2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth End Semester Examinations-2021)

MTMH-C502

Full Marks: 60 ***Time: 02 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Group - A

Partial differential equation

1. Answer any SIX questions: **6x2=12**

a) Define 'Domain of dependence' of one dimensional wave equation.

b) Find a solution of the P.D.E. $u = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2$

c) Classify the P.D.E $y^3 u_{xx} - \left(x^2 - 1\right) u_{yy} = 0$

d) Consider the P.D.E $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x \le 1, t > 0$

If $u\left(\frac{1}{3},0\right) = 1,\ u\left(\frac{1}{3},\frac{2}{3}\right) = \frac{1}{2},\ u\left(0,\frac{1}{3}\right) = \frac{3}{4}$

then find $u\left(2/3,\frac{1}{3}\right)$

e) Let, $u(x, t)$ be solution of the P.D.E

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ -\infty < x < \infty, t > 0$$

IC: $u(x,0) = \cos x,\ u_t(x,0) = 1$

Then find $u\left(\pi,\frac{\pi}{2}\right)$

f) Define complete solution and singular solution

g) Give the geometric interpretation of Language's solution of a linear partial differential equation

h) Solve the P.D.E px + qy =z

i) Eliminate the arbitrary constants a and b from $z = ax + by + a^2 + b^2$ and obtain the P.D.E.

2. Answer any TWO questions **2x5=10**

a) Obtain the equivalent canonical form of the equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ and hence solve it.

b) Solve the P.D.E :

$$\left(x^3 + 3xy^2\right)p + \left(y^3 + 3x^2 y\right)q = 2z\left(x^2 + y^2\right)$$

c) Obtain the characteristics of the equation $pq = xy$ and determine the integral surface which passes through the curve $z = x,\ y = 0$

3. Answer any TWO question **10x2=20**

a) i) Find the integral surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 0,\ x + z = 2$

ii)Find the solution of the Cauchy problem 6+4

$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0,\ -\infty < x < \infty,\ t > 0$ subject to the initial conditions $u(x,0) = \begin{cases} x+1, if\ -1 \le x \le 0 \\ 1-x,\ if\ 0 \le x \le 1 \\ 0\ ;\ elsewhere \end{cases}$

and $u_t(x,o) = \begin{cases} 1, if\ -1 \le x \le 1 \\ 0,\ elsewhere \end{cases}$

b) i) Find the solution of the heat equation $\frac{\partial T}{\partial t} - 3\frac{\partial^2 T}{\partial x^2} = 0,\ 0 \le x \le 2,\ t > 0$ subject to the boundary conditions $T(0,t) = T(2,t) = 0,\ t \ge 0$ and initial condition $T(x,0) = x,\ 0 \le x \le 2$ where $T(x,t) < \alpha$ as $t \to \infty$ 5

ii) Find the complete and singular solution of $2xz - px^2 - 2qxy + pq = 0$ 5

c) i) Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx},\ 0 < x < 2,\ t \ge 0$ under the following conditions

i) $u(0,t)=0,\ u(2,t)=0$

ii) $u(x,0)=Sin^3\frac{\pi x}{2},\ u_t(x,0)=0$ 5

ii) Solve the Cauchy's problem

$u_{tt}-c^2u_{xx}=x+t$ with the initial conditions

$u(x,0)=0,\ u_t(x,0)=Sinhbx$ 5

Group - B

Metric Space - II

4. Answer any Four questions: 4x2=8

a) Show that the function $f(x)=\frac{1}{x}$ mapping the real line into it self is continuous evere where on the real line except at the origin.

b) If in a matric space (X,d) the distance between two sets A and B is a positive real number then show that the sets are separated.

c) Show that the collection $A=\{A_n=(-n,n):n\in\mathbb{N}\}$ is an open cover of the real line.

d) Define sequential compactness of a metric space with an example.

e) Define compact matric space with an example.

f) Define component of a metric space with an example.

5. Answer any ONE question 10x1=10

a) i) Prove that continuous image of a connected set is connected.

ii) State and prove the Banach's fixed point theorem. 5+5

b) i) Show that the composition of two continuous functions is continuous.

ii) State and prove Heine-Borel theorem. 2+(2+6)

[The End]