1
Introduction

English and other natural languages contain plural expressions, which allow us to talk about many objects simultaneously, for example:

(1.1) The students cooperate.

(1.2) The natural numbers are infinite.

How should such sentences be analyzed? In recent years, there has been a surge of interest in plural logic, a logical system that takes plurals at face value. When analyzing language, there is thus no need to eliminate the plural resources of English in favor of singular resources. Rather, the plural resources can be retained as primitive, not understood in terms of anything else.

Plural logic has emerged as a new tool of great potential significance in logic, philosophy, linguistics, and beyond. What is this new tool, and what is it good for? We wish to provide a more nuanced discussion than has been given so far.

Three questions run through our discussion. First:

The legitimacy of primitive plurals

Should the plural resources of English and other natural languages be taken at face value or be eliminated in favor of the singular?

Different considerations pull in different directions. On the one hand, there is the tremendous success of set theory, which shows how to represent many objects by means of a single complex object, namely their set. This is a powerful theory, which has proven to be of great theoretical value. Why bother with the many when we have a supremely successful theory of complex "ones"? On the other hand, there is a strong case for taking plurals at face value. English and many other natural languages allow us to talk about the many, apparently without any detour via complex "ones" such as

The Many and the One: A Philosophical Study of Plural Logic. Salvatore Florio and Øystein Linnebo, Oxford University Press.
DOI: 10.1093/oso/9780198791522.003.0001

sets. Why not utilize these expressive resources in our systematic theorizing? Moreover, attempts to eliminate the plural in favor of the singular appear to lead to paradoxes. We are all familiar with Russell's paradox of the set of all sets that are not elements of themselves. While this set leads to paradox, its many elements—considered as many, not one—do not. It thus appears to follow that talk about the many elements cannot be eliminated in favor of talk about their set. These considerations encourage the view encapsulated in Bertrand Russell's trenchant remark that "the many are only many, and are not also one" (Russell 1903, Section 74).

While we end up favoring a "pluralist" view, which takes plural resources at face value, this book tries to give the opposing "singularist" view a fair hearing. Our reasons for endorsing pluralism are somewhat unconventional. We reject many of the usual arguments against singularism and, in particular, argue that linguists are often entitled to their predominantly singularist approach. We place greater weight on a less familiar argument for pluralism, namely that primitive plurals are of great value for the explanation of sets and set theory.

Suppose we accept primitive plurals. This gives rise to our second overarching question.

> How primitive plurals relate to the singular
>
> What is the relation between the plural and the singular? We are particularly interested in the circumstances under which many objects correspond to a single, complex "one" and whether any such correspondence can shed light on the complex "ones".

Consider all the students at the nearest university. Presumably, they are very many. It is natural, however, to think that they also correspond to various single objects, such as a single group, or set, of students. The question thus arises what kinds of singularizing transformations there are and whether such transformations might be used to shed light on the resulting "ones". Following Georg Cantor and others, we find it illuminating to explain a set as an object that is somehow "constituted" by its many elements. This suggests a non-eliminative reduction of certain "ones" to the corresponding "many"; that is, we retain the "ones" as objects in good standing but seek an account of them in terms of the corresponding "many". It is important to notice that this non-eliminative reduction would proceed in the opposite direction of the singularists' proposed elimination of the plural in favor of the singular.

Thus, our proposal is not to eliminate the many but, on the contrary, to put them to use in explaining certain complex "ones".

As is well known, however, singularizing transformations are fraught with danger. If you know Cantor's theorem, you won't be surprised to learn that traditional plural logic enables us to prove that there are more pluralities of objects than single objects. (If you don't know the theorem, don't worry—it will be explained in due course.) This generalization of Cantor's theorem appears to show that it is impossible for every "many" to correspond to a unique "one". For there are more "manys" than there are "ones"! This result appears to limit severely what singularizing transformations can exist—and thus also to threaten the explanatory value that such transformations might have.

When examining the relation between the plural and the singular, we face conflicting logical and metaphysical pressures. On the one hand, the traditional and most intuitive plural logic severely restricts what singularizing transformations there can be. On the other hand, such transformations are intuitively plausible in their own right and (more importantly) promise to be of great theoretical value. How are we to negotiate these conflicting pressures? Following an approach recently defended by Timothy Williamson (2013, 2014), we reject a "logic first" orientation according to which we *first* choose a plural logic and *then* require every other theory to conform to this logic. Instead, we argue that the choice of a plural logic is entangled with commitments in metaphysics, semantics, and the philosophy of mathematics. We must therefore choose between various "package deals" that include not only a plural logic but also commitments far beyond.

Three such package deals will be examined. One is based on *generality relativism*, which rejects the possibility of quantification over absolutely everything. This surprising rejection of absolute generality has the benefit of reconciling traditional plural logic with the availability of singularizing transformations. When we apply such transformations, the range of our quantifiers expands in a way that enables us to avoid paradox. The other two package deals hold on to absolute generality but differ on how to address the conflicting pressures identified above. The more familiar version of absolute generality retains traditional plural logic and therefore limits what singularizing transformations there can be. We also explore a less familiar version of absolute generality which is more liberal concerning singularizing transformations and instead restores consistency by developing a more "critical" plural logic. In the final part of the book, we argue that the first

two package deals suffer from analogous expressibility problems and should therefore be rejected in favor of the third package deal.

Finally, there is our third overarching question.

THE SIGNIFICANCE OF PRIMITIVE PLURALS

What are the philosophical and (more broadly) scientific consequences of taking plurals at face value?

The very fact that primitive plural resources are available in thought and language is itself highly significant. Many recent writers on this subject, especially philosophers, have claimed that there are major further consequences as well. For example, we encounter claims to the effect that primitive plurals: (i) help us eschew problematic ontological commitments, thus greatly aiding metaphysics and the philosophy of mathematics; (ii) ensure the determinacy of higher-order quantification; and (iii) require us to reformulate the semantics of natural language using primitive plurals not only in the object language but also in the metalanguage. We argue that these claims are severely exaggerated. While primitive plurals are indeed legitimate and often very useful (especially for the explanation of sets), many other debates are unaffected by our choice of whether or not to accept primitive plurals. In particular, we argue that (i) the use of plural quantifiers incurs a form of commitment analogous to ontological commitment as traditionally understood; (ii) primitive plurals provide no additional assurance of the determinacy of higher-order quantification; and (iii) linguists are, for the most part, fully within their rights to continue in their old "singularizing" ways.

The title of our book might entice some readers who ponder the ancient question of whether reality is fundamentally a unity or a multiplicity. Parmenides famously views reality as a unity, asserting of it:

> Nor is it divisible, since it is all alike, and there is no more of it in one place than in another, to hinder it from holding together, nor less of it, but everything is full of what is. Wherefore it is wholly continuous; for what is, is in contact with what is. (Fragment 8, translated in Burnet 1920, 262)

Russell vehemently disagrees:

> Academic philosophers, ever since the time of Parmenides, have believed that the world is a unity. […] The most fundamental of my intellectual beliefs is that this is rubbish. I think the universe is all spots and jumps,

> without unity, without continuity, without coherence or orderliness or any of the other properties that governesses love. (Russell 1949, 98)

We shall not take a stand on Parmenides's question about the fundamental nature of reality. But we fully endorse the ancient view that the relation between the many and the one is of profound philosophical importance. As Russell observes, there are many objects (whether fundamental or not). Our discussion—and book title—therefore start with the many. But as we shall see, there are some surprisingly hard puzzles and problems concerning the relation between the many and the one. Our analysis of these puzzles and problems leads us to propose an unconventional solution, namely to replace the traditional plural logic with a more "critical" alternative.