2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Third End Semester Examinations-2021)

MTMH-C301

Ordinary Differential Equations & Applications of Dynamics

Full Marks: 50 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group – A
[*Ordinary differential equation*]

1. **Answer any EIGHT questions:** **8x2=16**

a) Determine the differential equation by climinating the parameters a and b from the primitives $xy = ae^{x} + be^{-x}$

b) Solve the differential equation $(4x^3y^3 - 2xy)dx + \left(3x^4y^2 - x^2\right)dy = 0$

c) Show that the equation $\frac{dy}{dx}=\frac{3y}{2x}$, $y(0)=0$ has more than one solution.

d) Find the general solution of the equation $(3D^2+2D-8)y=5\cos x, D\equiv\frac{d}{dx}$

e) Show that the point of infinity is a regular singular point of the differential equation. $x^2\frac{d^2y}{dx^2}+(3x-1)\frac{dy}{dx}+3y=0$

f) Find the particular integral of the differential equation $(D^3-5D^2+8D-4)y=e^{2x}+e^x+3e^{-x}$

g) Locate and classify the singular points of the differential equation $x^3(x-2)\frac{d^2y}{dx^2}-(x-2)\frac{dy}{dx}+3xy=0$

h) If $y=\frac{ax+b}{cx+d}$ (a, b, c, d are arbitrary constants) then prove that $2\frac{dy}{dx}.\frac{d^3y}{dx^3}=3\left(\frac{d^2y}{dx^2}\right)^2$

i) If $\frac{d\gamma}{d\theta}+2\gamma\tan\theta=Sin\theta$ then find the maximum value of γ when $\gamma=0$ for $\theta=\pi/3$

j) $x^\alpha y^\beta$ be the integrating factor of $2ydx-3xy^2dx-xdy=0$, then find the value of α and β

k) Is x *Sinx* and x *Cosx* the solution of the differential equation $(D^4+2D^2+1)y=0$? If so find its general solution.

l) State the existence and uniqueness theorem of the differential equation $\frac{dy}{dx}=f(x,y), y(x_0)=y_0$

2. Answer any twoquestions **5x2=10**

a) Solve the differential $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$ by the method of variation of parameter

b) Show that the point nofinifinity is an irregularsingular point of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$

c) Solve the following set of simultaneous equation

$$\frac{dx}{dt}=3x-y$$

$$\frac{dy}{dt}=4x-y$$

3. Answer any ONE question **10x1=10**

a) i) Find the series solution of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-1)y=0$ near the point$x$=0

ii) Find the particular integral of the differential equation $\left(D^2-7D+6\right)y=\left(x-2\right)e^x$ 8+2

b) i) Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^{-x}\,Sec\,2x$

ii) Find the general solution of the equation $x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=4\log x$ 5+5

Group – B

[*Application of Dynamics*]

4. Answer any TWO questions **2x2=4**

a) Aparticle describes $\sqrt{r}Cos\frac{\theta}{2}=\sqrt{a}$ such that its cross radial velocity is constant, show that $\ddot{r}$ is constant

b) A point moves along the arc of a cycloid in such a manner that the tangent at it rotates with constant angular velocity; find the acceleration of the moving point.

c) Aparticle describe a central orbit $r^n=a^n\,Sin\,n\vartheta$ under a force directed towards the pole. Then find the velocity at any point.

5. Answer any TWO questions **5x2=10**

a) A particle moving under a constant central force from the centre is projected in a direction perpendicular to the radius vector with a velocity acquired in falling to the point of

projection from the centre. Show that the equation of the path is $a^3=r^3\cos^2\frac{3}{2}\theta$

b) If a body moves under a central force P in a medium which exerts a resistance equal to k times the square of the velocity per unit of mass, prove that $\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2}e^{2ks}$, where h is twice the initial moment of momentum about the centre of force.

c) Find the apsidal angle when a particle is describing a path under a central force μu^n which is nearly a circle.

6. Answer any ONE question **1x10=10**

a) Calculate the time of description of the planet of an are of an elliptic orbit.

b) A ball of mass m is moving under gravity in a medium which deposits matter on it at a constant rate ρ. show that the equation to the trajectory referred to horizontal and vertical axes through a point on itself, may be written in the form $k^2uy=kx\left(g+kv\right)+gu\left(1-e^{\frac{kx}{u}}\right)$, where u,v are horizontal and vertical velocities at the origin and $mk=2\rho$

[The End]