2021

Mathematics

[HONOURS]

(CBCS)

(B.Sc. Fifth End Semester Examinations-2021)

MTMH-DSE-501

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary

Linear Programming Problem

1. Answer any TEN questions: 2x10=20

a) Explain what is meant by a linear programming problem.

b) Do the vectors (1,2,3), (4,5,6) and (3,6,9) form a basis for E^3?

c) State Fundamental theorem of LPP.

d) Under what condition on LPP will have unbounded solution and infinite solution ?

e) Define hyper plane and convex set.

f) What are the difference between transportation problem and assignment problem ?

g) Express $x=[4,5]$ as a linear combination of $a=[1,3]$, $b=[2,2]$

h) Describe a two-person Zero Sum game.

i) Explain the following terms :

i) Pure strategies ii) Mixed strategies

j) For what value of K, the game with the following pay off matrix is strictly determinable ?

Players B

$$\text{Player A} \quad \begin{matrix} & B_1 & B_2 & B_3 \\ A_1 & k & 6 & 2 \\ A_2 & -1 & k & -7 \\ A_3 & -2 & 4 & k \end{matrix}$$

k) Show that the feasible solution $x_1=1, x_2=1, x_3=0$ and $x_4=2$ to the system :

$$x_1+x_2+x_3=2$$
$$x_1+x_2-3x_3=2$$
$$2x_1+4x_2+3x_3-x_4=4$$
$$x_i \geq 0, i=1,2,3,4$$

is not basic.

l) Find the half spaces defined by the hyperplane $2x_1+3x_2+4x_3+5x_4=7$ in which the points (1,2,3,4) and (1,2,3,-4) lie.

m) Find the initial basis feasible solution of the transportation problem by North-West- corner method.

	Destination					a_i
Origin	2	11	10	3	7	4
	1	4	7	2	1	8
	3	9	4	8	12	9
B_j	3	3	4	5	6	

n) Find the dual of the following problem

Max $Z=6x_1-3x_1$

Sub to $3x_1+2x_2 \leq 25$

$x_1 \geq 4$ $x_1, x_2 \geq 0$

o) Find the Extreme point if any of the set

$$X=\{(x,y)/|x| \leq 1, |y| \leq 1\}$$

2. Answer any FOURquestions **4x5=20**

a) Solve graphically the following L.P.P

Minimize. $z=2x_1+3x_2$

Subject to $5x_1+9x_2 \leq 45$

$2x_1+3x_2 \geq 6$

$x_2 \leq 4$, and $x_1, x_2 \geq 0$

b) Prove that the set of all feasible solutions of an L.P.P. is a convex set :

c) Use duality to solve the following L.P.P

$Max\, Z = 3x_1 + x_2$

Subject to $x_1 + x_2 \le 1$

$2x_1 + 2x_2 \ge 2, x_1, x_2 \ge 0$

d) Determine the optimal basic feasible solution to the following transportation problem :

	1	2	3	4	a_i
1	2	3	11	7	6
2	1	0	6	1	1
3	5	8	15	9	10
B_j	7	5	3	2	

e) Using dominance principle, solve the game whose pay-off matrix is given by

	Player B			
Player A	1	7	3	4
	5	6	4	5
	7	2	0	3

f) Define optimal strategies in a $m \times n$ matrix game. Show that the set of optimal strategies for each player is a matrix game in a convex set.

g) Solve the following L.P.P. by using two-phase simplex method.

$$\begin{aligned} \text{Min} \quad & z = x_1 + x_2 \\ \text{Subject to} \quad & 2x_1 + 4x_2 \ge 4 \\ & x_1 + 7x_2 \ge 7 \\ & x_1, x_2 \ge 0 \end{aligned}$$

3. Answer any TWO questions **10x2=20**

a) i) If for any basic feasible solution X_B of an L.P.P, at any iteration of simplex algorithm $z_j - c_j \ge 0$ for all the non-basic vectors of A, then X_B is an optimal solution.

ii) Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ of the system of equations

$x_1 + 4x_2 - x_3 = 5$

$2x_1 + 3x_2 + x_3 = 8$

to a basic feasible solution

b) i)Define L.P.P form of Transportation problem. Find optimal solution and corresponding cost of Transportation in the following T.P.

	D_1	D_2	D_3	D_4	a_i
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

ii) Show that the set of all feasible solution to a L.P.P is a closed convex set.

c) i)Every extreme point of the convex set of feasible solutions of the system

$Ax = b,\ x \geq 0$

corresponds to a basic feasible solution.

ii) Obtain the dual of the following primal L.P.P.

Max $z = 3x_1 + x_2 + 2x_3 - x_4$

Subject to $2x_1 - x_2 + 3x_3 + x_4 = 1$

$x_1 + x_2 - x_3 + x_4 = 3$

$x_1, x_2 \geq 0,\ x_3$ and x_4 are unrestricted in sign.

d) i) A furniture manufacturer plans to make only two products – chairs and tables from his available resources. He has 400 board feet of wood and 450 man-hours A chair requires 5 board feet of wood and 10 man-hours and yields a profit of Rs. 40, while one table requires 20 board feet of wood and 15 man-hours and yields a profit of Rs. 60. Pose an L.P.P, so that he can maximize his profit.

ii) Solve the following game graphically :

	B_1	B_2	B_3	B_4
A_1	2	2	3	-1
A_2	4	3	2	6

[The End]