objective chance constrained programming problem in fuzzy environment. An approach is proposed where instead of objective functions optimistic/pessimistic returns of the objective functions are optimized. Also a simulation approach is proposed to determine these optimistic/pessimistic returns in fuzzy environment. So, from the economical point of view, the proposed model will be useful to the business houses in the present context as it gives better inventory control system. Further extension of this model can be done considering some realistic situation like quantity discount policy. Also, the present MOGA can be applied to other inventory models with price dependent demand, probabilistic demand, time-dependent demand, fixed time horizon, etc., along with quantity discount formulated in stochastic and fuzzy-stochastic environments.

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