End Semester Examination, 2022

Semester - V (Hons.)

Physics

PAPER - DSE-1T

Full Marks : 60

Time : 3 Hours

The Figures in the right hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. *Answer any ten questions :* ***10x2=20***

a) If $\{p_i, q_i\}, \{q_i, p_i\}$ are the Lagranges brackets and $[p_i, p_j], [q_i, p_j]$ are the poissons brackets then prove the following identity. 2

$$\sum_{i=1}^{n} \{p_i q_j\} [p_i, p_j] + \sum_{i=1}^{n} \{q_i, q_j\} [q_i, p_j] = 0$$

b) Show that phase trajectory of one dimensional harmonic oscillator is an ellipse. 2

c) A bead moves on a circular wire. Specify the type of constraint. 2

d) A spaceship is receding from earth at a speed of 0.21c. A light from the spaceship appears as yellow $(\lambda = 589.3nm)$ to an observer on earth. What would be its colour as seen by the passenger of the spaceship ? 2

e) Find the relation between relativistic energy and momentum. 2

f) In a ventury meter, usually the length of the divergent outlet part is made longer compared to that of the converging inlet part. Why ? 2

g) If the kinetic energy of a particle of rest mass m_0 is equal to its rest mass energy. What is the momentum of the particle in units of m_0c.
Where c→ speed of light in vacuum. 2

h) Given that the linear transformation of a generalised co-ordinate q and the corresponding momentum p, $Q = q + 4ap$, $P = q + 2p$ is canonical, what is the value of the constant a ? 2

i) If the half life of an elementary particle moving with speed 0.9c in the laboratory frame is 5×10^{-8} sec, then what is the proper half life of the particle ? 2

j) Give the expression for angular momentum in terms of inertia tensor. 2

k) State the Bernouli's theorem. 2

l) Consider the Lagrangian $L=\frac{1}{2}(\alpha\dot{q}^2-\beta q^2)$, where α,β are constants. Find the Hamiltonian of the system. 2

m) What are the benefits of using Lagrangian and Hamiltonian mechanics over the Newtonian mechanics ? 2

n) What is four vector ? What is the difference between four vector and usual vectors ? 2

o) Why for small oscillation, the corresponding potential becomes harmonic in nature ? 2

Group - B

Answer any <u>four</u> questions from the following : 4 x 5 = 20

2. Consider a particle of mass m executing motion in 3D under a central force potential given by,
$V(r) = Br^a$
Where a and B are constants and r denotes the radial distance from the origin of the co-ordinate system. The angular momentum of the particle is J and the total energy is E.
a) Calculate the minimum permissible value of energy E for a given J.
b) Derive the expression for the radius of a stable circular orbit.
c) Calculate the frequency of small oscillation (radial) about the stable circular orbit. 2+2+1

3. Explain Hamilton Jacobi's method taking the example of a 1D harmonic oscillator. 5

4. Describe the Hamiltonian and Hamilton's equation of motion for a charged particle moving in an electromagnetic field. 5

5. What do you know by 'proper time' ? Show that the proper time is an invariant quantity. 2+3

6. A meson of mass Π comes to rest and disintegrate into meson of mass μ and neutrino of zero rest mass. Show that the kinetic energy of motion of μ meson is $T = \frac{(\Pi - \mu)^2}{2\Pi} c^2$. 5

7. i) Write down the difference between laminar flow and turbulent flow. 2

   ii) What do you mean by viscosity ? 1

   iii) What is continum hypothesis of fluid parcel ? 1

   iv) What is the definition of fluid ? 1

Group - C

Answer any <u>two</u> questions : **2x10=20**

8. i) Explain what is meant by "virtual displacement". 2

   ii) State the D'Alembert principle. 2

   iii) Write down the Lagrangian of a particle moving under a central force. Find the equation of motion. Is there any cyclic co-ordinate in the system ? 2+3+1

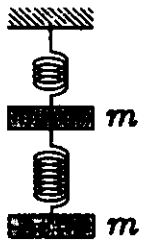
9. i) Write down the postulates of Special Theory of Relativity. 2

   ii) Consider the potential energy V(x) of a particle as given by $V(x) = 3x^4 - 8x^3 - 6x^2 + 24x$

   Determine the points of stable and unstable equilibrium. 3

iii) Two similar springs with spring constant k, hang vertically downyard from a rigid support with two equal masses m attached to them as shown in the figure. Show that the normal mode frequencies for small oscillation (vertically) are given by

$\omega\pm = \sqrt{\frac{k}{2m}(3 \pm \sqrt{5})}$. 5



Vertical motion of spring-mass system

10. a) Given , $\vec{L} = \vec{r} \times \vec{p}$ calculate the poisson brackets between the components of $\vec{L}$, viz, $\{Li, Lj\}_{P.B.}$

    b) A particle in 3D, is subjected to a potential of the form $V(\gamma, t) = \frac{\alpha(t)}{\gamma^6} + \frac{\beta(t)}{\gamma^{12}}$ Which of the following are conserved quantities ? Argue in each case.

    i) Energy of the particle.

    ii) Linear momentum of the particle.

    iii) Angular momentum of the particle.

    iv) The Laplace-Runge-Lenz vector. 6+4

11. A classical system with one degree of freedom is described by the lagrangian,

$$L = \frac{1}{2}mx^2 + \frac{a}{x} - \frac{b}{x^2}$$

Where both a and b are positive.

i) Obtain the Hamiltonian for this system.

ii) Find the value of x for which the system has a stable equilibrium point.

iii) Consider tiny perturbations about the equilibrium position and obtain the frequency of the small amplitude oscillations. 2+2+6