2022
MATHEMATICS
[HONOURS]
(B.Sc. Sixth Semester End Examination-2022)
PAPER-MTM-DSE-602
[BIO-MATHEMATICS]

Full Marks: 60 ***Time: 03Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

1. Answer any ten questions: **2x10= 20**

a) If the system $\frac{dx}{dt} = x\left(1-x^2\right) - a\left(1-e^{-bx}\right)$ undergoes transcritical bifurcation, then obtain the relation between the parameters a and b.

b) Discuss about the bifurcation that occur in $\frac{dx}{dt} = ax - x^3$.

c) What do you mean by "bifurcation" of a system?.

d) Find the nature and stability of the fixed points of $\dot{x} = -ax + y, \dot{y} = -x - ay$. for different values of parameter.

e) Write down the assumptions of Malthusian population model

f) When Logistic model cannot be used to describe a population model?

g) Describe discrete Predator – Prey population model.

h) Find the range of r for which the population model $x_{n+1} = rx_n(1 - x_n)$ is stable.

i) If the contact rate be 0.002 per population per weak and the initial susceptible population is 2500 then find the susceptible and infected population after 3 weeks.

j) Draw the cobweb diagram of the population model $x_{n+1} = rx_n$ when $r < 1$

k) Describe Nicholson – Bailey model..

l) Let x^* be a fixed point of the difference equation $x_{n+1} = rf(x_n), X(t_0) = X_0$. When x^* is said to be stable and attracting.

m) Define intertopic and intratropic predation.

n) What do you mean by environmental carring capacity of an environment?

o) Let x(t) denote the size of an animal population at time t. Suppose that the growth rate in this population is proportional to the size, with the constant of proportionality equal to -0.01 per year. If the initial population is 2000 then find the time after which population to be 1000.

Answer any four question: **4x5 = 20**

2. Consider the system of equations $\frac{dx}{dt} = 1 - (a+1)x + bx^2 y$

$$\frac{dy}{dt} = ax - bx^2 y$$

where a and b are positive parameters. Then linearize the system about its critical point.

3. Consider the system of equations $\dot{x} = y, \dot{y} = x^2 - y - \alpha$ where α is a parameter. Show the system observes saddle-node bifurcation with respect to the parameter α .

4. Consider the linear system $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ where a, b, c and d are positive constants. Given that two the eigen values of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ are real, unequal and have same sign. Then discuss in details about the nature of the equilibrium point.

5. Find the population at time t of a population model which satisfies the differential equation $\frac{dx}{dt} = bx - dx + e - f$ where b, d, e, f>0 What will be the behaviour of the population in long run when b< d and f>e, and also state under what condition total population will be dies out. 3+1+1

6. The population model will be describe by the equation $\frac{dx}{dt} = ax - bx^2$ where $a, b > 0$

i) What are the physical significance of first and second term of right hand side of the expression?

ii) Find the population at time t

iii) What will be the maximum population in long run

3+1+1

Answer any two questions: **2x10 = 20**

7. Find the equilibrium points and stability of the model $x_{n+1} = rx_n(1 - x_n)$ and also find the value of r for which bifurcation occurs. 2+2+1

8. a) Describe a logistic population model with constant harvesting rate and also find the condition under which population will be extinct. Find the equilibrium points and discuss their stability.

b) Find the non-negative equilibrium point a population model governed by the equation $x_{n+1} = \frac{2x_n^2}{x_n^2 + 3}$ and check the stability

(2+3+2)+3

9. a) Describe a predator-prey model with necessary assumptions. Find solution under different cases when prey population exists only or predator population exists only or both population exist. Interprect the solution graphically when both population exist

and indicate the point where maximum prey and predator population exist

b) What is the Allee effect in a population model and also its general characteristics. (2+5)+(2+1)

10. a) Write down the Susceptible-Infected epidemic model with necessary assumption. Find the solution and interpret it.

b) A population model is governed by the equation $\frac{dx}{dt} = x\left(e^{4-x} - 1\right)$. Find all equilibrium points and discuss their stability. (2+5)+(2+1)
