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Training Research Mathematicians *circa* 1900: The Cases of the United States, Germany, France, and Great Britain

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Introduction

In 1891, the three-year-old, New York Mathematical Society began publishing its *Bulletin* in an effort to communicate with a small but growing constituency of American research-level mathematicians. At least symbolically, this local group united a national community after 1894 when it changed its name to the *American* Mathematical Society. Its *Bulletin*, published ten times a year,[1] was primarily a venue for short research articles and book reviews, but it also aimed to keep its readers abreast of news of the emerging profession through its 'Notes' department. There, America's mathematicians could read of each other's promotions and movements from school to school as well as of mathematical news from abroad. In particular, they could stay informed, essentially semester by semester, about the research-oriented courses of study being offered at institutions both at home and, in particular, in Germany, France, and England. It was about programs in these countries that members of the emergent American mathematical research community most wanted up-to-date information. These were the countries that they viewed as the primary centers for advanced training open to and most viable for them around 1900. How, then, were would-be mathematicians trained in these four countries—Germany, the United States, France, and Great Britain, especially England—at the turn of the twentieth century?

Interestingly, this is not a question that could even have been asked a quarter-century earlier. Prior to the nineteenth century, there was no

[1] The *Bulletin* came out monthly, except during the two summer months of July and August.

Karen Hunger Parshall, *Training Research Mathematicians* circa *1900* In: *A Global History of Research Education: Disciplines, Institutions, and Nations, 1840–1950*. Edited by: Ku-ming (Kevin) Chang and Alan Rocke. *History of Universities* XXXIV/1, Oxford University Press (2021).
DOI: 10.1093/oso/9780192844774.003.0005

formal, research-level training in mathematics.[2] In Europe, mathematical talent was fostered, for example, in the context of scientific societies—the Berlin Academy, the Paris Académie des sciences, the Royal Society of London—while in the United States, a country born only in the late eighteenth century, it was scarcely fostered at all. In some sense, advanced training in mathematics only began in the United States in 1876 with the founding of the Johns Hopkins University on what its first president, Daniel Coit Gilman, interpreted as the German model.[3] How and when did other centers for mathematical training at the higher level develop? What, if any, were their interrelations? Considering these questions from the perspective of would-be, turn-of-the-twentieth-century American mathematicians serves not only naturally to unite the United States, Germany, France, and particularly England in Great Britain for the first time in a comparative analysis of research-oriented training, but also to provide interesting insights into the implementation of that level of training on both sides of the Atlantic.[4]

The Prussian Universities as a Model[5]

As is well known, the opening decade of the nineteenth century sent shockwaves through a Prussia defeated in 1806 at Jena at the hands of Napoleon's French army. A series of political, socioeconomic, and

[2] Indeed, this was the case in other subjects as well. See, for example, Joseph Ben-David, 'The Universities and the Growth of Science in Germany and the United States', *Minerva*, 7 (1968–1969), 1–35 on 7.

[3] As Gert Schubring has argued, however, there was no one German model. Students of higher education like Gilman were actually informed by the example of the Prussian universities, particularly the University of Berlin and, after its incorporation into Prussia in 1866, Göttingen University. See Gert Schubring, 'Pure and Applied Mathematics in Divergent Institutional Settings in Germany: The Role of Felix Klein', in David E. Rowe and John McCleary (eds.), *The History of Modern Mathematics*, 2 vols. (Boston, 1989), ii. 171–220.

[4] Using the United States as the lens through which to structure this comparative analysis should in no way suggest that the United States, Germany, France, and Great Britain were the only countries at the turn of the twentieth century where educational developments in graduate-level mathematics were taking place. China, Italy, Japan, Russia, Spain, Sweden, and elsewhere could also be included in a fuller discussion. For a sense of developments outside the four countries examined here, see Karen Hunger Parshall, 'Mathematics in National Contexts (1875–1900), An International Overview', in *Proceedings of the International Congress of Mathematicians: Zürich*, 2 vols. (Basel/Boston/Berlin, 1995), ii. 1581–91 on 1582–3 and the more elaborated version 'How We Got Where We Are: An International Overview of Mathematics in National Contexts (1875–1900)', *Notices of the American Mathematical Society* 43 (1996), 287–96 on 288–9 as well as the various chapters in Karen Hunger Parshall and Adrian C. Rice (eds.), *Mathematics Unbound: The Evolution of an International Mathematical Community, 1800–1945*, AMS/LMS Series in the History of Mathematics, 23 (Providence and London, 2002).

[5] For the account here of the situation in Germany, compare Parshall, 'Mathematics in National Contexts', 1582–3 and Parshall, 'How We Got Where We Are', 288–9.

educational reforms ensued that aimed at reorganizing, strengthening, and modernizing the kingdom. Perhaps the biggest educational reform was the founding of the University of Berlin in 1810 at the suggestion of the Prussian educational reformer, Wilhelm von Humboldt, elder brother of the celebrated traveler, geographer, and naturalist, Alexander von Humboldt. In addition to overhauling primary and secondary education in the kingdom, the elder Humboldt—influenced by idealist philosophy and neohumanism—crafted a vision of higher education in which professors should both teach and engage in pure and disinterested research free from outside political or religious influences.[6] Moreover, they should add to the store of knowledge through their own efforts—at the same time that they actively trained future researchers—in order to perpetuate the advancement of knowledge. As it came to be implemented first at the University of Berlin and then elsewhere, Humboldt's vision rested on the twin principles of *Lehr- und Lernfreiheit*, the freedom of the faculty to teach and of the faculty and the students to learn and to research unencumbered. Beginning with philosophy and philology and soon extending to the natural sciences, to mathematics, and to other areas, teaching *and* research came to define the dual mission of members of Prussian and other German-speaking university faculties by midcentury.

Interestingly, in mathematics as in history (see Chapter 2 in this volume), higher-level training was first institutionalized not at Berlin, but at the University of Königsberg. The instigator in the case of mathematics was the Berlin-trained mathematical prodigy, Carl Gustav Jacob Jacobi. When Jacobi attended the University of Berlin in the early 1820s, it still offered only elementary mathematics instruction, so Jacobi taught himself from the texts of such eighteenth- and early-nineteenth-century luminaries as Leonhard Euler, Joseph-Louis Lagrange, and Pierre Simon Laplace.[7] On finishing his doctoral dissertation in 1825, Jacobi became a *Privatdozent* at Berlin before moving on to Königsberg in 1826. There, longer-term job prospects seemed better. Indeed, he almost immediately secured an associate professorship, and a full professorship followed in 1832. At Königsberg, Jacobi fully embraced the twin ideals of research and teaching. He produced prodigious amounts of new mathematical ideas, particularly in the theory of elliptic functions but also in the more applied areas of the calculus

[6] For more on this, see Lewis Pyenson, *Neohumanism and the Persistence of Pure Mathematics in Wilhelmian Germany* (Philadelphia, 1983) and Fritz Ringer, *The Decline of the German Mandarins: The German Academic Community* (Cambridge, MA, 1969).

[7] Christoph Scriba, 'Jacobi, Carl Gustav Jacob', in Charles Gillispie (ed.), *Dictionary of Scientific Biography*, 16 vols. and 2 supps. (New York, 1970–1990), vii. 50–5 on 50. On Jacobi's student days in mathematics at Berlin, see Kurt-R. Biermann, *Die Mathematik und ihre Dozenten an der Berliner Universität 1810–1933* (Berlin, 1988), 33–5.

of variations, mechanics, and the theory of first-order partial differential equations. He also enthusiastically taught the fruits of his mathematical labors to his students. It was, moreover, Jacobi who, in bringing together his most mathematically inclined colleagues and the most advanced of his students, inaugurated, in 1834, the first mathematical-physical seminar in Germany. Modeled on the philological seminar he had attended while a student in Berlin, Jacobi's seminar aimed both to expose students to open problems and to train them actively to solve them.[8] Would-be mathematicians Carl Borchardt and Ludwig Otto Hesse, both of whom went on to become mid-nineteenth-century mathematical 'names', not only matured in Jacobi's seminar but also embraced the ideal that providing such training should be part of the university professor's mission.

They were not alone in this. The seminar notion spread from mathematicians in Prussia to those in other German states over the course of the nineteenth century, again, just as it had done in the case of historians. A seminar for mathematics and the natural sciences was founded in 1839 at Halle in the then Prussian Province of Saxony; a mathematical-physical seminar started up in 1850 at Göttingen, which was then in the Kingdom of Hanover; a mathematical-scientific seminar was finally established at Berlin in Prussia in 1860; and a mathematical seminar began at Leipzig in Saxony in 1881.[9] In all of these settings, the seminar provided a key supplement to lecture courses in areas of mathematical research interest. It was a venue in which students were charged both with mastering and presenting mathematical results from the most recent literature in a given area and with actively fielding questions on those results from both their professor and their peers. Unlike the lecture hall, then, the seminar room witnessed active engagement, mathematical give-and-take designed not only to bring students to the threshold of mathematical research but also eventually to see them successfully over that threshold through the production of their own original results.[10]

At Berlin, for example, the mathematical seminar was inaugurated by Ernst Kummer and Karl Weierstrass, two of the nineteenth century's leading mathematicians. Almost immediately perceived, in the words of a key

[8] Biermann, *Die Mathematik und ihre Dozenten an der Berliner Universität*, 60.

[9] Ibid, 97–8. On the mathematical seminar at Leipzig, in particular, see Herbert Becker and Horst Schumann, *100 Jahre Mathematisches Seminar der Karl-Marx-Universität Leipzig* (Berlin, 1981).

[10] For a general description of the mathematical seminar as implemented in German universities, see Karen Hunger Parshall and David E. Rowe, *The Emergence of the American Mathematical Research Community, 1876–1900: J. J. Sylvester, Felix Klein, and E. H. Moore*, HMATH, 8 (Providence and London, 1994), 190–1. See also Wilhelm Lorey, *Das Studium der Mathematik an den deutschen Universitäten seit Anfang des 19. Jahrhunderts*, Abhandlungen über den mathematischen Unterricht in Deutschland, Band III, Heft 9 (Leipzig, 1916).

government official, as 'the most propitious development for mathematical instruction' in Prussia,[11] the seminar was augmented in 1861 by a Mathematics Club (*Mathematischer Verein*) for lectures, discussion, and the posing and solving of mathematical exercises. By 1864, with Weierstrass's elevation to a full professorship, with the effective addition (after 1862) to the teaching staff of another of the nineteenth-century mathematical greats, Leopold Kronecker, with a full complement of lecture courses, with the seminar, and with the Mathematics Club, the program in mathematics at the University of Berlin entered into what has been called its 'heroic era'.[12] It was able to provide, in the context of a two-year-long course of study, what Weierstrass termed 'an important series of lectures on the most important mathematical disciplines'.[13] Mathematicians and mathematical aspirants not only in Germany but also internationally concurred in Weierstrass's assessment. By the mid-1880s, this program was attracting upwards of 250 students a year from all over Europe, from Russia, and from the United States.[14]

The program in Berlin was, however, soon rivaled by that in Göttingen, where, after his move there from Leipzig in 1886, Felix Klein drew increasing numbers of mathematics students, both male and female, especially from the United States.[15] A first-rate researcher and a master teacher, Klein not only trained a significant percentage of what might be termed the 'first generation' of American mathematical researchers but also served as a role model for those students as they returned to the United States to animate graduate programs of their own.

In his lecture courses, Klein exposed his students to the incredibly rich world of nineteenth-century analysis and geometry as fashioned by the likes of Carl Friedrich Gauss, Bernhard Riemann, Niels Abel, Alfred Clebsch, and Karl Weierstrass. These were among the giants of nineteenth-century mathematics, and their works could be notoriously difficult to penetrate, especially for the novice. Still, in Klein's view, theirs was the work on which the future of mathematics would be built, so theirs was the work that future researchers had first to encounter in lectures and then to master both in associated seminars and as actual assistants in writing up the official sets of notes (*Ausarbeitungen*) for each of the courses. Between 1890 and 1896, for example, Klein ran seminars on topics as diverse as 'Partial Differential Equations of Physics, on Cyclides and Lamé Functions',

[11] Biermann, *Die Mathematik und ihre Dozenten an der Berliner Universität*, 99 (quoting the Kultusminister, August von Bethmann-Hollweg; my translation).

[12] Ibid, 102 (quoting the mathematician, Adolf Kneser; my translation).

[13] Ibid, 102 (my translation).

[14] Ibid, 103.

[15] On Klein at Göttingen, and especially on his role in training a generation of American mathematicians, see Parshall and Rowe, *Emergence*, 189–259.

'Hypergeometric and Automorphic Functions', 'Linear Differential Equations and Spherical Functions', the 'Foundations of Analysis for Functions of a Single Variable', and the 'Foundations of Analysis for Functions of Several Variables'. In all of them, the students prepared lectures on specific topics that often required the mastery of research papers recently published in the German mathematical literature.[16]

That this model for the advanced teaching of mathematics and for the training of future researchers was deemed exemplary to those in the emergent American mathematical community was reflected in the fact that, in 1893, the *Bulletin of the New York Mathematical Society* carried a lengthy translation of a circular written by Klein and several of his colleagues on 'The Teaching of Mathematics at Göttingen'. There, would-be American mathematicians were presented with 'a detailed scheme of the lectures and exercises which they should attend during each semester' of their higher mathematical education.[17] The Göttingen curriculum had an 'unusually great' number of courses in mathematics and mathematical physics and treated not only those subjects 'which in the present state of science, have a recognized place in academic instruction; but [also] numerous courses [which] extend into those special departments of science which have only recently been established and are still actually in process of construction'.[18] The curriculum was thus both pure and applied. Indeed, Klein, in particular, was an advocate for applied mathematics even if his more pure-mathematically oriented students tended to find this advocacy less than compelling. This mix of courses was supplemented by seminars explicitly designed 'to lead the students to independent work and to instruct them in the application of what they have learned in the lectures'.[19] By 1893 when this account of the system at Göttingen was published, however, not only had a number of American mathematicians earned their doctoral degrees there but aspects of the broader educational model that Göttingen reflected had already been imported to the United States.

The Importation of 'the German Model' to the United States

The last quarter of the nineteenth century represented a transformative period in the history of American higher education.[20] Whereas earlier in

[16] For a list of Klein's seminars between 1881 and 1896 in which American students spoke as well as for the titles of their lectures, see Parshall and Rowe, *Emergence*, 255–7.

[17] Felix Klein et al., 'The Teaching of Mathematics at Göttingen', trans. Thomas S. Fiske, *Bulletin of the New York Mathematical Society*, 3 (1893), 80–8 on 80.

[18] Ibid, 84. [19] Ibid, 87.

[20] Much has been written on the history of American higher education in this period, but Laurence R. Veysey's *The Emergence of the American University* (Chicago, 1965) remains one of the most cogent and comprehensive analyses.

the century the mission of professors at the nation's colleges had been to impart knowledge, by the century's closing quarter, the professionalization of various academic fields had brought with it a strong sense that research should also be part of that mission (compare Chapter 1 in the present volume). Concurrently, the American Civil War, fought between 1861 and 1865, had witnessed, in addition to much bloodshed, the amassing of a number of great personal fortunes that were soon directed toward philanthropic causes, among them, higher education.

In particular, railroad magnate Johns Hopkins made provisions in his will for a previously unprecedented endowment of $7,000,000 for the founding of a new university with an associated medical school in Baltimore, Maryland.[21] When, two years after his death in 1874, the Johns Hopkins University opened, it represented a new American educational experiment in the hands of its first president, Daniel Coit Gilman. A geographer by training but long a student of higher education both at home and abroad, Gilman found himself essentially free to create an institution of higher education new to the United States, an institution of a sort that America was, in his view, sorely lacking, and an institution that would make the country more competitive with Europe. His vision for the new experiment in higher education had been fundamentally shaped by his experiences in the American colleges and by his observations particularly of the Prussian and some other German-speaking universities.[22]

Gilman's Hopkins would be, first and foremost, a graduate school, modeled on Gilman's interpretation of how advanced training was fostered in the Prussian universities, but adapted to the American educational climate. It would start out small. He would begin by assembling first-rate researchers specifically in mathematics and classics—two critical areas, yet two areas that required little infrastructure beyond books—and they would be joined by the best laboratory scientist he could secure. This initial faculty would grow as suitably strong researchers in other fields were identified and secured, but from the very beginning, the explicit mission of the faculty would be to pursue its research agenda and actively to train and engage graduate students, a number of whom would be supported with fellowships from university funds. The support of research and of the training of future researchers would allow Americans more efficiently and effectively to contribute to the store of knowledge and thus to make their mark in the intellectual arena.

[21] For the early history of the Johns Hopkins University, see Hugh Hawkins, *Pioneer: A History of the Johns Hopkins University, 1874–1889* (Ithaca, 1960).

[22] On the shaping of Gilman's ideas regarding higher education, see, in addition to Ibid, Francesco Cordasco's *Daniel Coit Gilman and the Protean Ph.D.: The Shaping of American Graduate Education* (Leiden, 1960).

In recognition, moreover, of the paucity of American publication outlets for original research in the 1870s, the university would also underwrite the publication of specialized journals in various fields. Nor was undergraduate instruction neglected. Gilman recognized that well-trained undergraduates would feed naturally into his institution's graduate programs. That lower-level work, however, was not supervised directly by the principal members of the research faculty.

For his program in mathematics, Gilman secured James Joseph Sylvester, an English mathematician with an international research reputation.[23] Given that graduate-, that is, research-level mathematics training was not yet institutionalized in Great Britain (see below), Sylvester had never had the opportunity to teach at that level, but he quickly fashioned a program that successfully drew his students into the ranks of the productive researchers. Among the topics on which he lectured were the theory of numbers, determinants and modern algebra, the theory of multiple quantity (or what would today be called matrix theory), the theory of substitutions, and partition theory.[24] Sylvester's curriculum thus focused primarily on the algebraic topics in which he was directly interested, whereas Klein's lecture courses and associated seminars treated aspects of mathematics both pure and applied. Like Klein, though, Sylvester introduced his students to active research, since he regularly challenged them to prove things that he either had not been successful in proving or that he had tossed out as open problems. Moreover, as did his colleagues at the University of Berlin, Sylvester augmented his lecture courses with an associated seminar and a Mathematics Club. Both of the latter met in the so-called Mathematical Seminary, a book- and mathematical-model-lined room dedicated to the program in mathematics in which students studied, researched, wrote, and otherwise interacted. It was, in a mathematical context, a *laboratory* for the production of new results.

From September 1876 when he arrived at Hopkins to December 1883 when he left to assume the Savilian Professorship of Geometry at Oxford, Sylvester oversaw the graduate work of some fifteen mathematics fellows, eight of whom earned the Ph.D. under him. Two of these, Thomas Craig and Fabian Franklin, remained at Hopkins, joining the mathematics

[23] Much has been written about the history of this program and its importance for the history of American mathematics. See, for example, Karen Hunger Parshall, 'America's First School of Mathematical Research: James Joseph Sylvester at The Johns Hopkins University', *Archive for History of Exact Sciences* 38 (1988), 153–96 and Parshall and Rowe, *Emergence*, 53–146. On Sylvester, his life and work, see Karen Hunger Parshall, *James Joseph Sylvester: Life and Work in Letters* (Oxford, 1998) and *James Joseph Sylvester: Jewish Mathematician in a Victorian World* (Baltimore, 2006).

[24] For a complete list of the mathematics courses taught at the Johns Hopkins University between 1876 and 1883, see Parshall and Rowe, *Emergence*, 95–6.

faculty there and trying to continue the graduate program that Sylvester had animated. Another, Irving Stringham, worked (ultimately unsuccessfully) to mount a graduate program in mathematics at the University of California, Berkeley following post-doctoral training in Germany under Felix Klein. Still another, Christine Ladd, had been allowed as an 'exception' to attend Sylvester's courses beginning in 1878 and, by 1882, had satisfied all of the requirements for the Ph.D. Owing to Hopkins's official 'males only' policy, however, she was only awarded her degree in 1926.

These and other Hopkins students and faculty, in addition to Sylvester himself, published the fruits of their mathematical labors in the *American Journal of Mathematics*. Underwritten by the University, launched in 1878, and edited by Sylvester, the *American Journal*, as it name suggested, served an emergent American mathematical research community. Fully a third of the articles appearing in it over the course of its first decade came from those associated in one way or another with Hopkins; another third came from other American contributors; and a final third came from mathematicians abroad.[25] The *American Journal* thus represented yet another novel training- and proving-ground for America's next generation of research mathematicians.

Following the Hopkins example, the trend of graduate training in mathematics gradually spread across the country.[26] Implemented initially, as the example of Sylvester's students illustrates, by a small number of home-grown and a larger number of German-trained mathematics PhDs, it ultimately took root at state-supported universities as well as at so-called land-grant universities that were partially financed by Federal funds. It also infiltrated the colonial, liberal arts colleges like Harvard, Yale, and Princeton that began their transformations into research universities in the

[25] For the percentages, see Karen Hunger Parshall, 'Eliakim Hastings Moore and the Founding of a Mathematical Community in America, 1892–1902', *Annals of Science*, 41 (1984), 313–33 on 324; reprinted in Peter L. Duren *et al* (eds.), *A Century of Mathematics in America—Part II* (Providence, 1989), 155–75.

[26] For more on this trend, see Parshall and Rowe, *Emergence,* 261–94. A substantial literature of case studies on the development of particular research-level programs in mathematics has also developed. See, for example, Parshall, 'E. H. Moore and the Founding of a Mathematical Community in America, 1892–1902'; William Aspray, 'The Emergence of Princeton as a World Center for Mathematical Research, 1896–1930', in William Aspray and Philip Kitcher (eds.), *History and Philosophy of Modern Mathematics* (Minneapolis, 1988), 346–66; Halsey Royden, 'A History of Mathematics at Stanford', in *A Century of Mathematics in America—Part II*, 237–77; Robin Rider, 'An Opportune Time: Griffith C. Evans and Mathematics at Berkeley', in *A Century of Mathematics in America—Part II*, 283–302; Gary Cochell, 'The Early History of the Cornell Mathematics Department: A Case Study in the Emergence of the American Mathematical Research Community', *Historia Mathematica* 25 (1998), 133–53; and Karen Hunger Parshall, 'Training Women in Mathematical Research: The First Fifty Years of Bryn Mawr College (1885–1935)', *The Mathematical Intelligencer*, 37 (2015), 71–83.

final quarter of the nineteenth century. And, it shaped new universities like Clark University and the University of Chicago which were created, like Hopkins, through private benefaction.

The primacy of graduate training was further reinforced by the mostly—but not exclusively—pure research orientation of the American Mathematical Society, the first specialized professional organization for mathematicians in the United States.[27] As the members of that society very quickly came to understand the notion, a *professional* mathematician was one who had earned a *doctoral* degree for an original piece of work and had then continued, in so far as circumstances allowed, both to add to the store of mathematical knowledge and to train—as his, or in rare cases, her[28] institutional circumstances permitted—succeeding generations in the field. By the close of the nineteenth century, then, higher education in the United States had at least two distinct steps: undergraduate training that led to a bachelor's degree and graduate training that led to a doctorate based on original research. In mathematics, as in other academic areas, this second step had thus been instrumental in creating a new category of academic professional.

Influences on France in the Aftermath of the Franco-Prussian War

An analogous notion of the professional mathematician also emerged in France, also in the closing quarter of the nineteenth century, also in the aftermath of war, also influenced by the Prussian example, but ultimately, and perhaps not surprisingly, in the context of very different local circumstances. France's loss in 1871 of the Franco-Prussian War had served as a kind of wake-up call to French intellectuals of all stripes, among them, French mathematicians. Indeed, at least some among the mathematical ranks had already sensed the need for change before the war. Writing to a fellow mathematician, Jules Houël, sometime between 1869 and 1871, Gaston Darboux had opined that 'we need to mend our [system of] higher education. The Germans get the better of us there as elsewhere. I think if that continues, the Italians will surpass us before too long'.[29]

[27] The New York, and later, American Mathematical Society had been preceded by specialized professional societies for research-level mathematics in a number of countries. The earliest, the Moscow Mathematical Society and the London Mathematical Society, were founded in 1864 and 1865, respectively; the Société mathématique de France and the Circolo matematico di Palermo followed in 1872 and 1884, respectively.

[28] See, in particular, Parshall, 'Training Women in Mathematical Research'.

[29] Gaston Darboux to Jules Houël, undated, in Hélène Gispert, 'La correspondence de G. Darboux avec J.Houël : Chronique d'un rédacteur (déc. 1869-nov. 1871)', *Cahiers du séminaire d'histoire des mathématiques* 8 (1987), 67–202 on 161. Also quoted in Hélène

At issue was the pervasive sense that French mathematics had entered a period of stagnation. While France had enjoyed what historian of mathematics Ivor Grattan-Guinness characterized as 'a remarkable dominance in mathematics from the 1780s until the 1820s, with Paris by far the leading center for the subject in the world'[30] in both pure and applied mathematics, it had entered into an era of perceived decline thereafter. At least as early as the 1850s, that is, from the earliest days of the Second Empire (1852–1870), France's strong, top-down control of higher education and its sense that 'to live the life of a *savant* was to engage in a public act as an obedient servant of the Empire'[31] had squelched scientific creativity in general and mathematical creativity in particular.[32] Before 1870, research, if it was supported at all within French higher education, was only fostered in the Collège de France and in the so-called *grandes écoles*, that is, the École polytechnique, the École normale supérieure, and the *écoles d'applications* such as the École des Ponts et Chaussées. Even in these institutions, however, it was expected neither that the faculty necessarily engage actively in research nor that it train future researchers. Within the Université de France with its various *facultés* such as the Sorbonne in Paris, research was even farther removed from the institutional mission.[33] Faculty members in the latter were charged with minimal lecturing—and then at the lower level of basic *cours* as opposed to at a higher, research

Gispert, *La France mathématique: La Société mathématique de France (1872–1914)* (Paris, 1991), 19 (my translation).

[30] Ivor Grattan-Guinness, 'The End of French Dominance: The Diffusion of French Mathematics Elsewhere, 1820–1870', in Karen Hunger Parshall and Adrian C. Rice (eds.), *Mathematics Unbound: The Evolution of an International Mathematical Research Community, 1800–1945*, HMATH, xxiii (Providence and London, 2002), 17–44 on 17. For a sense of the French mathematical scene—both pure and applied—in the first half of the nineteenth century, see Ivor Grattan-Guinness, *Convolutions in French Mathematics, 1800–1840: From the Calculus and Mechanics to Mathematical Analysis and Mathematical Physics*, 2 vols., Science Networks Historical Studies (Basel, 1990).

[31] Robert Fox, 'Science, the University and the State in Nineteenth-Century France', in Gerald L. Geison (ed.), *Professions and the French State, 1700–1900,* (Philadelphia, 1984), 66–145 on 90.

[32] As Hélène Gispert has convincingly argued, French mathematicians in the 1860s, instead of engaging in the latest developments in their field, were still pursuing research agendas reflective of the state of the discipline in the first half of the century. See Hélène Gispert, 'L'Enseignement scientifique supérieure et ses enseignants, 1860–1900: Les mathématiques', *Histoire de l'éducation*, 41 (1989), 44–78 on 50–2.

[33] George Weisz, 'Le corps professoral de l'enseignement supérieur et l'idéologie de la réforme universitaire en France, 1860–1885', *Revue française de sociologie* 18 (1977), 201–32 on 227. Relative to mathematics, in particular, see Ivor Grattan-Guinness, 'Grandes Écoles, Petite Université: Some Puzzled Remarks on Higher Education in Mathematics in France, 1795–1840', in *History of Universities*, 7 (Oxford, 1988), 197–225.

level—as well as with training, examining, and officially certifying secondary school teachers.[34]

Beginning in the mid-1860s, however, calls for the reform of this entrenched system, in which the Université served as a kind of learned bureaucracy, began to be heard not only in the French press but also from the Ministry of Public Instruction (compare Chapters 1 and 6 in the present volume). The latter thus initiated a number of reports on systems of higher education outside of France in order to provide a means of comparison.[35] In particular, as Darboux's statement to Houël attests, a strong sense emerged that, in order to be competitive in mathematics as well as in other fields, France needed to follow the German example, which not only coupled teaching and research but also adopted research as an explicit criterion for professional success.[36]

With the establishment of the Third Republic (1870–1940) and in the aftermath of the Franco-Prussian War, educational reforms were gradually implemented over the closing quarter of the nineteenth century that aimed to address these concerns albeit in a much larger political context. The new leaders of the Third Republic sought to neutralize the old political élite that had been associated with the *grandes écoles* and to create a new one. They did this both by strengthening the *facultés* and by embracing the idea 'that the growth of knowledge was crucial to social improvement and material progress'.[37] In particular, they provided funding to increase the size of the *facultés*, and they severed the administrative ties between the *facultés* and secondary education.[38] As a result, by the end of the nineteenth century, the Faculté des sciences at the Sorbonne in Paris had become a principal training ground for post-baccalaureate students desirous of earning a doctorate. Moreover, original research—as opposed to the demonstration through set examinations of encyclopedic knowledge of one's chosen field—had become the principal evaluative standard.[39]

The explicitly graduate training that post-baccalaureate students received was made possible, to some extent, by the doubling in the sizes of

[34] Terry Shinn, 'The French Science Faculty System 1808–1914: Institutional Change and Research Potential in Mathematics and the Physical Sciences', *Historical Studies in the Physical Sciences*, 10 (1979), 271–332 on 291.

[35] Fox, 'Science, the University and the State in Nineteenth-Century France', 92.

[36] Weisz, 'Le corps professoral de l'enseignement supérieur', 227–9 and Fox, 'Science, the University and the State in Nineteenth-Century France', 94.

[37] Shinn, 'The French Science Faculty System', 302.

[38] Ibid, 303.

[39] Craig Zwerling, 'The Emergence of the École Normale Supérieure as a Centre of Scientific Education in the Nineteenth Century', in Robert Fox and George Weisz (eds.), *The Organization of Science and Technology in France 1808–1914* (Cambridge and Paris, 1980), 31–60 on 35–7.

the science faculties to some 220 instructors and the concomitant increase in numbers of courses during the final four decades of the century.[40] Of especial importance, the augmentation of the faculties with newly minted doctoral degree holders in the position of *maître de conférences* not only opened the curriculum to the possibility of courses in individual research specialties but also made it possible for the professors to spend more of their time directing doctoral students and running actual research seminars on the German model.[41]

Relative to mathematics, these changes took place, just as in the United States, in conjunction with the formation of a specialized professional society for mathematics—the Société mathématique de France founded in 1872—as well as of new specialized journals such as its *Bulletin* founded in 1873.[42] Whereas before 1870, most of those who went into mathematics received their training at the engineering-oriented École polytechnique, in the century's final decades, such students were increasingly likely to have gotten their initial mathematical training at the ostensibly more pedagogically-oriented École normale supérieure.[43] And, whereas before 1870, if a student proceeded to a doctoral degree, it was for a relatively perfunctory piece of exposition, beginning in the decade of the 1880s, they pursued, in a *faculté* of the *Université*, actual doctoral studies aimed at the ultimate production of an original piece of research.[44] The latter level of instruction involved lecture courses and seminars—which were given by the chaired professors—as well as (sometimes) more specialized courses—which were given by the *maîtres de conférences*.[45] It was made possible, in part, by the fact that, with the doubling of the science faculties in general, the number of mathematics instructors also doubled nationwide from

[40] For the numbers, see Gispert, *La France mathématique*, table 1.2, 165.

[41] Shinn, 'The French Science Faculty System', 306–7. See also Gispert, 'L'Enseignement scientifique supérieure et ses enseignants', 59.

[42] On the professionalization of French mathematics and its symbiotic relationship with educational reforms, see Gispert, *La France mathématique,* and Hélène Gispert (ed.), *La France mathématique de la Troisième République avant la Grande Guerre* (Paris, 2015).

[43] For more on the place of the École normale in the development of mathematics in France, see David Aubin, *L'Élite sous la mitralle: Les normaliens, les mathématiques et la grande Guerre 1900–1925* (Paris, 2018).

[44] Hélène Gispert, 'The Effects of War on France's International Role in Mathematics, 1870–1914', in *Mathematics Unbound*, 105–21 on 109. On the École normale supérieure in particular, see Zwerling, 'The Emergence of the École Normale Supérieure'. The Sorbonne in Paris, but increasingly the *facultés* outside the capital city, attracted would-be mathematicians for their doctoral training.

[45] Shinn, 'The French Science Faculty System', 306–7. According to American mathematician, James Pierpont, however, at the Sorbonne, the *maîtres de conférences* were limited to conducting what were tantamount to recitation sections associated with the lecture courses given by the professors. See James Pierpont, 'Mathematical Instruction in France', *Bulletin of the American Mathematical Society*, 6 (1900), 225–49 on 235–6.

thirty-two in 1860 to sixty-four in 1900.[46] For example, as a student in the 1890s, Élie Cartan, one of France's leading mathematicians of the first half of the twentieth century, took courses at the Sorbonne on analysis from Paul Appell, the Chair of Mechanics, on elliptic functions under Charles Hermite, the Chair of Analysis, and on group theory from Gaston Darboux, the Chair of Higher Geometry and Dean of the Faculté des Sciences, while at the École normale supérieure, he studied function theory under Émile Picard, the Chair of Differential Calculus.[47] The thesis that Cartan presented to the Faculté des Sciences for his doctoral degree in 1894, 'Sur la structure des groupes de transformations finis et continus', represented a major breakthrough, namely, a classification of the simple complex Lie algebras.[48] By 1904, mathematician and then Dean of the Paris Faculty of Sciences, Paul Appell, could legitimately state that 'beyond their mission of making the sciences known and understood, the institutions of higher education…have another [mission], nobler than all the others, that of advancing science and of continually initiating new generations of researchers into the methods of invention and discovery'.[49] Graduate training in mathematics had been institutionalized; cutting edge courses in the field had been introduced into the curriculum; first-rate research had been produced.[50]

Writing on 'Mathematical Instruction in France' in the *Bulletin of the American Mathematical Society* in 1900, Yale mathematician James Pierpont recognized this evolution in French higher mathematics education. In his article, he sought to counterbalance what he deemed the 'excessive German influence' on American mathematics not only by describing the French system in detail to his American audience but also by encouraging American students to opt for post-baccalaureate or, in fact, post-doctoral study in France and not just in Germany.[51] Still, between 1891 and 1906 only fourteen Americans—or some 4.4% of those members of the American mathematical community who studied abroad—had pursued advanced studies in France, whereas almost

[46] Gispert, *La France mathématique*, table 1.2, 165.

[47] Cartan's course notebooks are held in the Fond Élie Cartan 38J, Archives de l'Académie des Sciences de Paris. See Carton 1: Cahiers 1.01, 1.02, 1.04, 1.08, and 1.09.

[48] See Élie Cartan, *Première Thèse: Sur la structure des groupes de transformations finis et continus* (Paris, 1894). On Cartan's early work, see Karen Hunger Parshall, 'Joseph H. M. Wedderburn and the Structure Theory of Algebras', *Archive for History of Exact Sciences* 32 (1985), 223–349 on 291–2 and Thomas Hawkins, 'Wilhelm Killing and the Structure of Lie Algebras', *Archive for History of Exact Sciences*, 23 (1977), 119–63.

[49] Quoted in Gispert, *La France mathématique*, 60 (my translation). In the papers cited in note 6 above, I inadvertently attributed this quote to Émile Picard.

[50] Gispert, *La France mathématique*, 62–3.

[51] Pierpont, 'Mathematical Instruction in France', 225.

100—or 30.6%—had done so in Germany.[52] Despite France's mathematical gains in the quarter-century after the Franco-Prussian War, Germany remained the top foreign destination for would-be American mathematicians until the 1920s.

Great Britain as a Late-Comer to Graduate Education in Mathematics

Behind even France, Great Britain had attracted a mere eleven American students of mathematics—just 3.4%—in the fifteen years from 1891 to 1906.[53] Given the common language and the fact that the colonial American colleges had largely been fashioned on the Oxbridge college model, it might have been expected that, at the very least, England would have represented more of a draw for American mathematical aspirants. The United States, however, had embraced its interpretation of the German system both in forming new institutions of higher education like the Johns Hopkins University and the University of Chicago as well as in eventually grafting research-oriented, graduate education onto some of the traditional liberal arts colleges like Harvard. Oxbridge, for its part, had been slow to react to calls for educational reform geared toward research and the production of future researchers (compare Chapter 5 in this volume). Instead, it had persisted in a college-oriented, examination- and cramming-dominated system that emphasized a traditional liberal education.[54]

Cambridge, long the more mathematically oriented of the two ancient universities,[55] was dominated by the Mathematical Tripos, the notorious examination that all 'reading men', regardless of their interests, had to take in order to obtain a bachelor's degree with honors. Training for the Tripos was done largely outside of the colleges and in the context of intensive

[52] Della Dumbaugh Fenster and Karen Hunger Parshall, 'A Profile of the American Mathematical Research Community: 1891–1906', in Eberhard Knobloch and David E. Rowe (eds.), *The History of Modern Mathematics*, 3 (San Diego, 1994), 179–227 on 205. The numbers given here represent a sort of greatest lower bound. According to Hélène Gispert, in 1901, only six Americans were studying mathematics in France, and that number had increased to only seven by 1914 and the outbreak of World War I. See Gispert, *La France mathématique*, 140.

[53] Fenster and Parshall, 'A Profile of the American Mathematical Research Community', 205. Again, this number represents a greatest lower bound.

[54] Renate Simpson, *How the Ph.D. Came to Britain: A Century of Struggle for Postgraduate Education* (Guildford, 1983), 52.

[55] On Cambridge and its strong tradition in the physics-oriented mathematical sciences, see Andrew Warwick, *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Chicago, 2003).

drilling and problem-solving under the supervision of private tutors.[56] Problem-solving and memorization, these were the main skills honed as a result of mathematical training at Cambridge throughout the nineteenth century.[57] The curriculum tested, moreover, was an increasingly antiquated one dominated by analytic geometry, conic sections, the differential and integral calculus, and Newtonian mechanics.[58] Physics thus largely defined what mathematics was deemed 'of interest'. Writing of Trinity College, Cambridge during his own student days there in the late 1890s, the noted early twentieth-century mathematician, G. H. Hardy confessed that it was only when he read Camille Jordan's 'remarkable' *Cours d'analyse de l'École polytechnique* (1909) that he 'learnt for the first time…what mathematics really meant'.[59] In his view, the official mathematical curriculum of his Cambridge education had done nothing truly to enlighten him.

Still, even in Hardy's student days at Cambridge, change had been and continued to be afoot in the context of higher education in England via a series of royal commissions. As early as 1850, the Oxford Commission had recommended that fellowships in (and funded by) the various Oxford colleges be converted into professorships in targeted, more specialized areas associated with the university. University professors would then be in a position to provide training outside the liberal arts confines of the college curriculum.[60]

From 1870 to 1875, the Devonshire Commission—with commissioners including such Victorian scientific worthies as the biologist Thomas Huxley, and the Cambridge mathematical physicist George Gabriel Stokes—cast an even wider net, examining scientific instruction nationwide. Among its many recommendations was the call for the institution of degrees awarded for the production of original research. This reflected the concurrent professionalization of science in Great Britain as exemplified in mathematics by the founding in 1865 of the London Mathematical Society. In 1885, the modern D.Sc., that is, a doctoral degree of just the sort advocated by the Devonshire commissioners, was adopted at the

[56] On perhaps the most famous of the nineteenth-century Cambridge mathematics tutors, see Alex D. D.Craik, *Mr Hopkins' Men: Cambridge Reform and British Mathematics in the 19th Century* (London, 2007).

[57] A. G. Howson, *A History of Mathematics Education in England* (Cambridge, 1982), 139–43.

[58] Ibid, 143 and Parshall, *James Joseph Sylvester: Jewish Mathematician in a Victorian World*, 44–8.

[59] G. H. Hardy, *A Mathematician's Apology*, with a foreword by C. P. Snow (Cambridge, 1967), 147. Hardy was a student at Trinity from 1896 to 1898, finishing Fourth Wrangler on the Mathematical Tripos in 1898.

[60] Simpson, *How the Ph.D. Came to Britain*, 24.

University of London, and schools like Newcastle, Manchester, and Leeds among the so-called 'red bricks' had begun actively to train students for the advanced London degree.[61] Although Oxford would only follow suit in 1900, Cambridge actually preceded London by three years in instituting a doctoral degree.

As far as actual graduate *instruction* was concerned, however, 'there was nothing' in Great Britain at the turn of the twentieth century that, in the words of historian of British education Renate Simpson, 'could as yet be even remotely described as systematic instruction for graduate students'.[62] Thomas Muir explicitly articulated this point relative to mathematics in a speech before the Mathematical Society of Edinburgh in 1884. Acknowledging that '[w]e recognize two of the functions of a University—instruction and research', he ruefully admitted that 'we ignore, so far as mathematics is concerned, a third equally important function—*instruction in research*'.[63]

Indeed, James Joseph Sylvester, who had so successfully animated the graduate program at Hopkins, tried to perform precisely that third function from his new position as Oxford's Savilian Professor of Geometry. During the course of a public lecture delivered in December 1885, he proposed to give, as he had done at Hopkins, 'lessons in the difficult art of mathematical thinking and reasoning—how to follow out familiar suggestions of analogy till they broaden and deepen into a fertilizing stream of thought—how to discover errors and to repair them'.[64] He sought, in short, to make Oxford a Hopkins on the Cherwell, and, at least initially, he attracted a 'class of 14 or 15 comprising several (5 or 6) of our college tutors to whom [he] lecture[d] twice a week on Reciprocants', the subject of his own then developing algebraic research.[65] By March 1887, however, ever-decreasing numbers in his classes had left him dejected. As he wrote to Gilman at Hopkins, 'I am out of heart in regard to my Professorial work

[61] Ibid, 48–50. The D.Sc. could be earned—for an original piece of research—after a minimum of two years following the bachelor's degree.

[62] Ibid, 66.

[63] Thomas Muir, *The Promotion of Research: with Special Reference to the Present State of the Scottish Universities and Secondary Schools: An Address Delivered before the Mathematical Society of Edinburgh, 8th February 1884* (London, 1884), 11 (his emphasis), as quoted in Simpson, *How the Ph.D. Came to Britain*, 43.

[64] James Joseph Sylvester, 'Inaugural Lecture at Oxford, on the Method of Reciprocants', in *The Collected Mathematical Papers of James Joseph Sylvester*, Henry F. Baker (ed.), 4 vols. (Cambridge, 1904–1912; reprint ed. New York, 1973), iv. 278–302 on 298. Quoted in Parshall, *James Joseph Sylvester: Jewish Mathematician in a Victorian World*, 299. See this same book (278–303, especially 296–303) for more on Sylvester's efforts at Oxford.

[65] James Joseph Sylvester to Arthur Cayley, 18 February, 1886, James Joseph Sylvester Papers, St. John's College, Cambridge, Box 12, as quoted in Parshall, *James Joseph Sylvester: Jewish Mathematician in a Victorian World*, 300.

in this University in which the real power of influencing the studies of the place lies in the hands of the College Tutors and in which I can see no prospect of doing any real good.'[66] In his view, 'this University except as a school of taste and elegant light literature is a magnificent sham. It seems to me that Mathematical science here is doomed and must eventually fall off like a withered branch from a Tree which derives no nutriment from its roots.'

Little wonder, then, that, according to Harvard President Charles Eliot in 1903, '[n]one of the higher degrees offered by Oxford University . . . could, I think, compare in attractiveness for American students with the German degree of Doctor of Philosophy'.[67] For mathematicians, the latter, as noted above, was earned in a rich and intense environment characterized by lecture courses, the seminar, and the *Mathematischer Verein*, while, at Oxford, as Sylvester's experience attested, that kind of mathematical atmosphere was simply not yet fostered. Oxford would only introduce in 1917 a doctoral degree, the D. Phil., comparable to the German or American doctoral degree. Cambridge and the University of London would follow two years later in 1919.[68]

A Comparative Assessment and a Broader Conclusion

Turn-of-the-twentieth-century American mathematicians found themselves at an interesting crossroad. Over the course of the final quarter of the nineteenth century, a community of research mathematicians had been emerging in the United States in the intertwined contexts of the professionalization of the field and the evolution of higher, graduate education in the nation. For them, teaching, research, and the training of future researchers came to define the *professional* mathematician. They were thus hard at work not only pursuing their mostly, although not exclusively, pure personal research agendas but also—when their institutional settings allowed—developing graduate programs in their field that would, one day they hoped, rival the German programs they were emulating. They fashioned high-level lecture courses; they instituted seminars; they founded mathematical clubs; and they did all of this in the context of a *university* as opposed to a *college* ethos that had evolved as their educational leaders shaped universities according to what they understood to be the

[66] James Joseph Sylvester to Daniel Coit Gilman, 11 March, 1887, Daniel Coit Gilman Papers, Coll #1 Corresp., Johns Hopkins University, in Parshall, *James Joseph Sylvester: Life and Work in Letters*, 263. The quote that follows is also from this letter.

[67] Charles Eliot to David B. Munro, Vice Chancellor of the University of Oxford, 28 July, 1903, as quoted in Simpson, *How the Ph.D. Came to Britain*, 77.

[68] Ibid, 147–59.

German model. Still, by 1900, they recognized that they had not yet reached their goal. What better way to train themselves at the research level, then, than actually to travel to Germany, to experience directly the instruction of those acknowledged masters, and then to import their newly gained knowledge to their programs at home? It was in this way that German, and especially Prussian, ideals of research-oriented training in mathematics were transplanted to the United States.

If Germany was widely perceived as enjoying mathematical hegemony around the turn of the twentieth century, the Americans were well aware that France and Great Britain were countries with long mathematical histories that continued to produce important mathematical work. As places for research-level study and training, however, they were decidedly less attractive than Germany. Indeed, first France and then especially England in Great Britain—in the context of very different national circumstances—reformed their systems of higher education to bring them more in line with standards and practices in place in Germany.[69] This comparative study reveals how, by the interwar period, all four of these countries had come to support advanced programs comparable in effectiveness relative to training future researchers. Members of all four—and others as well—were engaged in a mathematical dialogue in person in the context of the quadrennial International Congresses of Mathematicians that had begun in Zürich in 1897 as well as via publication in research-level periodicals that transcended national boundaries. Indeed, in coming to share the same notion of what it meant to be a professional mathematician—key aspects of which were graduate-level training, the attainment of the Ph.D., and the production of original research—all four had begun to transcend the merely national and were taking part in what was increasingly becoming an international mathematical research community.[70]

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[69] For a sense of the programs in place in all four countries in 1900, see the course offerings listed in the *Bulletin of the American Mathematical Society*, 6 (1900), 355–8 (for Columbia, Cornell, and Harvard); 6 (1900), 409–10 (for the University of Chicago and Yale); 6 (1900), 464–5 (for the University of California, Berkeley and the Johns Hopkins University); 7 (1900), 40–4 (for Berlin, Göttingen, and other German universities); 7 (1900), 103–6 (for Cambridge University); and 7 (1900), 150–1 (for the University of Paris and Oxford University).

[70] For more on this process of internationalization, see Parshall and Rice, *Mathematics Unbound* and Karen Hunger Parshall, 'The Internationalization of Mathematics in a World of Nations: 1800–1960', in Eleanor Robson and Jackie Stedall (eds.), *The Oxford Handbook of the History of Mathematics* (Oxford, 2009), 85–104.