M.Sc. First Semester End Examination, 2022

Applied Mathematics with Oceanology and Computer Programming

PAPER-MTM-101

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[REAL ANALYSIS]

Answer question no. 1 and any four from the rest

1. Answer any four questions: $2\times4=8$

a) Define σ - algebra with an example.

b) Define Borel set.

c) Evaluate $\int_{1}^{2} x^{5}\, d(|x|^{3})$

d) Show that Cantor set is a null set.

(e) Prove that every Cauchy sequence in a metric space is bounded. Is converse of the theorem is true? Justify.

(f) Is the subset S= $\{(x,y)\in\Re^2 : x^2+y^2=1 \; and xy\neq 0\}$ of $\Re^2$ compact? Justify your answer.

(g) If α is continuous function and β is of bounded variation on $[a,b]$ show that $\int_a^b \alpha \, d\beta$ exists.

2. (a) Let (X, d) be a metric space and $A \subseteq X$. If $p \in A'$ then prove that there exists a sequence of distinct points of A which is converging to *p*.

(b) Show that the sequence $\{x_n\}$ is convergent in $\Re^2$ with the Euclidean distance where

$$x_n = \begin{cases} (2^n, \frac{1}{n}), & n \le 9 \\ (2^{10}, -\frac{1}{n}), & n \ge 10 \end{cases}$$

4+4

3. (a) If f is continuous function and α is monotone on [a,b] then show that

$$\int_a^b f \, d\alpha = f(b)\,\alpha(b) - f(a)\,\alpha(a) - \int_a^b \alpha \, df$$

(b) Evaluate the R-S integral $\int_0^2 (3e^{4x} + 2x^2 - 7x + 5)\, d(2|x| + 7)$

5+3

4. (a) Let $x_1, x_2, \quad , x_n, \ldots\ldots$ be an enumeration of all rational points in [0, 1] and let $f:[0,1] \to \Re$ be defined by

$$f(x_n) = \begin{cases} \frac{1}{n^2}, & n = 1,2,3,\ldots\ldots\ldots \\ 0, & elsewhere \end{cases}$$

Prove that f is function of bounded variation on [0,1]

(b) Let $f:[a,b] \to \Re$ be a function of bounded variation on [a, b] and V(x) be the variation function of f on [a, b]. Then show that V+f is a monotonic increasing function on [a, b].

4+4

5. (a) Show that if a metric space X is compact then it is closed and bounded..

(b) Show that every finite sum of real numbers can be expressed as the R-S integral over some interval. 4+4

6. (a) Let μ be a positive measure on a σ algebra m. Then show that

$\mu(A_n) \to \mu(A)$ as $n \to \infty$ if A= $\bigcup_{n=1}^{\infty} A_n$,

$A_n \in m \; and \; A_1 \subset A_2 \subset A_3 \subset \ldots\ldots\ldots$

(b) Every bounded measurable function on [a,b] is Lebegue integrable on [a,b]. 3+5

7. (a) Give an example of a function which is not Reimann integrable but Lebegue integrable.

What is the value of the integral?

(b) Let $f:[0,1]\to\Re$ be defined by $f(x)=\begin{cases}1, & x\ be\ rational\\ 0, & x\ be\ irrational\end{cases}$.

Check whether function f is a function of bounded variation. 5+3

Internal Assessment - 10
