2021

Applied Mathematics with Oceanology and Computer Programming

[P.G.]

(CBCS)

(M.Sc. First Semester EndExaminations-2021)

MTM – 105

(*CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS*)

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Attempt Question No. 1 and any four from the rest:

1. **Attemptany FOUR questions** **4x2=8**

   a) Define cyclic coordinate and give one example.

   b) Obtain the mathematical expression of D'Alembert's principle.

   c) Differentiate between Lagrange formulation and Hamilton formulation.

   d) What do you mean by bifurcation of a system ?

e) Prove that, if the transformation does not depend explicitly on time then the Hamilton represents the total energy.

f) Suppose a rigid body is rotating about a fixed point. Prove that the kinetic energy is conserved throughout the motion.

2. a)Show that with respect to a uniformly rotating reference frame Newton's second law for a particle of mass m acted upon by real force $\vec{F}$ can be expressed as

$\vec{F}_{eff} = \vec{F} - 2m\vec{w}\times\vec{V}rot - m\vec{w}\times(\vec{w}\times\vec{r})$

b) The Hamiltonian of a dynamical system is given by $H = q_1p_1 - q_2p_2 - aq_1^{\,2} + bq_2^{\,2}$ where *a, b* are constants. Solve the problem. 5+3

3. a) Construct the Lagrangian and equations of motion of a coplanar double pendulum placed in a uniform gravitational field.

b) Show that the transformation $Q = \log\left(\frac{Sinp}{q}\right)$, $p = q\cot p$ is canonical. Find the generating function $G(q,Q)$. 4+4

4. a) Determine the parh for which the functional $\int_{-l}^{l}\left(\frac{1}{2}ay''^2 + by\right)dx$ subject to

$y(-l) = 0, y'(-l) = 0,\ y(l) = 0,\ y'(l) = 0$ is extremum.

b) Discuss 'time dilation' effect in special theory of relativity. 4+4

5. a) Prove that the equation of a curve for which surface area is minimum is a catenary $x = a\cosh\left(\frac{y-b}{a}\right)$, where a and b are constants.

b) A heavy bead of mass m is freely movable on a smooth circular wire of radius a which is a made to rotate about a vertical diameter with a spin w, prove that the action will be

$A = ma^2\int_{\theta_1}^{\theta_2}\left(\frac{2H}{ma^2} + \frac{2g}{a}\cos\theta + w^2\sin^2\theta\right)^{1/2} d\theta$ 4+4

6. a) Prove that the Poisson bracket of two constants of motion is itself a constant even when the constants depend on time explicitly.

b) What is the effect of the Coriolis force on a particle falling freely under the action of gravity ?. 4+4

7. a) Consider the following nonlinear dynamical system,

$$\dot{x} = x^2y - x^5$$
$$\dot{y} = y - x^2$$

Then study the stability at the origin.

b) Prove that phase volume is invariant under canonical transformation. 4+4

[Internal Marks – 10]