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Two storage inventory model with random planning horizon

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Abstract

An inventory model with stock-dependent demand and two storage facilities under inflation and time value of money is developed where the planning horizon is stochastic in nature and follows exponential distribution with a known mean. The model is a order-quantity reorder-point problem where shortages are not allowed. Two rented storehouses are used for storage – one (say RW_1) at the heart of the market place and the other (say RW_2) little away from the market place. At the beginning, the item is stored at both RW_1 and RW_2. The item is sold from RW_1 and as the demand is stock-dependent, the units are continuously released from RW_2 to RW_1. Replacement of the item occurs when its inventory level reaches its reorder point (Q_r). The model is formulated to maximize the total expected proceeds out of the system from the planning horizon. A genetic algorithm (GA) is developed based on entropy theory where region of search space is gradually decreases to a small neighborhood of the optima. This is named as region reducing genetic algorithm (RRGA) and is used to solve the model. The model is illustrated with some numerical examples and some sensitivity analyses have been done.

Keywords: Region reducing genetic algorithm; Random planning horizon; Two storage inventory

1. Introduction

In the existing literature, inventory models are generally developed under the assumption of finite or infinite planning horizon. Extensive research work has been done in this direction and are available in the standard literatures [1,14,23,26]. But there are many real life situations where these assumptions are not valid, e.g., for a seasonal product, though planning horizon is normally assumed as finite and crisp in nature, but, in every year it fluctuates depending upon the environmental effects and it is better to estimate this horizon as a stochastic parameter with some feasible distribution. In 1983, Gurnani [13] pointed out that an infinite planning horizon is of rare occurrence because the costs are likely to vary disproportionately and because of change in product specifications and design or its abandonment or substitution by another product due to rapid development of

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technology. Chung and Kim [6] also suggested that the assumption of the infinite planning horizon is not realistic and called for a new model which relaxes the assumption of the infinite planning horizon. Moon and Yun [21] developed an EOQ model where the planning horizon is a random variable following exponential distribution. Moon and Lee [22] developed an EOQ model for an item with random lifetime of the product under inflation and time value of money. They assumed exponential and normal distribution for the life time of the item and proposed simulation approach for solution when expected value of the objective is difficult for calculation.

In an inventory management, different types of demand are considered – constant, stock-dependent, time-dependent, probabilistic demand, etc. According to Levin et al. [16], "the presence of inventory has a motivational effect on the people around it". In the present competitive market, the inventory/stock is decoratively displayed through electronic media to attract the customers and thus to push the sale. Schary and Baker [25] and Wolfe [28] also established the impact of product availability for simulating demand. Mandal and Phaujder [17], Datta and Pal [7] and other considered linear form of stock-dependent demand, i.e. $D = c + dq$ where as Urban [27], Giri et al. [11], Mandal and Maiti [18] and Maiti and Maiti [19] and others took the demand of the form $D = dq^{\beta}$.

During the last few decades, the monetary situation of most of the countries, affluent or otherwise, has changed a lot due to large scale inflation and consequent sharp decline in the purchasing power of money. As a result several efforts have been made by researchers to reformulate the optimal inventory management policies taking into account inflation, time value of money, etc. The initial attempt in this direction was made by Buzacott [5] in 1975. He dealt with an EOQ model with inflation subject to different types of pricing policies. In the subsequent year, Bierman et al. [4] shows that the inflation rate does not affect the optimal order-quantity perse; rather, the difference between the inflation rate and the discount rate affects the optimum order quantity. Several authors then extended these works to make the more realistic inventory model under inflation. Datta and Pal [8] studied the effect of internal and external inflation in an inventory model with time-dependent demand rate, Dey and Maiti [10] considered inflationary effect when lead time is fuzzy. But none has considered two storage inventory model under inflation, especially when demand is stock-dependent and planning horizon is random.

Now-a-days due to globalization of market with the introduction of multi-nationals in the business, there is a trend among the business houses specially middle order retailers and small retailers of different multi-national products to compete with each other for sale and as a result, they use the decorative showroom at the selling point to boost their items in addition to a separate warehouse for storage. As a result, in the important market places like super markets, municipality markets, etc., it is almost impossible to have a big showroom/shop due to the scarcity of space and very high rent. Normally, moderate and big business houses operate through two rented houses – one, smaller in size, is in the heart of the market place and other with large capacity little away from the market place. During last two decades, two-warehouse inventory models have been developed and solved by many researchers [3,15,24]. Till now, most of the two storage problems have been formulated with one own house and another rented one. But normally, both are rented houses, rent at the market place being higher than that at far-away places [19].

Here, a two storage inventory model for an item is developed whose demand depends upon the displayed inventory level. It is assumed that planning horizon is stochastic in nature and follows exponential distribution with a known mean and optimal decision is made considering the effect of inflation and time value of money on different inventory costs. The model is a order-quantity reorder-point problem where shortages are not allowed. Two rented storehouses are used for storage – one (say RW_1) at the heart of the market place from where the item is sold and the other (say RW_2) little away from the market place where most of the inventory is stored. Here, the size of RW_1 is finite but that of RW_2 has unlimited capacity. The units are sold from RW_1 and are continuously released from RW_2 to RW_1 (as the demand is stock-dependent). Replacement of the item occurs when its inventory level reaches its reorder point (Q_r). The model is formulated to maximize the total expected profit from the system for the planning horizon. A genetic algorithm (GA) is developed based on entropy theory where region of search space is gradually decreases to a small neighborhood of the optima. This is named as region reducing genetic algorithm (RRGA) and is used to solve the model. The model is illustrated with some numerical examples and some sensitivity analyses have been performed.

2. Region reducing genetic algorithm

Genetic algorithms are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation, etc.) and have been developed by Holland, his colleagues and students at the University of Michigan (cf. [12]). Because of its generality and other advantages over conventional optimization methods it has been successfully applied to different decision making problems.

Generally a GA starts with a single population [12,20], randomly generated in the search space. One of the difficulties of GAs is that they often converge too quickly and tend to make quickly uniform the population of the chromosomes. Consequently they are easily trapped into local optima of the objective function. This difficulty is mainly due to the premature loss of diversity of the population during the search. To overcome this difficulties, Bessaou and Siarry [2] proposes a genetic algorithm where initially more than one population of solutions are generated. Genetic operations are done on every populations a finite number of times to find a promising zone of optimum solution. Finally a population of solutions is generated in this zone and genetic operations are done on this population a finite number of times to get a final solution. Again the theoretical convergence towards the global optima of a GA, operating with a constant probability of crossover p_c, is ensured if the probability of mutation $p_m(k)$ follows a given decreasing law, in function of the generation number k [9]. Following Bessaou and Siarry [2] a genetic algorithm is developed using the entropy generated from information theory, where promising zone is gradually reduces to a small neighborhood of the optimal solution. This algorithm is named as RRGA and is used to solve our model. The algorithm is given below:

RRGA Algorithm

1. Initialize probability of crossover p_c and probability of mutation p_m.
2. Set iteration counter $T = 0$.
3. Generate M sub-populations of solutions, each of order N, from search space of optimization problem under consideration, such that the diversity among the solutions of each population is maintained. Diversity is maintained using the entropy originating from information theory (cf. Section 2.1(b)). Solutions for each of the population are generated randomly from the search space in such a way that the constraints of the problem are satisfied. Let $P_1, P_2, \ldots, P_M$ be these populations.
4. Evaluate fitness of each solution of every populations.
5. Repeat
   A. Do for each sub-populations P_i.
      a. Select N solutions from P_i, for mating pool using Roulette-wheel selection process [20] (one solution may be selected more than once). Let this set be P_i^1.
      b. Select solutions from P_i^1, for crossover and mutation depending on p_c and p_m respectively.
      c. Make crossover on selected solutions for crossover.
      d. Make mutation on selected solutions for mutation.
      e. Evaluate fitness of the child solutions.
      f. Replace the parent solutions with the child solutions.
      g. Replace P_i with P_i^1.
   B. End Do
   C. Reduce probability of mutation p_m.
6. Until number of generations < Maxgen1, where Maxgen1 represents the maximum number of generations to be made on initial populations.
7. Select optimum solutions from each sub-populations and S^* be the best among these solutions.
8. Select a neighborhood $V(T)$ of S^*.
9. Repeat
   a. Generate a population of solutions of size N in $V(T)$. Let it be P.
   b. Evaluate fitness of each solutions.
   c. Initialize probability of mutation p_m.
   d. Repeat

(i) Select N solutions from P for mating pool using Roulette-wheel selection process. Let this set be P^1.
(ii) Select solutions from P^1 for crossover and mutation depending on p_c and p_m respectively.
(iii) Make crossover on selected solutions for crossover.
(iv) Make mutation on selected solutions for mutation.
(v) Evaluate fitness of the child solutions.
(vi) Replace the parent solutions with the child solutions.
(vii) Replace P with P^1.
(viii) Reduce probability of mutation p_m.

e. Until number of generations < Maxgen2, where Maxgen2 represents the maximum number of generations to be made on this population.
f. Update S^* by the best solution found.
g. Reduce the neighborhood $V(T)$.
h. Increment T by 1.

10. Until T < Maxgen3, where Maxgen3 represents the maximum number of times for which the search space to be reduced.

2.1. RRGA procedures for the proposed model

(a) *Representation:* A 'K-dimensional real vector' $X = (x_1, x_2, \ldots, x_K)$ is used to represent a solution, where $x_1, x_2, \ldots, x_K$ represent different decision variables of the problem such that constraints of the problem are satisfied.

(b) *Inialization:* At this step M sub-populations, each of size N are randomly generated in the search space in such a way that diversity among the solutions of each of the populations is maintained and the constraints of the problem are satisfied. Let $X_{l1}, X_{l2}, \ldots, X_{ln}$, are the solutions of lth population P_l, $l = 1, 2, \ldots, M$. Diversity can be maintained using the entropy originating from information theory. Entropy of jth variable for the lth population P_l can be obtained by the formula: $E_j(P_l) = \sum_{i=1}^{N}\sum_{k=i+1}^{N} - p_{ik}\log(p_{ik})$, where p_{ik} represents the probability that the value of jth variable of ith solution ($X_{li}(j)$) is different from the one of the jth variable of the kth solution ($X_{lk}(j)$) and is determined by the formula: $p_{ik} = 1 - \frac{|X_{li}(j) - X_{lk}(j)|}{U_j - L_j}$, where $[L_j, U_j]$ is the variation domain of the jth variable. The average entropy $E(P_l)$ of the lth sub-population P_l is taken as the average of the entropies of the different variables for the population, i.e. $E(P_l) = \frac{1}{K}\sum_{j=1}^{K}E_j$. It is clear that if P_l is made-up of same solutions, then $E(P_l)$ vanishes and more varied the solutions, higher the value of $E(P_l)$ and the better is its quality. So to maintain diversity, every time a new solution is randomly generated for P_l from the search space, the entropy between this one and the previously generated individuals for P_l is calculated. If this value is higher than a fixed threshold E_0, fixed from the beginning, the current chromosome is accepted. This process is repeated until N solutions are generated. Following the same procedure all the sub-populations P_l, $l = 1, 2, \ldots, M$ are generated. This solution sets are taken as initial sub-populations.

(c) *Fitness value:* Value of the objective function due to the solution X, is taken as fitness of X. Let it be $f(X)$.

(d) *Selection process for mating pool:* The following steps are followed for this purpose:

(i) For each population P_i, find total fitness of the population $F = \sum_{j=1}^{N} f(X_{ij})$.
(ii) Calculate the probability of selection pr_{ij} of each solution X_{ij} by the formula $\mathrm{pr}_{ij} = f(X_{ij})/F$.
(iii) Calculate the cumulative probability qr_{ij} for each solution X_{ij} by the formula $\mathrm{qr}_{ij} = \sum_{k=0}^{j}\mathrm{pr}_{ik}$.
(iv) Generate a random number 'r' from the range $[0, \ldots, 1]$.
(v) If $r < \mathrm{qr}_{i1}$ then select X_{i1} otherwise select $X_{ij}(2 \leqslant j \leqslant N)$ where $\mathrm{qr}_{ij-1} \leqslant r < \mathrm{qr}_{ij}$.
(vi) Repeat step (iv) and (v) N times to select N solutions for mating pool. Clearly one solution may be selected more than once.
(vii) Selected solution set is denoted by P_i^1 in the proposed FSRRGA algorithm.

(e) *Crossover:*

(i) Selection for crossover: For each solution of P_i^1 generate a random number r from the range $[0, \ldots, 1]$. If $r < p_c$ then the solution is taken for crossover, where p_c is the probability of crossover.

(ii) Crossover process: Crossover taken place on the selected solutions. For each pair of coupled solutions Y_1 and Y_2 a random number c is generated from the range $[0,\dots,1]$ and Y_1 and Y_2 are replaced by their offspring's Y_{11} and Y_{21} respectively where $Y_{11} = cY_1 + (1-c)Y_2$, $Y_{21} = cY_2 + (1-c)Y_1$.

(f) *Mutation:*

(i) Selection for mutation: For each solution of P_i^1 generate a random number r from the range $[0,\cdots,1]$. If $r < p_{\mathrm{m}}$ then the solution is taken for mutation, where p_{m} is the probability of mutation.

(ii) Mutation process: To mutate a solution $X = (x_1, x_2, \dots, x_K)$ select a random integer r in the range $[1,\dots,\mathrm{K}]$. Then replace x_r by randomly generated value within the boundary of rth component of X.

(g) *Reduction process of p_m:* Let $p_{\mathrm{m}}(0)$ is the initial value of p_{m}. $p_{\mathrm{m}}(T)$ is calculated by the formula $p_{\mathrm{m}}(T) = p_{\mathrm{m}}(0)\exp(-T/\alpha)$, where α is calculated so that the final value of p_{m} is small enough (10^{-2} in our case). So $\alpha = \mathrm{Maxgen1}/\log[\frac{p_{\mathrm{m}}(0)}{10^{-2}}]$ for initial populations P_i, $i = 1, 2, \dots, M$ and $\alpha = \mathrm{Maxgen2}/\log[\frac{p_{\mathrm{m}}(0)}{10^{-2}}]$ for the population $P(T)$ in the promising zone.

(h) *Reduction process of neighborhood:* $V(0)$ is the initial neighborhood of S^*. $V(T)$ is calculated by the formula $V(T) = V(0)\exp(-T/\alpha)$, where α is calculated so that the final neighborhood is small enough (10^{-2} in our case). So $\alpha = \mathrm{Maxgen3}/\log[\frac{V(0)}{10^{-2}}]$.

3. Assumptions and notations for the proposed models

The following notations and assumptions are used in developing the models.

(i) Inventory system involves only one item.
(ii) Two rented warehouses RW_1 and RW_2 are used to store the item.
(iii) Location of RW_1 is at the heart of the market place and RW_2 is little away from the market place.
(iv) Item is sold from RW_1 and are continuously filled up from RW_2.
(v) $q(t)$ is the inventory level at time t.
(vi) Q_0 is the capacity of RW_1 and capacity of RW_2 is unlimited.
(vii) H is the planning horizon and is stochastically governed by exponential distribution with parameter λ, its density function $f(H) = \lambda e^{-\lambda H}$.
(viii) Demand $D(q)$ of the item is dependent on the stock of RW_1 and is of the form:

$$D(q) = \begin{cases} a + bQ_0 & \text{for } q > Q_0, \\ a + bq & \text{for } q \leqslant Q_0. \end{cases}$$

(ix) Q is the order quantity and Q_{r} the reorder point.
(x) N is the number of fully accommodated cycles to be made during the time horizon and T is the length of a cycle.
(xi) Time horizon (H) ends during the $(N+1)$th cycle.
(xii) t_l is the last cycle length. So $t_l = H - NT$.
(xiii) c_{p} is the purchase cost per unit quantity in \$.
(xiv) c_{s} is the selling price per unit quantity in \$ which is a mark-up m of the purchase cost c_{p}, i.e, $c_{\mathrm{s}} = mc_{\mathrm{p}}$.
(xv) c_{o} is the ordering cost in \$, which depends on order size and is of the form $c_{\mathrm{o}} = c_{\mathrm{o}1} + c_{\mathrm{o}2}Q$.
(xvi) $c_{\mathrm{h}1}$ and $c_{\mathrm{h}2}$ are the holding costs per unit quantity per unit time in \$ at RW_1 and RW_2 respectively.
(xvii) c_{sr} is the reduced selling price per unit quantity in \$ which is a mark-up m_0 of purchase cost, i.e., $c_{\mathrm{sr}} = m_0 c_{\mathrm{p}}$.
(xviii) i and r are the inflation and discount rates respectively and $R = r - i$.
(xix) c_{t} represents transportation cost in \$ per unit item from RW_2 to RW_1.
(xx) T_{j-1} is the ordering time of jth cycle where $T_j = jT$ for $j = 1, 2, \dots, N+1$.
(xxi) t_0 is the depletion time of the item at RW_2.
(xxii) Lead time is negligible.
(xxiii) Here Q_{r} and T are decision variables and Z is the expected total profit during H.

4. Model development and analysis

In the development of the model, we assume that the business starts with an amount $Q_r + Q$ and at the beginning of every intermediate cycle company purchases an amount Q units of the item. Among these units at first RW_1 is fulfilled and the remaining units are stored at RW_2. Demands of the item are met using the stocks of RW_1 and are continuously filled up from RW_2. When stock level drops to reorder level Q_r at RW_1 after exhausting the stock at RW_2, order for next cycle is placed (cf. Figs. 1a and 1b). Item remains at the end of planning horizon H is sold at a reduced price in a lot. It is clear from the assumption that $Q_r \leqslant Q_0$.

4.1. Formulation for N full cycles

Duration of jth $(1 \leqslant j \leqslant N)$ cycle is $[(j-1)T, jT]$ and at the beginning of the cycle inventory level at RW_1 is Q_0 and at RW_2 is $Q + Q_r - Q_0$. So instantaneous state $q(t)$ of the item during $(j-1)T \leqslant t \leqslant jT$ is given by

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = \begin{cases} -(a + bQ_0) & \text{for } q(t) > Q_0, \\ -(a + bq) & \text{for } q \leqslant Q_0. \end{cases} \tag{1}$$

Solving the above differential equation using the initial condition at $t = T_j$, $Q = Q_r$, we get

$$q(t) = \begin{cases} (Q + Q_r) - (a + bQ_0)(t - T_{j-1}) & \text{for } T_{j-1} \leqslant t \leqslant T_{j-1} + t_0, \\ \frac{1}{b}\left[(a + bQ_0)\mathrm{e}^{-b(t - T_{j-1} - t_0)} - a\right] & \text{for } T_{j-1} + t_0 \leqslant t \leqslant T_j, \end{cases} \tag{2}$$

where

$$t_0 = \frac{Q + Q_r - Q_0}{a + bQ_0}.$$

Present value of holding cost at RW_2 for jth $(1 \leqslant j \leqslant N)$ cycle, $H2_j$, is given by

$$H2_j = c_{h2} \int_{T_{j-1}}^{T_{j-1}+t_0} \{q(t) - Q_0\}\mathrm{e}^{-Rt}\,\mathrm{d}t = K_{h2}\mathrm{e}^{-RT_{j-1}}, \tag{3}$$

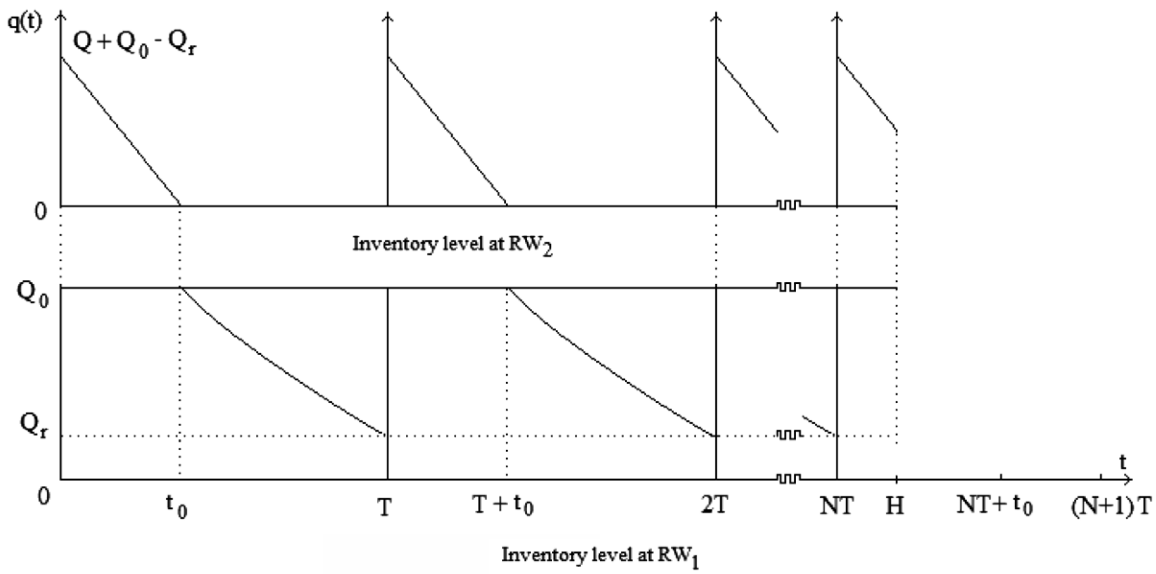

Fig. 1a. Inventory situation when $NT \leqslant H \leqslant NT + t_0$.

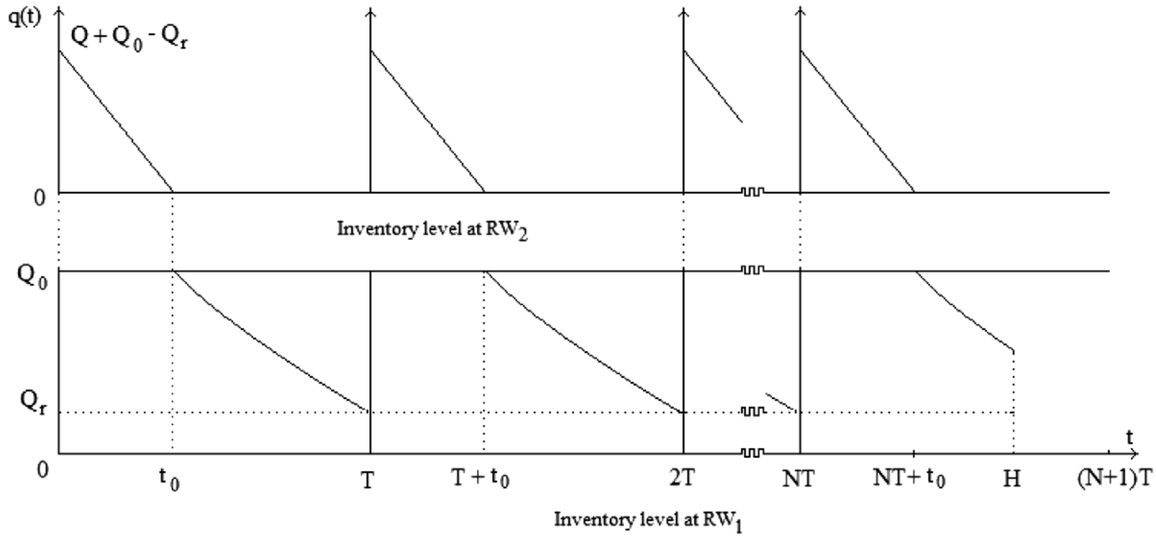

Fig. 1b. Inventory situation when $NT + t_0 \leqslant H < (N+1)T$.

where

$$K_{h2} = c_{h2}\left[\left(\frac{Q + Q_r - Q_0}{R}\right)(1 - e^{-Rt_0}) - (a + bQ_0)\left\{\frac{1 - e^{-Rt_0}}{R^2} - \frac{t_0 e^{-Rt_0}}{R}\right\}\right]. \tag{4}$$

So present value of holding cost at RW_2 for first N cycles, H2N is given by

$$\mathrm{H2N} = \sum_{j=1}^{N} \mathrm{H2}_j = \sum_{j=1}^{N} K_{h2} e^{-RT_{j-1}} = K_{h2} S_1, \tag{5}$$

where

$$S_1 = \sum_{j=1}^{N} e^{-RT_{j-1}} = \frac{1 - e^{-NRT}}{1 - e^{-RT}} \tag{6}$$

and present value of expected holding cost at RW_2 for first N cycles, EH2N is given by

$$\mathrm{EH2N} = K_{h2}\mathrm{ES}_1, \tag{7}$$

where ES_1 is the expected value of S_1 and is calculated in Appendix A.

Present value holding cost at RW_1, $\mathrm{H1}_j$, is given by

$$\mathrm{H1}_j = c_{h1}\left[\int_{T_{j-1}}^{T_{j-1}+t_0} Q_0 e^{-Rt}\,dt + \int_{T_{j-1}+t_0}^{T_j} q(t) e^{-Rt}\,dt\right] = K_{h1} e^{-RT_{j-1}}, \tag{8}$$

where

$$K_{h1} = \frac{c_{h1} Q_0}{R}\left(1 - e^{-Rt_0}\right) + \frac{c_{h1}}{b}\left[\left(\frac{a + bQ_0}{b + R}\right)\left\{e^{-Rt_0} - e^{-b(T-t_0)-RT}\right\} - \frac{a}{R}\left\{e^{-Rt_0} - e^{-RT}\right\}\right]. \tag{9}$$

So present value of holding cost at RW_1 for first N cycles, H1N is given by

$$\mathrm{H1N} = \sum_{j=1}^{N} \mathrm{H1}_j = \sum_{j=1}^{N} K_{h1} e^{-RT_{j-1}} = K_{h1} S_1 \tag{10}$$

and the present value of expected holding cost at RW_1 for first N cycles, EH1N is given by

$$\mathrm{EH1N} = K_{h1}\mathrm{ES}_1. \tag{11}$$

Present value selling price SP_j is given by

$$\mathrm{SP}_j = c_\mathrm{s}\left[\int_{T_{j-1}}^{T_{j-1}+t_0}(a+bQ_0)\mathrm{e}^{-Rt}\,\mathrm{d}t + \int_{T_{j-1}+t_0}^{T_j}(a+bq(t))\mathrm{e}^{-Rt}\,\mathrm{d}t\right] = K_\mathrm{sp}\mathrm{e}^{-RT_{j-1}}, \tag{12}$$

where

$$K_\mathrm{sp} = \frac{c_\mathrm{s}(a+bQ_0)}{R}\left(1-\mathrm{e}^{-Rt_0}\right) + \frac{c_\mathrm{s}(a+bQ_0)}{b+R}\{\mathrm{e}^{-Rt_0} - \mathrm{e}^{-(b+R)T+bt_0}\}. \tag{13}$$

So present value of selling price for first N cycles, SPN is given by

$$\mathrm{SPN} = \sum_{j=1}^{N}\mathrm{SP}_j = \sum_{j=1}^{N}K_\mathrm{sp}\mathrm{e}^{-RT_{j-1}} = K_\mathrm{sp}S_1 \tag{14}$$

and present value of expected selling price for first N cycles, ESPN is given by

$$\mathrm{ESPN} = K_\mathrm{sp}\mathrm{ES}_1. \tag{15}$$

Present value transportation cost TC_j, is given by

$$\mathrm{TC}_j = c_\mathrm{t}\int_{T_{j-1}}^{T_{j-1}+t_0}(a+bQ_0)\mathrm{e}^{-Rt}\,\mathrm{d}t = K_\mathrm{tc}\mathrm{e}^{-RT_{j-1}}, \tag{16}$$

where

$$K_\mathrm{tc} = \frac{c_\mathrm{t}(a+bQ_0)}{R}\left(1-\mathrm{e}^{-Rt_0}\right).$$

So present value of transportation cost for first N cycles, TCN is given by

$$\mathrm{TCN} = \sum_{j=1}^{N}\mathrm{TC}_j = \sum_{j=1}^{N}K_\mathrm{tc}\mathrm{e}^{-RT_{j-1}} = K_\mathrm{tc}S_1 \tag{17}$$

and present value of expected transportation cost for first N cycles, ETCN is given by

$$\mathrm{ETCN} = K_\mathrm{tc}\mathrm{ES}_1. \tag{18}$$

Present value purchase cost PC_j is given by

$$\mathrm{PC}_j = \begin{cases} c_\mathrm{p}Q\mathrm{e}^{-RT_{j-1}} & \text{for } j = 2,\ldots,N, \\ c_\mathrm{p}(Q+Q_\mathrm{r}) & \text{for } j = 1. \end{cases} \tag{19}$$

So the present value of purchase cost for first N cycles, PCN is given by

$$\mathrm{PCN} = \sum_{j=1}^{N}\mathrm{PC}_j = c_\mathrm{p}Q_\mathrm{r} + c_\mathrm{p}\mathrm{QS}_1. \tag{20}$$

So the present value of expected purchase cost for first N cycles, EPCN is given by

$$\mathrm{EPCN} = c_\mathrm{p}Q_\mathrm{r} + c_\mathrm{p}\mathrm{QES}_1. \tag{21}$$

Present value ordering cost OC_j is given by

$$\mathrm{OC}_j = \begin{cases} (c_{\mathrm{o}1}+c_{\mathrm{o}2}Q)\mathrm{e}^{-RT_{j-1}} & \text{for } j = 2,\ldots,N, \\ c_{\mathrm{o}1}+c_{\mathrm{o}2}(Q+Q_\mathrm{r}) & \text{for } j = 1. \end{cases} \tag{22}$$

So present value of ordering cost for first N cycles, OCN is given by

$$\mathrm{OCN} = \sum_{j=1}^{N}\mathrm{OC}_j = c_{\mathrm{o}2}Q_\mathrm{r} + (c_{\mathrm{o}1}+c_{\mathrm{o}2}Q)S_1. \tag{23}$$

So the present value of expected ordering cost for first N cycles, EOCN is given by

$$\mathrm{EOCN} = c_{o2}Q_r + (c_{o1} + c_{o2}Q)\mathrm{ES}_1. \tag{24}$$

So total expected profit from these N cycle $Z1$ is given by

$$Z1 = \mathrm{ESPN} - \mathrm{EPCN} - (\mathrm{EH1N} + \mathrm{EH2N}) - \mathrm{EOCN} - \mathrm{ETCN}. \tag{25}$$

4.2. Formulation for last cycle

Duration of last cycle is $[TN, H]$. Now two cases may arise – Case-1: $TN \leqslant H \leqslant TN + t_0$, Case-II: $TN + t_0 \leqslant H \leqslant T(N+1)$. In both the cases at the beginning of the cycle inventory level at RW_1 is Q_0 and at RW_2 is $Q + Q_r - Q_0$. So instantaneous state $q(t)$ of the item during $NT \leqslant t \leqslant H$ is given by

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = \begin{cases} -(a + bQ_0) & \text{for } q(t) > Q_0, \\ -(a + bq) & \text{for } q \leqslant Q_0. \end{cases} \tag{26}$$

Solving the above differential equation we get

$$q(t) = \begin{cases} (Q + Q_r) - (a + bQ_0)(t - T_N) & \text{for } TN \leqslant t \leqslant TN + t_0, \\ \frac{1}{b}\left[(a + bQ_0)\mathrm{e}^{-b(t - T_N - t_0)} - a\right] & \text{for } TN + t_0 \leqslant t \leqslant T(N+1). \end{cases} \tag{27}$$

Case-1: $TN \leqslant H \leqslant TN + t_0$

Present value of holding cost at RW_2 for last cycle in this case, $\mathrm{H2L}_1$, is given by

$$\begin{aligned} \mathrm{H2L}_1 &= c_{h2}\int_{TN}^{H}\{q(t) - Q_0\}\mathrm{e}^{-Rt}\,\mathrm{d}t \\ &= c_{h2}\left[\frac{Q + Q_r - Q_0}{R}(\mathrm{e}^{-RNT} - \mathrm{e}^{-RH}) + (a + bQ_0)\left\{\frac{(H - NT)\mathrm{e}^{-RH}}{-R} + \frac{1}{R^2}(\mathrm{e}^{-RNT} - \mathrm{e}^{-RH})\right\}\right]. \end{aligned} \tag{28}$$

Present value of holding cost at RW_1 for last cycle in this case, $\mathrm{H1L}_1$, is given by

$$\mathrm{H1L}_1 = c_{h1}\int_{TN}^{H} Q_0\mathrm{e}^{-Rt}\,\mathrm{d}t = \frac{c_{h1}Q_0}{R}(\mathrm{e}^{-RNT} - \mathrm{e}^{-RH}). \tag{29}$$

Present value of selling price at RW_1 for last cycle in this case, SPL_1, is given by

$$\mathrm{SPL}_1 = c_s\int_{TN}^{H}(a + bQ_0)\mathrm{e}^{-Rt}\,\mathrm{d}t = \frac{c_s(a + bQ_0)}{R}(\mathrm{e}^{-RNT} - \mathrm{e}^{-RH}). \tag{30}$$

Present value of expected selling price at RW_1 for last cycle in this case, ESPL_1, is given by

$$\mathrm{ESPL}_1 = \frac{c_s(a + bQ_0)}{R}(\mathrm{ES}_3 - \mathrm{ES}_4). \tag{31}$$

Present value of transportation cost for last cycle in this cases, TCL_1, is given by

$$\mathrm{TCL}_1 = c_t\int_{TN}^{H}(a + bQ_0)\mathrm{e}^{-Rt}\,\mathrm{d}t = \frac{c_t(a + bQ_0)}{R}(\mathrm{e}^{-RNT} - \mathrm{e}^{-RH}). \tag{32}$$

Stock in hand at the end of the cycle QH_1, is given by

$$\mathrm{QH}_1 = (Q + Q_r) - (a + bQ_0)(H - TN). \tag{33}$$

Present value of selling price of QH_1 at $t = H$, SPQH_1, is given by

$$\mathrm{SPQH}_1 = c_{sr}[(Q + Q_r) - (a + bQ_0)(H - TN)]\mathrm{e}^{-RH}. \tag{34}$$

Case-II: $TN + t_0 \leqslant H \leqslant T(N+1)$

Present value of holding cost at RW_2 for last cycle in this case, $\mathrm{H2L}_2$, is given by

$$\begin{aligned}\mathrm{H2L}_2 &= c_{\mathrm{h2}} \int_{TN}^{TN+t_0} \{q(t) - Q_0\} \mathrm{e}^{-Rt}\,\mathrm{d}t \\ &= c_{\mathrm{h2}} \left[\frac{Q + Q_{\mathrm{r}} - Q_0}{R} (1 - \mathrm{e}^{-Rt_0}) + (a + bQ_0) \left\{ \frac{t_0 \mathrm{e}^{-Rt_0}}{R} + \frac{1}{R^2} (\mathrm{e}^{-Rt_0} - 1) \right\} \right] \mathrm{e}^{-RNT}. \end{aligned} \tag{35}$$

Present value of holding cost at RW_1 for last cycle in this case, $\mathrm{H1L}_2$, is given by

$$\begin{aligned}\mathrm{H1L}_2 &= c_{\mathrm{h1}} \left[\int_{TN}^{TN+t_0} Q_0 \mathrm{e}^{-Rt}\,\mathrm{d}t + \int_{TN+t_0}^{H} q(t) \mathrm{e}^{-Rt}\,\mathrm{d}t \right] \\ &= \frac{c_{\mathrm{h1}} Q_0}{R} (1 - \mathrm{e}^{-Rt_0}) \mathrm{e}^{-RNT} + \frac{c_{\mathrm{h1}}}{b} \left[(a + bQ_0) \frac{\mathrm{e}^{-Rt_0} \mathrm{e}^{-RNT} - \mathrm{e}^{-bt_0} \mathrm{e}^{-b(H-NT)-RH}}{b+R} - \frac{a}{R} (\mathrm{e}^{-Rt_0} \mathrm{e}^{-RNT} - \mathrm{e}^{-RH}) \right]. \end{aligned} \tag{36}$$

Present value of selling price for last cycle in this case, SPL_2, is given by

$$\begin{aligned}\mathrm{SPL}_2 &= c_{\mathrm{s}} \left[\int_{TN}^{TN+t_0} (a + bQ_0) \mathrm{e}^{-Rt}\,\mathrm{d}t + \int_{TN+t_0}^{H} (a + bq(t)) \mathrm{e}^{-Rt}\,\mathrm{d}t \right] \\ &= \frac{c_{\mathrm{s}}(a + bQ_0)}{R} (1 - \mathrm{e}^{-Rt_0}) \mathrm{e}^{-RNT} + \frac{c_{\mathrm{s}}(a + bQ_0)}{b+R} \mathrm{e}^{bt_0 - RNT} \{ \mathrm{e}^{-(b+R)t_0} - \mathrm{e}^{-(b+R)T} \}. \end{aligned} \tag{37}$$

Present value of transportation cost for last cycle in this cases, TCL_2, is given by

$$\mathrm{TCL}_2 = c_{\mathrm{t}} \int_{TN}^{TN+t_0} (a + bQ_0) \mathrm{e}^{-Rt}\,\mathrm{d}t = \frac{c_{\mathrm{t}}(a + bQ_0)}{R} (1 - \mathrm{e}^{-Rt_0}) \mathrm{e}^{-RNT}. \tag{38}$$

Stock in hand at the end of the cycle QH_2 is given by

$$\mathrm{QH}_2 = \frac{1}{b} [(a + bQ_0) \mathrm{e}^{bt_0} \mathrm{e}^{-b(H-NT)} - a]. \tag{39}$$

So present value of reduced selling price of QH_2 at $t = H$, SPQH_2, is given by

$$\mathrm{SPQH}_2 = \frac{c_{\mathrm{sr}}}{b} [(a + bQ_0) \mathrm{e}^{bt_0} \mathrm{e}^{-b(H-NT)} - a] \mathrm{e}^{-RH}. \tag{40}$$

Expected costs in the last cycle

Present value of expected holding cost at RW_2 for last cycle, EH2L, is given by

$$\begin{aligned}\mathrm{EH2L} &= \sum_{N=0}^{\infty} \left[\int_{NT}^{NT+t_0} \mathrm{H2L}_1 \lambda \mathrm{e}^{-\lambda H} \mathrm{d}H + \int_{NT+t_0}^{(N+1)T} H2L_2 \lambda \mathrm{e}^{-\lambda H}\,\mathrm{d}H \right] \\ &= c_{\mathrm{h2}} \left[\frac{Q + Q_{\mathrm{r}} - Q_0}{R} (\mathrm{ES}_3 - \mathrm{ES}_4) + (a + bQ_0) \left\{ \frac{\mathrm{ES}_5 - \mathrm{TES}_2}{-R} + \frac{1}{R^2} (\mathrm{ES}_3 - \mathrm{ES}_4) \right\} \right] \\ &\quad + c_{\mathrm{h2}} \left[\frac{Q + Q_{\mathrm{r}} - Q_0}{R} (1 - \mathrm{e}^{-Rt_0}) + (a + bQ_0) \left\{ \frac{t_0 \mathrm{e}^{-Rt_0}}{R} + \frac{1}{R^2} (\mathrm{e}^{-Rt_0} - 1) \right\} \right] \mathrm{ES}_7, \end{aligned} \tag{41}$$

where expressions of ES_2, ES_3, ES_4, ES_5 and ES_7 are given in Appendix A.

Present value of expected holding cost at RW_1 for last cycle in this case, $\mathrm{EH1L}_1$, is given by

$$\begin{aligned}\mathrm{EH1L}_1 &= \sum_{N=0}^{\infty} \left[\int_{NT}^{NT+t_0} \mathrm{H1L}_1 \lambda \mathrm{e}^{-\lambda H}\,\mathrm{d}H + \int_{NT+t_0}^{(N+1)T} \mathrm{H1L}_2 \lambda \mathrm{e}^{-\lambda H}\,\mathrm{d}H \right] \\ &= \frac{c_{\mathrm{h1}} Q_0}{R} (\mathrm{ES}_3 - \mathrm{ES}_4) + \frac{c_{\mathrm{h1}} Q_0}{R} (1 - \mathrm{e}^{-Rt_0}) \mathrm{ES}_7 \\ &\quad + \frac{c_{\mathrm{h1}}}{b} \left[(a + bQ_0) \frac{\mathrm{e}^{-Rt_0} \mathrm{ES}_7 - \mathrm{e}^{-bt_0} \mathrm{ES}_9}{b+R} - \frac{a}{R} (\mathrm{e}^{-Rt_0} \mathrm{ES}_7 - \mathrm{ES}_8) \right], \end{aligned} \tag{42}$$

where expressions of ES_8 and ES_9 are given in Appendix A.

Present value of expected selling price for last cycle, ESPL, is given by

$$\begin{aligned}\mathrm{ESPL} &= \sum_{N=0}^{\infty}\left[\int_{NT}^{NT+t_0} SPL_1\lambda e^{-\lambda H}\,dH + \int_{NT+t_0}^{(N+1)T} SPL_2\lambda e^{-\lambda H}\,dH\right]\\ &= \frac{c_s(a+bQ_0)}{R}(\mathrm{ES}_3-\mathrm{ES}_4) + \frac{c_s(a+bQ_0)}{R}(1-e^{-Rt_0})\mathrm{ES}_7\\ &\quad + \frac{c_s(a+bQ_0)}{b+R}e^{bt_0}\{e^{-(b+R)t_0}-e^{-(b+R)T}\}\mathrm{ES}_7. \end{aligned} \tag{43}$$

Present value of expected transportation cost from RW_2 to RW_1 for last cycle, ESPL_1, is given by

$$\begin{aligned}\mathrm{ETCL} &= \sum_{N=0}^{\infty}\left[\int_{NT}^{NT+t_0} \mathrm{TCL}_1\lambda e^{-\lambda H}\,dH + \int_{NT+t_0}^{(N+1)T} \mathrm{TCL}_2\lambda e^{-\lambda H}\,dH\right]\\ &= \frac{c_t(a+bQ_0)}{R}(\mathrm{ES}_3-\mathrm{ES}_4) + \frac{c_t(a+bQ_0)}{R}(1-e^{-Rt_0})\mathrm{ES}_7. \end{aligned} \tag{44}$$

Present value of expected reduced sell revenue of the quantity at hand at the end of the last cycle, ESPQH, is given by

$$\begin{aligned}\mathrm{ESPQH} &= \sum_{N=0}^{\infty}\left[\int_{NT}^{NT+t_0} \mathrm{SPQH}_1\lambda e^{-\lambda H}\,dH + \int_{NT+t_0}^{(N+1)T} \mathrm{SPQH}_2\lambda e^{-\lambda H}\,dH\right]\\ &= c_{sr}[(Q+Q_r)\mathrm{ES}_4-(a+bQ_0)(\mathrm{ES}_5-\mathrm{TES}_2)] + \frac{c_{sr}}{b}[(a+bQ_0)e^{bt_0}\mathrm{ES}_9 - a\mathrm{ES}_8]. \end{aligned} \tag{45}$$

Present value of ordering cost for last cycle in both the cases, OCL, is given by

$$\mathrm{OCL} = (c_{o1}+c_{o2}Q)e^{-RTN}. \tag{46}$$

Present value of expected ordering cost for last cycle, EOCL, is given by

$$\mathrm{EOCL} = (c_{o1}+c_{o2}Q)\mathrm{ES}_6, \tag{47}$$

where ES_6 represents expected value of e^{-RTN} and is calculated in Appendix A.

Present value of purchase cost for last cycle in both the cases, PCL, is given by

$$\mathrm{PCL} = c_p Q e^{-RTN}. \tag{48}$$

Present value of expected purchase cost for last cycle, EPCL, is given by

$$\mathrm{EPCL} = c_p \mathrm{QES}_6. \tag{49}$$

Thus, total expected profit from last cycle Z2 is given by

$$\mathrm{Z2} = (\mathrm{ESPL}+\mathrm{ESPQH}) - \mathrm{EPCL} - (\mathrm{EH1L}+\mathrm{EH2L}) - \mathrm{EOCL} - \mathrm{ETCL}. \tag{50}$$

4.3. Total expected profit

Incorporating all the cases and subcases, the total expected profit during the planning horizon H, Z is given by

$$Z = \mathrm{Z1}+\mathrm{Z2}, \tag{51}$$

where Z1 and Z2 are given by Eqs. (25) and (50) respectively.

5. Numerical Illustration

Following parametric values are assumed to illustrate the proposed model.

$$Q_0=30,\quad a=75,\quad b=1.15,\quad \lambda=0.1,\quad i=0.07,\quad r=0.15,\quad c_p=30,\quad c_s=42,$$
$$c_{sr}=20,\quad c_{h2}=1.5,\quad c_{h1}=2.5,\quad c_t=1,\quad c_{o1}=200,\quad c_{o2}=1.2.$$

Table 1
Results obtained for the model via RRGA

T	Q_r	Z
0.86	7.89	5176.03

Table 2
Results obtained for the model for different R

R	T	Q_r	Z
0.08	0.86	2.80	4807.98
0.09	0.85	0.06	4488.63
0.10	0.83	0.04	4204.26
0.11	0.81	0.04	3948.29
0.12	0.79	0.04	3716.77
0.13	0.77	0.04	3506.43
0.14	0.75	0.04	3314.57
0.15	0.74	0.04	3138.91
0.16	0.72	0.04	2977.53

Table 3
Results obtained for the model for different λ

λ	T	Q_r	Z
0.11	0.86	3.53	4861.82
0.12	0.86	0.06	4588.03
0.13	0.85	0.06	4344.48
0.14	0.83	0.04	4124.81
0.15	0.82	0.04	3925.64
0.16	0.81	0.04	3744.34
0.17	0.79	0.04	3578.65
0.18	0.78	0.04	3426.72

For the above parametric values, results are obtained via RRGA and presented in Table 1.

For the above parametric values, results are obtained for different values of R, i.e., for different resultant effect of inflation and time value of money on expected profit and presented in Table 2. It is observed that as R increases expected profit Z decreases, which agrees with reality.

Again for the above parametric values, results are obtained for different values of λ, i.e., for different mean values of stochastic planning horizon H and presented in Table 3. It is observed that as λ increases expected profit Z, decreases. It happens because as λ increases mean value of H, $1/\lambda$ decreases, which in turn decreases the expected length of planning horizon. Here, the total expected profit during the planning horizon is optimized. So as expected value of planning horizon decreases, expected value of total profit decreases.

6. Conclusion

For the first time an inventory model with stock-dependent demand with two storage facilities is developed for a random planning horizon considering the effect of inflation and time value of money on different inventory costs. To solve the model a genetic algorithm is developed where diversity among the solutions of the populations is maintained using entropy theory and as iteration proceeds search space gradually decreases to the optima. The reason for adaptation of this model is twofold.

Firstly it is very difficult to define time horizon of an inventory problem precisely – specially for seasonal goods, which are normally stochastic in nature, which render stochastic planning horizon for the model. Another reason for the adaptation of the model is the crisis of having large space in the important market place, which did not exist few years ago. Earlier two-warehouse models deal with one own warehouse

(OW) at the market place and one rented storehouse (RW) at a distant away. The holding cost at OW is assumed to be less than that of RW. But in real life, now-a-days, it is the reverse as normally, both warehouses are hired. Hence the holding cost at the main marketing place is more than that at the distant storage house. Such a realistic situation has been considered in this model. So, from the economical point of view, the proposed model will be useful to the business houses in the present context as it gives a better inventory control system.

Appendix 1

$$\mathrm{ES}_1=\sum_{N=0}^{\infty}\int_{NT}^{N(T+1)}\frac{1-\mathrm{e}^{-NRT}}{1-\mathrm{e}^{-RT}}\lambda\mathrm{e}^{-\lambda H}\mathrm{d}H=\frac{\mathrm{e}^{-\lambda T}}{1-\mathrm{e}^{-(\lambda+R)T}},\tag{52}$$

$$\begin{aligned}\mathrm{ES}_2&=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_0}N\mathrm{e}^{-RH}\lambda\mathrm{e}^{-\lambda H}\mathrm{d}H\\&=\sum_{N=0}^{\infty}\frac{\lambda}{\lambda+R}\{1-\mathrm{e}^{-(\lambda+R)t_0}\}N\mathrm{e}^{-(\lambda+R)NT}\\&=\frac{\lambda}{\lambda+R}\{1-\mathrm{e}^{-(\lambda+R)t_0}\}S_N,\quad \text{where}\end{aligned}$$

$$S_N=\sum\nolimits_{N=0}^{\infty}N\mathrm{e}^{-(\lambda+R)NT}=\frac{\mathrm{e}^{-(\lambda+R)T}}{1-\mathrm{e}^{-(\lambda+R)T}},\tag{53}$$

$$\mathrm{ES}_3=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_0}\mathrm{e}^{-RNT}\lambda\mathrm{e}^{-\lambda H}\mathrm{d}H=\sum_{N=0}^{\infty}(1-\mathrm{e}^{-\lambda t_0})\mathrm{e}^{-(\lambda+R)NT}=\frac{1-\mathrm{e}^{-\lambda t_0}}{1-\mathrm{e}^{-(\lambda+R)T}},\tag{54}$$

$$\mathrm{ES}_4=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_0}\mathrm{e}^{-RH}\lambda\mathrm{e}^{-\lambda H}\,\mathrm{d}H=\frac{\lambda}{\lambda+R}\frac{\{1-\mathrm{e}^{-(\lambda+R)t_0}\}}{\{1-\mathrm{e}^{-(\lambda+R)T}\}},\tag{55}$$

$$\begin{aligned}\mathrm{ES}_5&=\sum_{N=0}^{\infty}\int_{NT}^{NT+t_0}H\mathrm{e}^{-RH}\lambda\mathrm{e}^{-\lambda H}\,\mathrm{d}H\\&=\lambda\left[\frac{TS_N\{1-\mathrm{e}^{-(\lambda+R)t_0}\}-\frac{t_0\mathrm{e}^{-(\lambda+R)t_0}}{\{1-\mathrm{e}^{-(\lambda+R)T}\}}}{\lambda+R}+\frac{1-\mathrm{e}^{-(\lambda+R)T}}{(\lambda+R)^2\{1-\mathrm{e}^{-(\lambda+R)T}\}}\right],\end{aligned}\tag{56}$$

$$\mathrm{ES}_6=\sum_{N=0}^{\infty}\int_{NT}^{N(T+1)}\mathrm{e}^{-RNT}\lambda\mathrm{e}^{-\lambda H}\,\mathrm{d}H=\frac{1-\mathrm{e}^{-\lambda T}}{1-\mathrm{e}^{-(\lambda+R)T}},\tag{57}$$

$$\mathrm{ES}_7=\sum_{N=0}^{\infty}\int_{NT+t_0}^{N(T+1)}\mathrm{e}^{-RNT}\lambda\mathrm{e}^{-\lambda H}\,\mathrm{d}H=\frac{\mathrm{e}^{-\lambda t_0}-\mathrm{e}^{-\lambda T}}{1-\mathrm{e}^{-(\lambda+R)T}},\tag{58}$$

$$\mathrm{ES}_8=\sum_{N=0}^{\infty}\int_{NT+t_0}^{N(T+1)}\mathrm{e}^{-RH}\lambda\mathrm{e}^{-\lambda H}\,\mathrm{d}H=\frac{\lambda}{\lambda+R}\frac{\mathrm{e}^{-(\lambda+R)t_0}-\mathrm{e}^{-(\lambda+R)T}}{1-\mathrm{e}^{-(\lambda+R)T}},\tag{59}$$

$$\mathrm{ES}_9=\sum_{N=0}^{\infty}\int_{NT+t_0}^{N(T+1)}\mathrm{e}^{-(b+R)H}\mathrm{e}^{bNT}\lambda\mathrm{e}^{-\lambda H}\,\mathrm{d}H=\frac{\lambda}{\lambda+b+R}\frac{\{\mathrm{e}^{-(\lambda+b+R)t_0}-\mathrm{e}^{-(\lambda+b+R)T}\}}{\{1-\mathrm{e}^{-(\lambda+R)T}\}}.\tag{60}$$

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