2022

Physics (Hons)

[First Semester]

Paper - CC1T

Full Marks : 40

Time : 2 hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group - A

1. Answer any five questions: 5×2=10

(a) State Euler's theorem. Give an example of it.

(b) Show that the functions e^{ax} sinbx and e^{ax} cosbx are linerly independent with the help of Wron Skian.

(c) Find the unit vector perpendicular to each of the vectors

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - \hat{k}$

(d) Prove that vectors $\hat{i}-2\hat{j}+3\hat{k}$, $-2\hat{i}+3\hat{j}-4\hat{k}$ and $\hat{i}-3\hat{j}+5\hat{k}$ are coplaner.

(e) Evaluate $\int_{-\infty}^{\infty} e^{-5t}\delta(t-2)dt$

(f) If $\frac{d\vec{a}}{dt}=\vec{u}\times\vec{a}$ and $\frac{d\vec{b}}{dt}=\vec{u}\times\vec{b}$ then prove that

$$\frac{d}{dt}\left[\vec{a}\times\vec{b}\right]=\vec{u}\times\left(\vec{a}\times\vec{b}\right)$$

(g) Prove that $\vec{\nabla}\times\left(\vec{a}\times\vec{r}\right)=2\vec{a}$

(h) Find the Binomial series for $\frac{1}{\sqrt{1+x}}$

Group - B

Answer any four questions :- **4×5=20**

2. (a) Using method of variation of parameters solve

$$\frac{d^2y}{dx^2}+4y=\tan x$$

b) Solve the differential equations $\frac{dy}{dx}+y=3e^{x}y^{3}$ 3+2

3. (a) If $u=\log(x^3+y^3+z^3-3xyz)$ then prove that

$$\left(\frac{d}{dx}+\frac{d}{dy}+\frac{d}{dz}\right)^2 u=-\frac{9}{(x+y+z)^2}$$

(b) Solve the differential equation

$(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$ 3+2

4. (a) If the thermodynamic variables pressure(P), Volume (v) and Temperature (T) are connected by the relation f(P,V,T)=0 then prove that

$$\left(\frac{dV}{dT}\right)_P\left(\frac{dT}{dP}\right)_V\left(\frac{dP}{dV}\right)_T=-1$$

(b) Expand Sinx in powers of $\left(x=\frac{\Pi}{2}\right)$ in Taylor series.

2+3

5. Find the constants p and q such that the surfaces $px^2-qyz=(p+2)x$ and $4yz^2+z^3=4$ are othogonal at the point (1,–1,2). 5

6. A fluid motion is given by $\vec{v} = (y+z)\vec{i} + (z+x)\hat{i} + (x+y)\hat{k}$.

Prove that the motion is irrotational and find the velocity potential. 3+2

7. Verify Green's theorem by evaluating

$$\int_c \left\{\left[x^3 - y^3 x\right]dx + \left(y^2 - 2xy\right)dy\right\}$$

where C is the square with vartices at points (0,0), (2,0), (2,2) and (0,2). 5

Group - C

Answer any one question **1×10=10**

8. (a) If ϕ is any scalar point function then

$$\iiint_v \vec{\nabla}\phi + dv = \iint_s \phi\hat{n}\, ds$$

(b) If $\vec{\nabla}.\vec{E} = 0, \vec{\nabla}.\vec{H} = 0, \vec{\nabla}\times\vec{E} = \frac{\partial\vec{H}}{\partial t}, \vec{\nabla}\times\vec{H} = -\frac{\partial\vec{E}}{\partial t}$ then show that $\vec{\nabla}\times\vec{E} = -\frac{\partial^2\vec{E}}{\partial t^2}$ and $\vec{\nabla}\times\vec{H} = -\frac{\partial^2\vec{H}}{\partial t^2}$

(c) Evaluate $\int_0^3 t^3\,\delta(t-5)dt$ 4+4+2

9. (a) Verify Stoke's theorem for $\vec{F}=\left(x^2+y^2\right)\hat{i}-2xy\hat{j}$ taken around the ractangle bounded by the lines x=$\pm$a, y=0, y=b.

(b) Find the shape of box which will minimise energy E given by $E=\frac{h^2}{8m}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$ if volume is constant. 5+5