2022

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth Semester End Examinations-2022)

MTMH-DSE-502

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Probability and Statistics

1. Answer any ten questions: **10x2=20**

(a) If $Var(x)=1$, $Ver(y)=3$ and x, y are uncorrelated, then find the value of $Var(2x+3y)$

(b) Let us consider three events A, B, C in a random experiment. If A and B are independent and B and C are independent. Does it follow that A and C are independent? Justify.

(c) Prove that distribution of sample is a statistical image of the distribution of the population.

(d) Prove that correlation coefficient is independent of choice of origin and units of measurement

(e) For what values of θ and c, is the function f_i is given by

$$f_i = \begin{cases} \dfrac{c\theta^i}{i}, & \text{Is a positive integer} \\ 0, & otherwise \end{cases}$$

a possible probability mass function.

(f) A card is drawn from a pack and repeated 260 times. Find the probability of obtaining queen of hearts 4 times.

(g) Find the variance of χ^2 (n) distribution from its moment generating function.

(h) Show that the first absolute moment about the mean for the normal (m, σ) distribution is $\sqrt{\dfrac{2}{\pi}}\ \sigma$

(i) For the random experiment of tossing a coin, a random variable X is defined by $X(H)=0$ & $X(T)=1$. Find the probability distribution function of X.

(j) Prove that $\lim_{n-\infty} A_n = (x=a)$ where $A_n = \left(a - \dfrac{1}{n} < X \le a\right)$

(k) If F(x) be the distribution function of a random variable X, then show that $P(a \le X \le b) = F(b) - F(a-0)$

(l) Let P be a point on a line segment of length $2a$. Find the probability of the event $(AP:PB<K)$. Assume that P is uniformly distributed on AB.

(m) Let T_1 and T_2 be two statistics with $E(T_1)=2\theta_1+\theta_2$ and $E(T_2)=\theta_1-2\theta_2$. Find the unbiased estimators of θ_1 *and* θ_2

(n) Define Alternative hypothesis and null hypothesis.

(o) Give an example of a distribution where the random variables X and Y are uncorrelated although X and Y are dependent.

2. Answer any four questions: **4x5=20**

(a) Let $F(x)$ be a probability distribution function of a random variable X. Then show that $G(x)$ is also function where $G(x)=\dfrac{1}{2h}\int_{x-h}^{x+h} f(t)dt, \quad h\neq 0$.

(b) A straight line is drawn through a fixed point (λ,μ) $(\lambda>0)$ making an angle X which is chosen at random in the interval $(0,\pi)$ with the y-axis. Prove that the intercept on the y-axis, Y has a Cauchy distribution with parameter (λ,μ).

(c) The numbers $X_1, X_2 \ldots\ldots X_n$ are independently chosen at random in the interval (a, b). Prove that the probability density function of the random variable X=min{X_1, X_2,.....,X_n}is given by

$$f(x)=\frac{n(b-x)^{n-1}}{(b-a)^n},\quad a<x<b$$

d) A random sample of size 10 was drawn from a normal population with an unknown mean and variance 44.1. The observations are 65, 71, 80, 76, 78, 82, 68, 72, 65, 81. Find the 95% confidence interval for the population mean. $\left[\frac{1}{\sqrt{2\pi}}\int_{1.96}^{\infty} e^{-t^2/2}dt=0.025\right]$

(e) For a normal (m,σ) distribution, show that $\mu_{2k}=1.3.5........(2k-1)\sigma^{2k}, k=1,2,3....$ and $\mu_{2k+1}=0,\ k=0,1,2,....$ where μ_k be the kthorder central moment of the distribution.

(f) State and prove Tchebycheff's inequality

3. Answer any two questions: **2x10=20**

(a) (i) Let X be normally distributed. Then show that $\frac{nS^2}{\sigma^2}$ is χ^2 distributed with (n-1) degrees of freedom, where n is the size of the sample, S^2 is the sample variance and $\sigma^2=\text{var}(X)$.

(ii) For a pair of variables X and Y, the transformation $(X,Y)\to(U,V)$ given by the rotation of axes through a constant angle α i.e. $U=X\cos\alpha+Y\sin\alpha$, $V=-X\sin\alpha+Y\cos\alpha$. If U and V are uncorrelated then prove that

$$\sigma_u^{\ 2}+\sigma_v^{\ 2}=\sigma_x^{\ 2}+\sigma_y^{\ 2}\text{ and }\sigma_u\sigma_v=\sigma_x\sigma_y\sqrt{1-\rho^2}$$ 3+2+5

(b) (i) Let X is normally distributed with $E(x)=\mu$ and Var(x)=4. We are going to test the hypothesis Ho: $\mu=-1$ against the hypothesis $H_1:\mu=1$ on the basis of a sample of size $10: x_1,x_2,...,x_{10}$. If the critical region w is given by

$W=\{:(x_1,x_2,...,x_{10})|x_1+2x_2+3x_3+...+10x_{10}\geq 0\}$ then find the power of the test. Given that $\frac{1}{\sqrt{2\pi}}\int_0^{1.4}e^{-\frac{u^2}{2}}du=0.4192$

(ii) If (X, Y) has the general bivariate distribution, show that

$$\left\{\left(\frac{X-m_x}{\sigma_x}\right)^2-2\rho\left(\frac{X-m_x}{\sigma_x}\right)\left(\frac{Y-m_y}{\sigma_y}\right)+\left(\frac{Y-m_y}{\sigma_y}\right)^2\right\}/(1-\rho^2)$$

has χ^2 distribution with 2 degrees of freedom. 5+5

(c) (i) If a linear relation exists between two variables x and y, then prove that $r=\pm 1$ where r is the correlation coefficient.

(ii) Let $x_1, x_2, \ldots\ldots\ldots, x_n$ sample of size n drawn from a population with variance σ^2. Find the variance of the sample mean and mean of the sample variance. Give a suitable statistic that is unbiased estimate of population variance. 3+(3+3+1)
