2022

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

[P.G.]

(M.Sc. Second Semester End Examination-2022)

PAPER-MTM 203

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

UNIT – I

Marks - 25

[Abstract Algebra]

Attempt Question No. 1 and any two from rest:

1. Attempt any **two** questions: $2 \times 2 = 4$

a) Show that the symmetric group S_3 has trivial centre.

b) Show that the abelian group of order 15 is cyclic.

c) Prove that if $|G| = 2907$, than G is not simple

d) Give an example of an infinite quotient group.

2. a) Let , K be a finite extension of F and L, a finite extension of K. Then show that L is a finite extention of F and

$$[L:F] = [L:K][K:F]$$

b) Let, G be a group and $O(G)$ 108. Show that there exists a normal subgroup of order 27 *or* 9. 4+4

3. a) Find the number of elements of order 5 in the group $Z_{15} \times Z_{10}$

b) Let, G be a group. Then show that Inn (G) is a normal subgroup of Aut (G). 4+4

4. a) suppose G be a group of order pq where p and q are primes with p<q . If p does not divide $(q-1)$, then show that G is cyclic and is isomorphic to $\mathbb{Z}_{pq}$.

b) If G is an abelian group living subgroups $H_1, H_2, \ldots, H_t$ such that $|H_i \cap H_j| = 1$ for all $i \neq j$, then K = $H_1 . H_2 \ldots H_t$ is a subgroup of G of order $|H_1| \times |H_2| \times \ldots \times |H_t|$ and $K \cong H_1 \times H_2 \times \ldots \times H_t$ 5+3

[Internal Asssment-5]

UNIT – II

Marks - 25

[Linear Algebra]

Attempt Question No. 1 and any two from rest:

1. Attempt any **two** questions: **2 × 2 = 4**

a) Let, W be the subplace of $\mathbb{R}^3$ spanned by (1,1,0) and (0,1,1) . Find a basis of the annihilator of W.

b) Find the minimal polynomial of the matrix $\begin{vmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ 3 & 6 & 4 \end{vmatrix}$

c) If $\{\alpha_1, \alpha_2, \alpha_3\}$ is a basis of $\mathbb{R}^3$, where . $\alpha_1 = (1,-1,3), \alpha_2 = (0,1,-1), \alpha_3 = (0,3,-2)$ Then find its dual basis.

2. a) prove that a necessary and sufficient condition that an nxn matrix A over F be diagonalizable is that A as n liunearly independent eigen vectors in $V_n(F)$.

b) Let, P_2 be a family of polynomials of 2 almost. Define an inner product on P_2 as $\langle f(x)/g(x) \rangle = \int_0^1 f(x)g(x)dx$ Let $\{1, x, x^2\}$ be a basis of the inner product space P_2. Find out an orthonormal basis from the basis. 4+4

3. a) Consider the vector space P_n of real polynomials in x of degree less than or equal to n. Define $T: P_2 \to P_3$ by $(Tf)(x) = \int_0^x f(t)dt + f'(x)$. Then obtain the matrix represent tation of T with respect to the bases $\beta = \{1, x, x^2\}$ and $\beta^1 = \{1, x, x^2, x^3\}$

b) Let, V be a vector space of dimention 6 over R and T be a linear operator whose minimal polynomial is $g(x) = (x^2 - 2x + 3)(x - 2)^2$. Then explain all the possible canonical forms

4. a) Alinear operator on $\mathbb{R}^2$ is defined by $T(x, y) = (x + 2y, x - y)$ Find the adjoint, i.e., T^* if the inner product is standard one. If $\alpha = (1,3)$ find $T^*(\alpha)$

b) Suppose T is a linear operator on an inner product space. Then T is normal if and only if its real and imaginary parts commute.

c) Give an example of two self-adjoint transformations whose product is not self-adjoint. 3+3+2

[Internal Assssment-5]