2022

Mathematics

[Honours]

(B.Sc. Third Semester End Examination-2022)

PAPER-MTMH C301

(Ordinary differential Equations and applications of Dynamics)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

(Use separate answer script for each group)

Group-A

[Ordinary differential Equations]

1. Answer any eight questions $2 \times 8 = 16$

a. Solve differential equation $\frac{dy}{dx} + \frac{y}{x} logy = \frac{y}{x^2}(logy)^2$

b. Find the singular integral of the ODE $(x\dot{y} - y)^2 = x^2(x^2 - y^2)$

c. Find the particular integral of the differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2$

d. Solve $\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$

e. Find the differential equation of all circles in the first quadrant which touch the co-ordinate axes.

f. Solve $9x^2\ddot{y} + 3xy + y = 0, x > 0$

g. Find the integrating faster for

$(Cosy\,Sin2x)dx + (Cos^2y - Cos^2x)dx = 0$

h. If $x^{\alpha}y^{\beta}$ be the integrating factor of $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$ then find α and β

i. Locate and classify the singular points $x^3(x^2-1)\ddot{y} + 2x^4\dot{y} + 4y = 0$

j. Show that the $IVP: \frac{dy}{dx} = 5x^2y^{2/3}$, $y(0) = 0$ has non-unique solution

k. Solve the differential equation $sinpx\ cosy = cospx\ siny + p$

l. Calculate $\frac{1}{D-ia}Sec\,ax$ where $D \equiv \frac{d}{dx}$

2. Answer any two questions $2\times5=10$

a. Solve the differential equation $(D^4 + 2D^2 + 1)y = x^2cos^2x$

b. Solve by method of variation of parameter $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = logx$

c. Solve the system of equations :

$\frac{d^2x}{dt^2} - 3x - 4y = 0$, $\frac{d^2y}{dt^2} + x + y = 0$

3. Answer any one question $1\times10=10$

a. (i) Find the series solution of the differential equation

$x(1-x)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$, near x=0

(ii) Show that the point of infinity is a regular singular point of the equation $x^2\frac{d^2y}{dx^2} + (3x-1)\frac{dy}{dx} + 3y = 0$ 7+3

b. (i) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + xy = e^{e^x}$

ii) Given system of liner differential equation

$\frac{dx}{dt} = x + 3y$, $\frac{dy}{dt} = x - y$

(i) Find the general solution of this system

ii) Justify the stability and unstability of critical point

Group-B

[Applications of Dynamics]

4. Answer any two questions $2\times2=4$

a) A partical describes the path $r = a\tan\theta$ under a force to the origin. Find the velocity is terms of r.

b) If the radial and transverse velocities are proportional to each other then find the path.

c) A particle describes an equiangular spiral $r = ae^{\theta}$ in such a way that its acceleration has no radial component. Prove that its angular velocity is constant and the magnitudes of the velocity and acceleration are each proportional to r.

5. Answer any one question $2\times 2=10$

a) A particle describes the equiangular spiral $r = ae^{m\theta}$ with a constant velocity. Find the components of the velocity and acceleration along the radius vector and perpendicular to it.

b) If a body moves under a central force P in a medium with exerts a resistance equal to k times the velocity per unit of mass, prove that $\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}e^{2kt}$, where h is twice the initial moment of momentum about the centre of force.

c) One end of an elastic string of un-stretched length a, is tied to a point on a smooth table and a particle is attached to the other end and can move freely on the table. If the path be nearly a circle of radius b, then show that its apsidal angle is approximately $\pi\sqrt{\frac{b-a}{4b-3a}}$

6. Answer any one question $1\times 10=10$

i) Calculate the time of description of the planet of an arc of an elliptic orbit. 10

ii) Derive the equation of motion of a varying mass particle. A spherical raindrop of radius a cms, falls from rest through a vertical height h, receiving throughout the motion an

accumulation of condensed vapour at the rate of k grammes per square cm. per second, no vertical force but gravity acting. Show that when it reaches the ground its radius will be $k\sqrt{\frac{2h}{g}}\left[1+\sqrt{1+\frac{ga^2}{2hk^2}}\right]$. $3+7$