Reversibility of α – asynchronous Cellular Autotmata: A simulation-based study

Anupam Pattanayak
Computer Science Department
Raja Narendra Lal Khan Women's College
Midnapore, West Bengal, India - 721102
anupam.pk@gmail.com

Subhasish Dhal
Computer Science and Engineering Department
IIIT Guwahati
Assam, India - 781015
subhasis.rahul@gmail.com

Abstract—**Cellular automata (CA) is a computing model. Change of CA cells are governed by the CA rules 0 – 255. When cells of CA are subject to change synchronously then that CA is referred as synchronous CA. But when all the cells do not participate in the process of change synchronously is called Asynchronous CA (ACA). The most complex type of ACA is α – asynchronous Cellular Automata where all the cells changes according to a probability. Reversibility is one important issue where we check if it's possible for CA with any initial state to comeback to this initial state after some time steps. We have attempted to study the reversibility of α – asynchronous cellular automata in this work which is so far not reported in the literature.**

Index Terms*- Cellular automata, Asynchronous cellular automata, Reversibility.*

I. INTRODUCTION

Among the many computation models proposed which are equivalent to Turing Machine is Cellular automata (CA). A simple CA is an one dimensional array (can be infinite at the both ends) of cells. each cell is in one of a finite set of possible states at every discrete time point. cells change state at every clock tick. If all the cells of CA can change it's state at the same time then the CA is termed as synchronous CA. On the other hand If only a set of cells can change state then the CA is said to be asynchronous CA. Elementary CA (ECA, or sometimes just CA) is a 1-D CA where state of every cell is either 0 or 1 and a cell has only two neighbors - left neighbor and right neighbor [4].

ECA has 256 rules to govern the state change. If we see the truth table of the left neighbor, the cell, and right neighbor, then the output column can be one of total 256 possibilities. We refer each of these 256 possibility of CA output as a rule. If each cell of a CA follows same rule then the CA is referred as uniform CA. When that is not the case then the CA is said non-uniform CA. In our work here, we have considered uniform ACA only. The rules are numbered according to the decimal equivalent of 8-bit binary of CA output column of the truth table. For example rule 55, which has binary equivalent 00110111, governs the change of cell of CA at time step t depending on its left and right neighbor cell to the value of cell at time step t+1 as per the table shown in the table 1:

$Left_t$	$Cell_t$	$Right_t$	$Cell_{t+1}$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Table 1. CA cell update under Rule 55

So we can express this change of a state as a local function, $f(Left_t, Cell_t, Right_t) = Cell_{t+1}$. For a given rule, if this function can be expressed using only XOR / XNOR logic functions then that CA rule is referred as linear / additive. The rules 15, 51, 60, 85, 90, 102, 105, 153, 165, 170, 195, 204, and 240 are linear. All other CA rules are nonlinear. There is one issue that points to the absence of left neighbor of leftmost cell and similarly absence of right neighbor of rightmost cell. If we consider these cells are 0 then that CA is referred as null boundary CA. If we consider that CA is circular then the CA is referred as periodic CA. In this work when we just mention CA , it means CA with periodic boundary [1].

Given a CA configuration, when all the cells undergo the transformation as per the governing rule, a new CA configuration is reached [2]. Given a CA configuration, if the CA reaches to this same configuration after two or more time steps, then the CA governing rule is said to be *reversible*. Reversibility of synchronous (and uniform) CA is well studied. Reversibility of ACA has attracted the few researchers to study its dynamics. But so far the existing results are available only for the fully asynchronous CA where just one cell undergo (possible) change. The α – asynchronous cellular automata is a more general case, where each cell undergo the local transformation with probability α. This is more realistic to nature. There is no previous work on reversibility of α – asynchronous Cellular Autotmata in the literature as of today to the best of our knowledge.

The rest of this paper is organized as follows. Works on irreversibility of asynchronous CA and fully asynchronous CA are discussed next, in section II. Next our simulation-based observation on reversible rules for ACA is reported in section III. Finally we discuss the future scope of work and the conclusion in section IV.

II. Related work

A. B. Reversibility of non-uniform CA

Das and Sikdar studied the reversibility of non-uniform CA with null boundary [3]. They have devised and used a tool named *reachability tree* to analyze and synthesize the reversible rules. They have listed total sixty two rules as reversible, given in the table 2.

B.
15, 23, 27, 30, 39, 43, 45, 51, 53, 54, 57, 58, 60, 75, 77, 78, 83, 85, 86, 89, 90, 92, 99, 101, 102, 105, 106, 108, 113, 114, 120, 135, 141, 142, 147, 149, 150, 153, 154, 156, 163, 165, 166, 169, 170, 172, 177, 178, 180, 195, 197, 198, 201, 202, 204, 210, 212, 216, 225, 228, 232, 240

Table 2. Reversible rules for non-uniform null boundary CA.

C. Irreversibility of Asynchronous CA

Sarkar, Mukherjee, and Das attempted to study reversibility of ACA [5]. In order to do that they first studied separately the irreversibility of periodic ACA and irreversibility of null-boundary ACA. In the following table 3 and table 4 we list these irreversible rules respectively.

D.
0, 2, 4, 6, 8, 10, 12, 14, 16, 20, 24, 28, 64, 66, 68, 70, 72, 74, 76, 78, 80, 84, 88, 92, 141, 143, 157, 159, 173, 175, 189, 191, 197, 199, 205, 207, 213, 215, 221, 223, 229, 231, 237, 239, 245, 247, 253, 255

Table 3. Irreversible rules for periodic boundary ACA.

E.
0, 4, 13, 15, 29, 31, 45, 47, 61, 63, 69, 71, 77, 79, 85, 87, 93, 95, 101, 103, 109, 111, 117, 119, 125, 127, 141, 143, 157, 159, 160, 168, 170, 173, 175, 189, 191, 197, 199, 205, 207, 213, 215, 221, 223, 224, 229, 231, 232, 234, 237, 239, 240, 245, 247, 248, 250, 253, 255

Table 4. Irreversible rules for null-boundary ACA.

So they were able to conclude that reversible ACA rules will not belong to this rule set. But they failed to deterministically point out the reversible rules for ACA. They remarked that "*it is hard to synthesize reversible 1-d ACAs*".

F. Reversibility of Fully Asyncronous CA

To overcome the difficulty of synthesizing the reversibility of 1-d ACA, a special type of ACA has been considered by Sethi, Fates, and Das [6]. They have considered the ACA where just one cell is chosen for possible update at a time step. This class of ACA is referred as *fully asynchronous CA*. They have studied the reversibility of this fully asynchronous CA. They found that some rules are always irreversible. Some rules are sometimes reversible but not other times, depending on the initial state of CA. And some rules are always reversible, irrespective of initial state of CA. They have referred the class of rules which are always reversible as recurrent rules. They have found total forty six recurrent rules for fully asynchronous CA, listed in the table 5.

G.
33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214

Table 5. Recurrent rules for fully asynchronous CA.

III. Reversibility of alpha–synchronous CA

We recall again that α – asynchronous CA is the CA where each cell makes a state transition with a probability α independent of other cells.

To simulate the α – asynchronous CA, we have developed C program and compile it using gcc compiler and run it in Ubuntu system. This program implements the α – asynchronous CA. The program takes size of CA and local state update probability as inputs and produces all the rules that are reversible as output. We randomly initialize the ACA cells initially and then for each rule we update each cell with given probability and continue evolving the ACA for maximum 20,000 time steps to check if the initial state is reached. The value 20,000 is taken as a reasonable threshold to stop iterations and come to conclude that the particular rule is not reversible if the initial state is not reached. This value could have been set to a higher value, but that would make the program execution time beyond a reasonable one. We take different probabilities such as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99, and 1.0 to see the results. Note that when the probability is 1.0, the ACA becomes synchronous CA.

In the following we list out our observations for ACA of size 5, 6, 7, 8, 9, and 10.

ACA Size	α	Reversible Rules for different update probability	Remarks
5	0.1	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.2	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules

	0.5	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.8	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.9	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.99	46, 51, 54, 57, 60, 62, 99, 102, 105, 108, 118, 134, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 212, 214	Total 27 rules
	1.0	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules

Table 6. Reversible rules for α – ACA with size 5

ACA Size	α	Reversible Rules for different update probability	Remarks
6	0.1	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.4	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.6	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.8	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121,	Total 46 rules

		123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	
	0.9	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.99	38, 51, 54, 57, 60, 99, 102, 105, 108, 134, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 212, 214	Total 23 rules
	0.999	51, 54, 57, 60, 99, 102, 105, 108, 142, 147, 148, 150, 153, 156, 195, 198, 201, 204, 212, 214	Total 20 rules
	1.0	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules

Table 7. Reversible rules for α – ACA with size 6

ACA Size	α	Reversible Rules for different update probability	Remarks
7	0.1	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.2	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.5	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.9	35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 43 rules
	0.99	51, 54, 57, 60, 99, 102, 105, 108, 134, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 212, 214.	Total 22 rules

	0.99	38, 51, 54, 57, 60, 99, 102, 105, 108, 134, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 212, 214	Total 23 rules
	1.0	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules

Table 8. Reversible rules for α – ACA with size 7

ACA Size	α	Reversible Rules for different update probability	Remarks
8	0.1	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.2	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.3	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.4	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.5	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 62, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 123, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 46 rules
	0.6	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 43 rules
	0.7	33, 35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 44 rules
	0.8	35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 131, 134, 139, 142, 145, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 42 rules
	0.9	35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 99, 102, 107, 108, 113, 115, 116, 118, 131, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 211, 212, 214.	Total 39 rules
	0.99	51, 54, 57, 60, 99, 102, 105, 108, 134, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 212, 214.	Total 22 rules
	0.999	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules
	1.0	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules

Table 9. Reversible rules for α – ACA with size 8

ACA Size	α	Reversible Rules for different update probability	Remarks
9	0.1	35, 38, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 134, 139, 142, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 38 rules
	0.2	35, 38, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 134, 139, 142, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 38 rules
	0.3	35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 42 rules
	0.4	35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 42 rules
	0.5	35, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108,	Total 41 rules

		113, 115, 116, 118, 121, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	
	0.6	35, 38, 41, 43, 46, 49, 51, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 134, 139, 142, 145, 147, 148, 150, 153, 155, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 41 rules
	0.7	35, 38, 41, 43, 46, 49, 51, 52, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 121, 134, 139, 142, 145, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 41 rules
	0.8	35, 38, 43, 46, 49, 51, 54, 57, 59, 60, 97, 99, 102, 105, 108, 113, 115, 116, 118, 134, 139, 142, 145, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 211, 212, 214.	Total 38 rules
	0.9	35, 51, 52, 54, 57, 59, 99, 102, 108, 118, 131, 134, 142, 145, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 212, 214.	Total 25 rules
	0.99	51, 54, 57, 60, 99, 102, 105, 108, 147, 148, 150, 153, 156, 195, 198, 201, 204, 212, 214.	Total 25 rules
	0.999	51, 54, 57, 60, 99, 102, 105, 108, 150, 153, 156, 195, 198, 201, 204.	Total 15 rules
	1.0	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules

Table 10. Reversible rules for α – ACA with size 9

ACA Size	α	Reversible Rules for different update probability	Remarks
10	0.1	35, 43, 46, 49, 51, 54, 57, 59, 60, 99, 102, 105, 108, 113, 115, 116, 139, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 212, 214.	Total 31 rules
	0.2	35, 43, 46, 49, 51, 54, 57, 59, 60, 97, 99, 102, 105, 108, 113, 115, 116, 139, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 212, 214.	Total 32 rules
	0.5	35, 41, 43, 46, 49, 51, 54, 57, 59, 60, 97, 99, 102, 105, 107, 108, 113, 115, 116, 118, 134, 139, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 212, 214.	
	0.6	35, 41, 43, 46, 49, 51, 54, 57, 59, 60, 97, 99, 102, 105, 108, 113, 115, 116, 118, 121, 134, 139, 142, 147, 148, 150, 153, 156, 158, 195, 198, 201, 204, 209, 212, 214.	Total 36 rules
	0.7	35, 43, 49, 51, 54, 57, 59, 60, 99, 102, 105, 108, 113, 115, 118, 142, 145, 147, 148, 150, 153, 156, 195, 198, 201, 204, 209, 212, 214.	Total 29 rules
	0.8	51, 54, 57, 60, 99, 102, 105, 108, 118, 134, 142, 145, 147, 148, 150, 153, 156, 195, 198, 201, 204, 209, 212, 214.	Total 24 rules
	0.9	51, 54, 57, 60, 99, 102, 105, 108, 142, 145, 148, 150, 153, 156, 195, 198, 201, 204, 209, 212, 214.	Total 21 rules
	1.0	51, 54, 57, 60, 99, 102, 105, 108, 147, 150, 153, 156, 195, 198, 201, 204.	Total 16 rules

Table 11. Reversible rules for α – ACA with size 10

IV. CONCLUSION

So we observed the behavior of α – ACA for different sizes and different probabilities. We have noted down the reversible rules as per our simulation result. The result we obtain does not allow us to deterministically list the reversible rules for α – ACA like the similar works for other kind of CA. What we conclude is that it's indeed hard to synthesize the reversible rules of α – ACA. But still, we have attempted and tried to note down the results of our simulation. This we believe will pave the path for further research in this area. As a future work, we can attempt to move one step forward than fully asynchronous CA where at a time at max. two cells can update and observe the reversibility of this kind of ACA.

ACKNOWLEDGMENT

The authors wish to acknowledge the immense help received from *Cellular Automata Research Group* of Information Technology department of IIEST Shibpur for introducing this area of research and insightful discussions during 2014-15.

REFERENCES

[1] P. Pal Chaudhury, D. Roy Chowdhury, S. Nandi, and S. Chatterjeee. *Additive Cellular Automata – Theory and Applications*, volume 1. IEEE Computer Society Press, USA, 1997.

[2] P. Sarkar, “A brief history of cellular automata,” *ACM Computing Service*, Volume 32, Issue 1., 2000.

[3] S. Das and B. K. Sikdar, “Analysis and synthesis of nonlinear reversible cellular automata in linear time”, arXiv preprint arXiv:1311.6879, 2013.

[4] S. Wolfram. *Cellular Automata and Complexity* – Collected Papers, Addision Wesley, 1994.

[5] A. Sarkar, A. Mukherjeee, and S. Das. “Reversibility in asynchronous cellular automata”, *Complex Systems*, 21(1:71-84, 2012.

[6] B. Sethi, N. Fates and S. Das, “Reversibility of Elementary Cellular Automata Under Fully Asynchronous Update”, Theory and Applications of Models of Computation, TATM 2014, pp 39-49, 2014.

Anupam Pattanayak is an Assistant Professor in the Department of Computer Science of Raja Narendralal Khan Women's College (Autonomous), Midnapore. He has obtained B. Sc. degree in Computer Science (Honours) from Vidyasagar University in 1999. He has completed MCA degree from Swami Ramanand Teerth Marathwadda University, Nanded. He has obtained M. Tech. in CSE from NIT Rourkela in 2009. He is continuing his Ph. D. in IIIT Guwahati. He has worked as project-linked personnel in Applied Statistics Unit, ISI Kolkata during June 2002 – March 2004, and August 2004 – March 2005. He has also worked as part-time lecturer in Sarsuna College during September 2004 – July 2007. He has served Haldia Institute of Technology as Assistant Professor in CSE during July 2009 – December 2016. His research interests are in the areas of Information Security, and Cellular Automata.

Subhasish Dhal is an Assistant Professor in the department of Computer Science and Engineering at Indian Institute of Information Technology Guwahati. He received B. Sc. degree in Computer Science (Honours) from Vidyasagar University in 2002. He has obtained his MCA degree from NIT Durgapur in 2005. He completed M. Tech. in CSE from NIT Rourkela in 2009. He has completed his PhD in IIT Kharagpur in 2016. From August 2005 till August 2007 he worked in Asutosh College, Kolkata as a lecturer and from August 2009 till December 2009, he worked in IE & IT, Durgapur as a lecturer. His research interests are in the areas of Security and privacy in IoT, Mobile Networks and Key Management and Distribution.