2021

Applied Mathematics with Oceanology and Computer Programming

[P.G.]

(CBCS)

(M.Sc. First Semester EndExaminations-2021)

MTM – 106

(*GRAPH THEORY*)

Full Marks: 25 ***Time: 01 Hr***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

1. **Answer any TWO questions** **2x2=4**

   a) Show that a graph G with degree sequence (2, 3, 4, 4, 5) is connected. Give a pictorial representation of this graph along with spanning tree. Find also the number of branches and number of chords of the spanning tree.

   b) Find thenumber of vertices of a 4-regular graph*G* with 10 edges.

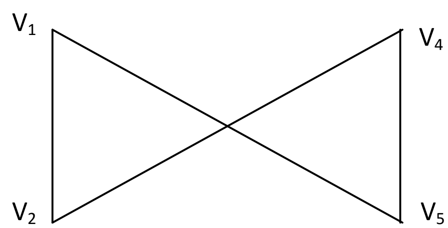
   c) Define fundamental cut-set of a graph *G*.

d) Draw the digraph G corresponding to adjacency matrix.

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

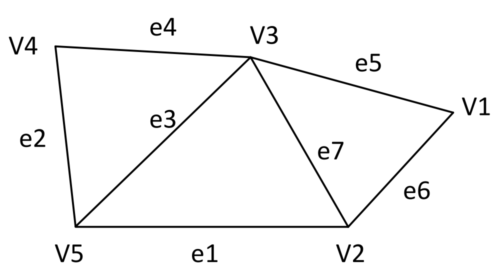
2. Answer any TWO questions **4x2=8**

a) Define walk, path and circuit with example. Consider the graph shown in figure, find the number of walks of length three from V_2 to V_4 and also check the connectedness of the graph.



b) Write the properties of dual graph. Find the geometrical dual of the graph given below.

1+3



c) Define spanning tree of a graph G. A tree has two vertices of degree 2, one vertices of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have ?

d) Define centre of a graph G. Show that every tree has either one or two centre.

3. Answer any ONE question **8x1=8**

a) i) Write down the statement of four-colour problem in graph theory.

ii) Define chromatic polynomial. Prove that every tree with two or more vertices is 2-chromatic.

iii) Prove that the chromatic polynomial of any cycle C_n of length n is $p_n(\lambda) = (\lambda - 1)^n + (-1)^n(\lambda - 1)$ 1+(1+2)+4

b) i) Define planar graph. State and prove Euler's theorem for a connected planar graph.

ii) If G is connected planar graph with $n(\geq 3)$ vertices and e edges, then prove that $e \leq 3n - 6$. Also, show that a simple connected planar graph with 6 vertices and 12 edges, each of the face is bounded by 3 edges. 4+4

[Internal Marks – 05]