2022

Mathematics

[Honours]

(B.Sc. Fourth Semester End Examination-2022)

PAPER-MTMH SEC401

Full Marks: 40 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Graph Theory]

Group-A

1. Answer any five questions: **5x2= 10**

a) Define out degree and in degree of a directed graph with example.

b) Prove that, $r(G) \le d(G) \le 2r(G)$ where $r(G)$ and $d(G)$ are the radius and diameter respectively corresponding to a graph G.

c) Define centre and radius of a graph. Show that every tree is either one or two centre.

d) Show that every complete graph K_n. for all n is a Hamiltonian graph.

e) Show that the height of a complete binary tree with n vertices is $\left[\log_2(n+1)-1\right]$

f) Find the rank and nulity of the bipartite graph $K_{2,3}$

g) Is it possible to construct a graph with 11 vertices such that 2 vertices has degree 3 and remaining vertices of degree 4? What will be the number of edges.

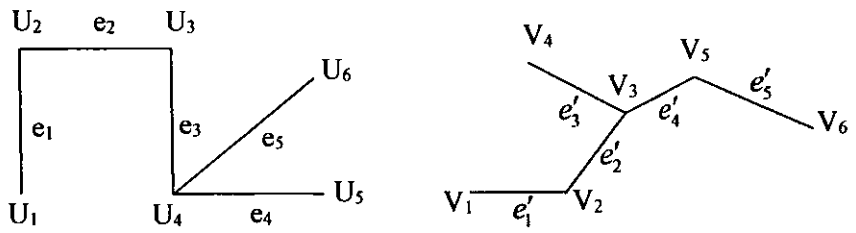
Group-B

2. Answer any four questions from the following questions **4x5= 20**

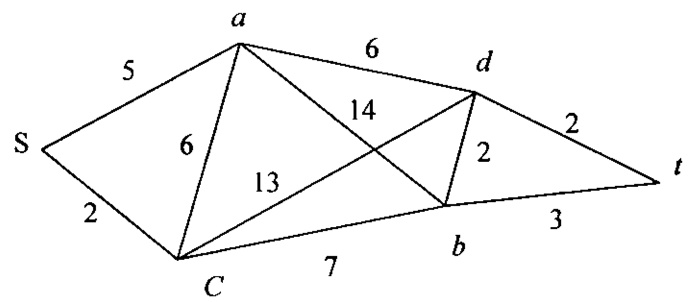
a) In a group of eleven students in your department is it possible for each student to shake hands with exactly five other students? Justify.

Define complete graph with degree sequence (2, 2, 3, 4, 5, 6) exists.

b) Examine whether the following two graphs G_1 and $G2$ are isomorphic. Define isomorphic graphs.



c) Using Dijkstra's algorithm, find the lengths of the shortest paths from the source vertex s to a, b, c, d and t of the following weighted graph.



Show that every connected graph has at least one spanning tree. 3+2

d) Show that the total number of different Hamiltonian circuits (or cycles) in a complete graph K_n is $\frac{1}{2}[(n-1)!]$

Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit. 3+2

e) Show that a graph having degree sequence (1, 3, 3, 3, 5) has spanning tree. Give a pictorial representation of this graph with a spanning tree.

Let T be a tree having even number of edges. Show that T must have at least one even vertex. 3+2

f) Draw a graph whose incidence matrix is given below.

$$I(G) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Define eccentricity and radius of a graph. Find the nature of the digraph represented by the relation ρ on the set $A=\{1,2,3,4,\ldots\ldots,10\}$ where, $\rho=\{(a,b)\in A\times A:5|2a+3b\}$

Group-C

4. Answer any one question: **10**

a) Show that a bipartite graph does not contain odd cycle.

Find the smallest positive integer n such that the complete graph K_n has at least 500 edges.

Define pseudo graph. Let G be a graph of order $n\geq 2$ such that $d(v)=\frac{n-1}{2}$ for all v in G. Then show that $d(G)\leq 2$

(3+3+1+3)

b) i) Solve the following Travelling Salesman Problem

Towns	A	B	C	D	E
A	-	30	17	11	22
B	30	-	19	16	11
C	17	19	-	17	9
D	11	16	17	-	23
F	22	11	9	23	-

ii) Show that a tree with n vertices has $(n-1)$ edges. 5+5