2021

Applied Mathematics with Oceanology and Computer Programming

[P.G.]

(CBCS)

(M.Sc. First Semester EndExaminations-2021)

MTM – 102

(*COMPLEX ANALYSIS*)

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Attempt Question No. 1 and any four from the rest:

1. Attemptany FOUR questions **4x2=8**

a) Is $f(z)=|z|^2$ analytic ?

b) Determine the region of *w*-plane when the region bounded by $x=0, y=0, y=3$ in *z*-plane under the map $w=z\sqrt{2}e^{i\pi/4}$

c) Evaluate $\int_C \frac{e^z}{z^2(z+1)^3}dz$ where $C: 9x^2+4y^2=36$

d) It is possible to evaluate the integral $\int_C f(z)dz$ where $f(z)=\dfrac{5z+2}{z(z-2)}$ and $C:|z|=1$ using the single residue of $\dfrac{1}{z^2}f\left(\dfrac{1}{z}\right)$ at $z=0$? justify.

e) Expand $f(z)=\dfrac{1}{z}$ as a series $z = 1$

f) What kind of singularity has the function $f(z)=\left(z^2+4\right)e^{-z}$ at $z=\infty$

2. a)Let $u=x^2-y^2$ and $v=-\dfrac{y}{x^2+y^2}$. Is $f(z)=u+iv$ analytic function ? Justify your answer. Also examine whether u and v are harmonic or not.

b) In the transformation $z=\dfrac{i-w}{i+w}$, show that half of w-plane given by $v\geq 0$ corresponds to the circle $|z|\leq 1$ in z-plane. 4+4

3. a) Show and prove Morera's theorem.

b) The only singularities of a single valued function $f(z)$are poles of order 1 and 2 at z = -1 and z = -2, with residues at these poles 1 and 2 respectively. If $f(0)=\dfrac{7}{4}, f(1)=5/2$ determine $f(z)$. 4+4

4. a) Apply the calculus of residues to evaluate the integral $\int_0^{\alpha}\dfrac{dx}{\left(x^2+4\right)^3}$

b) If$f(z)$ has a pole at z = a then prove that $|f(z)|\to\infty$ as $z\to a$

5+3

5. a) Let C_R denotes upper half of the circle $|z|=r>3$ taken in the counter clockwise direction. Show that

$$\left|\int_{C_R} f(z)dz\right|\leq\frac{\pi r\left(3r^2+1\right)}{\left(r^2-4\right)\left(r^2-9\right)}$$

b) If the mapping $w=f(z)$ is conformal then show that $f(z)$ is an analytic function of z. 4+4

6. a) If $f(z)=u+iv$ is an analytic function and $u-v=\dfrac{-\cos x+\sin x-e^y}{2\cosh y-\cos x}$ find $f(z)$ when $f(\pi)=1/2$

b) Evaluate the integral $\int_C\dfrac{f(z)+f(-1/z)}{(z-i)^2}dz$ where C is the simple close contour $|z-i|=\dfrac{1}{2}$ in counter clockwise sense and $f(z)$ is analytic in $|z-i|\leq 1$. 4+4

7. a) If a function $f(z)$ is analytic for all finite values of z and as $|z| \to \infty |f(z)| = A|Z|^k$, then prove that f(z)is a polynomial of degree less and equal to k.

b) Represent the function $f(z) = \dfrac{4z+3}{z(z-3)(z+2)}$ in Laurent's series when $2 < |z| < 3$ 4+4

[Internal Marks – 10]