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2022

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

[P.G.]

(M.Sc. Fourth SemesterEnd Examination-2022)

PAPER-MTM 401

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[FUNCTIONAL ANALYSIS]

Group - A

Answer anytwo questions from the following: 2x10= 20

1. a) Show that the set of all convergent sequences is a normed linear space with norm . $\|\{x_n\}\| = \underset{n}{Sup}|x_n|$

   b) Let χbe the linear space of all bounded real continuous real functions $f(x)$ defined on the closed interval $[a,b]$. Show that the mapping $T : X \to \Box$ defined by $T(f) = \int_a^b f(x)dx$ is a linear functional. 6+4

2. a) Show that for any bounded linear operator $T : X \to Y$

$\underset{x \neq 0}{Sup} \frac{\|Tx\|}{\|x\|} = \underset{x \neq 0}{Sup} \|Tx\|$ where X and Y are normed linear spaces.

b) Let the integral operator $T : C[0,1] \to C[0,1]$ be defined by

$Tx(t) = \int_0^t K(t,s).x(s)dx$ where $K(t,s)$ is a given continuous function on the closed sequence $[0,1]\times[0,1]$

Show that there exist a real constant M such that $\|Tx\| \leq M\|x\|$ for all $x \in C[0,1]$ 4+6

3. a) Let X any Y be Banach spaces and $A \in BL(X,Y)$. Show that there is a constant $c > 0$ such that $\|Ax\| \geq c\|x\|$ for all $x \in X$ if and only if $Ker(A) = \{0\}$ and $Ran(A)$ is closed in X.

b) Let the space $l^2(□)$ be defined as the space of all two-sided squaresummable sequences and the bilateral shift is the operator W on $l^2(□)$ defined by

$W(\ldots, a_{-2}, a_{-1}, \hat{a}_{-1}, a_1, a_2, \ldots) = (\ldots, a_{-3}, a_{-2}, \hat{a}_{-1}, a_0, a_1, \ldots)$ Prove that W is a unitary, 5+5

4. a) Let X and Y be normed linear spaces and Let $T : X \to Y$ be a linear continuous transformation. Show that the kernel of T is a closed linear subspace.

b) If X is a normed space, M is a closed subspace of $X, x_0 \in X \setminus M$ and $d = dist(x_0, M)$, show that there is an $f \in X^*$ such that $f(x_0) = 1, f(x) = 0$ for all $x \in M$ and $\|f\| = d^{-1}$ 5+5

Group - B

Answer anytwoquestions: **2x6 = 12**

5. a) Define parallelogram law in an inner product space.

b) Show that parallelogram law does not hold in $C[a,b]$ 2+4

6. a) Define unitary and isometric operators.

b) Show that a bounded linear operator T on a complex Hillbertspace X is unitary iff T is isometric and surjective.

2+4

7. a) Let $X = C[0,1]$ with the supremum norm. Consider the sequence $x_n(t) = \frac{t^n}{n^3}$, $t \in [0,1]$ Check whether the series $\sum_{n=1}^{\infty} x_n$ is summable in X.

b) State the relation between the continuous property.and boundedness property of a linear function. 4+2

8. a) Let Y be a normed space and Y_0 be a dense subspace of Y .Suppose Z is a Banch space and $T \in BL(Y_0, Z)$. Prove that there exist a unique $\tilde{T} \in BL(Y, Z)$ such that $\tilde{T} | Y_0 = T$

b) Let $T \in BL(H)$ be self –adjoint. Show that $Ker(T) = Ker(T^*)$ 4+2

Group - C

Answer any Four questions of the following: **4x2 = 8**

9. If X and Y are normed linear spaces then state the Condition on the space Y so that $B(x,y)$ becomes a Banach space.

10. Let X and Y be normed spaces. If X is finite dimensional, then show that every linear map from X to Y is continuous.

11. Prove that $T^{*}T$ is a positive operator.

12. If M is a closed subspace of a Hillbest space X and $x \in X$ then we know there exists element y in M and Z in $M^{\perp}$ such that $x = y + z$. Prove that such decomposition is unique.

13. Let $\{e_i\}$ be a nonvoidorthonormat Set in an inner product space X. State the condition on $\{e_i\}$ so that Bassels inequality reduces to Parseval's identity

14. Let X be a normed space. Show that $x_n \to x$ weakly in X does not imply $x_n \to x$ in X in general.

15. Let $\{x_n\}$ be a weakly Convergent sequence in a normed linear space X and x be the weak limit of $\{x_n\}$. Prove that this weak limit is unique.

16. Let X be a normed space and $\Phi(x) = 0$ for every $\Phi \in X^{*}$. Show that $x = 0$

Internal Assessment - 10