2022

MATHEMATICS

[HONOURS]

(B.Sc. Sixth Semester End Examination-2022)

PAPER-MTMH C-601

Full Marks: 60 ***Time: 03Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[Graph Theory – III]

1. Answer any ten questions: **10x2= 20**

a) Consider the group $(\mathbb{Z},+)$ Show that $\mathbb{Z}\times\mathbb{Z}$ is not a cyclic group.

b) Find the number of elements of order 9 in $Z_3 \times Z_9$.

c) Define internal direct product of two groups with an example.

d) Find all abelian group of order 800 upto isomorphism.

e) Let G_1, G_2 be two groups. Show that $Z(G_1 \times G_2) = Z(G_1)\times Z(G_2)$ where $Z(G)$ is the center of G.

f) What do you mean by group acting on itself by conjugation?

g) Define orbit and stabilizer.

h) If H is normal in G and P is a sylow p-subgroup of H, then show that $G = N_G(P)H$.

i) If every sylow subgroup of a group G is normal and abelian, then show that G is abelian.

j) Let H and K be normal subgroups of G such that $H \cong K$. Is $\frac{G}{H} \cong \frac{G}{K}$?

k) Find all the non-isomorphic abeliam groups of order 20.

l) Define Kernal of an action.

m) Show that there is no simple group G of order 216.

n) Let G be a group and S=G. Show that $*$ defined by $a * x = ax\overline{a}^1, a, x \in G$ is a group action.

o) Let G act on G by Conjugation i.e., $g * a = ga\overline{g}^1, a, g \in G$ then show that $Ker(*) = Z(G)$

2. Answer any four questions: 4x5 = 20

a) Let G_1, G_2 be two finite cyclic groups of order p,q respectively. Show that $G_1 \times G_2$ is a cyclic group iff $gcd(p,q) = 1$

b) Let G be a group and S be a non-empty set. Show that every action of G on S determine a homomorphism G to A(s).

c) If H, K are normal subgroups of G, Show that $\frac{G}{H \cap K}$ is isomorphic to a subgroup of $\frac{G}{H} \times \frac{G}{K}$.

d) Show that $Z_{mn} \simeq Z_m \times Z_n$ iff $gcd(m,n) = 1$

e) Let G be a finite group and H be a subgroup of G with [G:H]=n and $\|G\| \dagger n!$. Then show that G has a non-trivial normal subgroup.

f) Let G be a finite group and P be a prime number. Show that G is a p-group iff $|G| = P^n$ for $\mathbb{N} \cup \{O\}$.

3. Answer any two questions: 2x10 = 20

a) i) Prove that the number of sylow p-subgroups of G is of the form $1+kp$ where $(1+kp)|o(G), K$ being a non-negative integer.

ii) Let G be a group and S be a non-empty set. Show that every action of G on S determine a homomorphism G to $A(S)$ 6+4

b) i) Let G be a group and $H \leq G$ and $S = \{aH : a \in G\}$ and $A(S)$= the group of all permutations on S. Then show that there exist a homomorphism $\psi : G \to A(S)$ such that ker $\psi \subseteq H$.

ii) Let G be a group of order p^2 where p is a prime number. Show that G is an abelian group. 7+3

c) Let $O(G)$=30, Show that

i) Either Sylow 3-Subgroup or Sylow 5-subgroup is normal in G.

ii) G has a normal subgroup of order 15.

iii) Both Sylow 3-subgroup and Sylow 5-subgroup are normal in G.
