6

Causation in Terms of *causae causantes*

We now turn to the task of analyzing causation in Branching Space-Times. Since BST has a 'pre-causal order' $<$ as one of its primitives, one might be tempted to pick $<$ as the causal relation. Yet this relation obviously falls short of being a fully fledged causal notion. Consider the pre-causal order of either Minkowski space-time or branching time. No one thinks that in all those cases in which the pre-causal order relation $<$ holds between two events, and in which therefore the first is in the 'causal past' of the second, the earlier event 'is a cause of' the later event. The sun rose six hours and four minutes ago, and a red car is passing the house right now, but the sun's rising is not a cause of the car's passing in any meaningful way. A useful BST theory of causation will have to be more elaborate.

6.1 Causation: Causes and effects as BST transitions

Working toward a more sensible account of causation that will still be based on BST's pre-causal relation, we begin with the uncontroversial assumption that causation is a two-place relation between cause and effect. Controversial, but crucial for any theory of causation, are responses to the following two questions.

Question 6.1. What is caused, that is, which entities are *effects*?

Question 6.2. What causes, that is, which entities are *causes*?

As is well known, there are numerous different answers to these questions. One main divide is between singular and generic causes and effects. Obviously, both notions are important—consider 'Bob's sudden move caused the boat to rock' vs. 'smoking causes cancer'. It seems both ontologically most promising and most in line with our framework to address questions of singular causation first. Thus we look for singular, concrete causes and effects that are anchored in the branching histories of a BST structure. What sorts

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of entities are causes and effects? A common intuition has it that causation involves change, but it is notoriously difficult to say what precisely constitutes a change. The notion of a transition, developed in Chapter 4.1, is a liberalized notion of change. Following this idea, in causation in Branching Space-Times the crucial causal notion is therefore that of a transition.

The answer to question 6.1, from the point of view of causation in Branching Space-Times, is accordingly:

Answer to Question 6.1: *Transitions* are caused.

Given this decision, one needs to ask for causes of a transition from an initial event to an outcome event. In other words, an effect always has to involve not just an outcome event, but also an initial event. This decision appears to be well aligned with our practice of responding to questions about causes. Typically, a context of uttering a causal question delineates how far into the past we are supposed to look for causes. We hardly ever mention the Big Bang or the deeds of our remote ancestors when addressing questions concerning the causes of, say, the Tunguska event, or Bob's sudden motion in the boat. In practice, we ignore most remote happenings, although they do come up as causes on some accounts of causation. On our BST approach, the context of a causal question excludes remote causal factors by providing an initial event, which is part of the transition constituting the effect that we are after. Given our formal machinery to be developed soon, it is provable that no BST cause of a transition can occur before any element of the initial defining the transition—see Fact 6.1(1).[1]

The crucial idea of causation in Branching Space-Times is that, in sharp contrast to other theories of causation, *non-trivial causation depends on indeterminism*. A deterministic transition $I \rightarrowtail \mathcal{O}^*$, whose outcome $\mathcal{O}^*$ is bound to occur given that I occurs, has no causes. In terms of occurrence propositions, for a deterministic transition, $H_{[I]} = H_{\langle \mathcal{O}^* \rangle}$. Such a transition does not need any causes, since it happens anyway. Causes in BST are thus understood as indeterministic originating causes (*causae causantes*). This seems to capture our actual everyday usage of the notion of causation better than other accounts.

In philosophy, however, the view that non-trivial causation needs indeterminism is a minority view, even though important philosophers such

[1] To link this idea to some existing literature, the initial of a transition plays the role of Spohn's (1983) concept of background or *Seinsgrund*, or of the background circumstances discussed in Xu (1997).

as Anscombe (1971) and von Wright (1974) have held that position. In contrast, there are—as far as we know—only a few philosophers who object to the intelligibility of causation in a deterministic universe, which suggests that the combination of causation with determinism is not widely perceived to be controversial.[2] It is, therefore, the view that causation requires indeterminism that needs to be defended.

The theory of causation that we will present here is a formal development of the view that objective causation as objective difference-making requires objective indeterminism. It may sound like a platitude to say that causes make a difference to how the world develops. Yet, how to understand the notion of difference-making, and consequently, how to capture it formally, remains contentious. Different accounts of causation employ relations like deducibility, counterfactual dependence, or manipulability in order to draw a distinction between factors that make a difference, and factors that do not make a difference. Now, if the aim is to give an account of objective causation, the notions used to capture the notion of difference-making need to be shown to be objective. This means to argue, for instance, that counterfactual dependence is relevantly independent of our linguistic practices, or that the notion of manipulation is not invariably tied to the technological and scientific development achieved in a given society at a given time. We do not claim that such arguments cannot be given, but we want to stress that they are genuinely required for accounts of objective causation. In contrast, local objective indeterminism, such as represented in BST, provides an inherent and clear-cut notion of objective difference-making, as there are critical junctions at which things can go differently (see Chapter 4.5). More precisely, at a critical junction, two or more alternative continuations are still possible, but immediately after that junction, some continuations are prohibited, while others are still possible. In other words, what happens at a critical junction makes an objective difference: some developments stop being possible, while other remain possible. Local objective indeterminism thus provides the sought-for notion of objective difference-making. In BST, we can spell this out formally in terms of choice points and non-trivial basic transitions.

[2] Philosophers who object to the combination of non-trivial causation and determinism include, as already mentioned, Anscombe and von Wright. A critical view of causation under determinism is also common in some approaches to agency theory. This is important, as an action is certainly a prime example of a causal happening. It is typical in theories of agency to assume that an outcome that is determined (or 'settled') cannot be the outcome of an action. Nobody, that is, sees to what is happening anyway. See Belnap et al. (2001) for a formal statement in terms of the *stit* ('seeing to it that') account of agency.

To continue our defense of the combination of causation and indeterminism, we observe that under determinism it is hard to ensure that a given concept of difference-making picks intuitively adequate candidates for causes: not too many, nor too few, just the right ones. A deterministic world simply does not provide enough useful resources for an objective notion of difference-making.

By way of illustration, we will focus on the concrete example of a deterministic Newtonian pendulum that is completely isolated from its surroundings. Consider a particular event of the bob being in a given position at a given instant. On a determinism-friendly account of objective causation as difference-making, that event has innumerably many causes. After all, the given position is derivable from any instantaneous dynamical state of the pendulum (a pair of the bob's position and momentum at a given instant) and Newton's law of gravitation. Thus, any pair of the bob's instantaneous position and momentum, at any instant—earlier or later than the one under consideration—counts as a cause of the bob's position in question, because any instantaneous position and momentum different from the actual one leads to a different current position of the bob.

One might think that the popular counterfactual approach to causation fares better under determinism, but we are also doubtful. On the counterfactual approach, a cause is an event whose non-occurrence would have prevented the occurrence of the effect. This is taken to mean, roughly, that from among all scenarios (possible worlds) in which the cause-event does not occur, in those that are most similar to actuality, the effect-event does not occur either. It appears that again, this account is overly generous: after all, under determinism, almost any purported change to an earlier state would have resulted in a change to a later state, so that (almost) any earlier state of a deterministic system counts as a cause of its present state. Returning to our Newtonian pendulum, it is plausible to keep Newton's law of gravitation fixed, as sameness of laws trumps other entries on the list of criteria of similarity for possible worlds. But then one has to assent to counterfactuals like "if the bob had had an instantaneous state different from the actual one, it would not have landed in the position that it has now", and so all the previous states count as causes. Hopefully, not every feature of our Newtonian bob is classified as a cause on the counterfactual analysis. Supposing that our bob has a certain color, say blue, we can show that its being blue is *not* a cause of its current position and momentum. Arguably, it is false to say that if the bob had not been painted blue, it would have had a different instantaneous position and momentum than it actually has.[3] Thus, being painted a particular color does not count as a cause of the bob's having a particular instantaneous position and momentum. This is a welcome result, but we do not think that it is satisfactory. The illustration shows that the counterfactual analysis provides a distinction between factors that are relevant for the bob's motion and factors that are irrelevant. But the factors thus relevant to a system's behavior still seem to form a much larger category than the category of properly causal factors. In physics, the family of factors relevant to a system's evolution is subsumed under the heading of 'dynamical states', so each dynamical state should be a cause. This, however, goes against our causal intuitions. We would not say each dynamical state is a cause of a system's behavior, while only irrelevant factors that are not mentioned in respectable physics can count as non-causes. In reaction, one may attempt to define a more discerning category of causal factors by giving a more detailed description of the pendulum, so that one could narrow down the candidates for causes via a relation of similarity. But this move raises the worry that the extra criteria that play a role are too epistemic.

We do not mean to dismiss accounts of deterministic causation out of hand. Our analysis shows, however, that in deterministic contexts, we will either have no causes (this will be the verdict of our own approach), or far too many, or we will have to take recourse to some non-objective criteria. We are after an objective, non-trivializing account in which the only resources that play a role are provided by the objective pre-causal ordering of our world. Such an account will accord with our assumption of indeterminism and with the fact that local transitions are the basic building blocks for objective difference-making in BST. The following answer to our second question, "what are the causes?", therefore seems reasonable:

Answer to Question 6.2: Causes are sets of (especially simple) transitions.

It is important to note that our analysis of causation is linked to indeterminism of a local kind which is captured by BST. If we have a non-trivial, indeterministic transition $I \rightarrowtail \mathcal{O}^*$, by which we mean that I can occur with or without the occurrence of $\mathcal{O}^*$, then even if I occurs, there is at least one local risky junction at which things can go wrong for $\mathcal{O}^*$, that is, at which $\mathcal{O}^*$

[3] But note that assenting to this counterfactual requires a specific context. As the counterfactual analysis proceeds in terms of events, we need to bring in an event of painting the bob, plausibly before the bob is set in motion. Then, to assent to the counterfactual, we need to picture the event of painting as not disturbing the bob's later motion in any way. It is unclear whether that is compatible with determinism.

could be prohibited from occurring. But, similarly, there is a development at this junction that keeps the occurrence of $\mathscr{O}^*$ possible. The causes of a non-trivial transition $I \rightarrowtail \mathscr{O}^*$ are, therefore, those developments at risky junctions that keep the occurrence of $\mathscr{O}^*$ possible.

To translate this idea into the formal framework of BST_{92}, a risky junction for a transition $I \rightarrowtail \mathscr{O}^*$ will be identified with a choice event consistent with the initial I and at which the bundle of histories $H_{\langle \mathscr{O}^* \rangle}$ splits off from a history in which $\mathscr{O}^*$ does not occur. To recall Def. 5.10, the set of such crucial choice points for an outcome event $\mathscr{O}^*$ we called the *cause-like loci* for the outcome, $cll(\mathscr{O}^*)$. The developments at a cause-like locus that keep $\mathscr{O}^*$ possible are identified as those basic outcomes of the cause-like locus that are consistent with the occurrence of $\mathscr{O}^*$. In what follows we will consider cause-like loci not just for outcome events, but for transitions, to be written $cll(I \rightarrowtail \mathscr{O}^*)$. The details will be provided via Def. 6.1. Given no MFB, a cause-like locus $e \in cll(I \rightarrowtail \mathscr{O}^*)$ must be in the past of $\mathscr{O}^*$ (see Fact 5.4 and Exercise 5.3(1)). In this case there is then a unique basic outcome $H \in \Pi_e$ of e that is consistent with $H_{\langle \mathscr{O}^* \rangle}$, namely, $H = \Pi_e \langle \mathscr{O}^* \rangle$ (see Fact 4.7(2) and Exercise 5.3(2)). Thus, in the absence of MFB, the individual local causes for $I \rightarrowtail \mathscr{O}^*$ will be the basic transitions $e \rightarrowtail \Pi_e \langle \mathscr{O}^* \rangle$ with $e \in cll(I \rightarrowtail \mathscr{O}^*)$. Each of these basic transitions will be called a *causa causans*, and together they will be called the *causae causantes*, of $I \rightarrowtail \mathscr{O}^*$ (see Def. 6.2).[4]

Our approach implies that a basic transition from a point event e to one of its outcomes, $e \rightarrowtail H$ with $H \in \Pi_e$, is its own cause. This is as it should be: basic indeterministic transitions constitute the irreducibly indeterministic building blocks of our indeterministic world, so that there can be no further account of why they occur. If we ask why such an indeterministic transition occurred, the only answer that we can give is that that is what happened. And if we ask why the specific outcome H of e occurred rather than one of e's other possible outcomes, there can be no answer—it is a conceptual truth that there can be no contrastive explanation of the occurrence of basic indeterminism.[5]

In the probability theory that we develop in Chapter 7, causes understood as basic transitions will form the building blocks for the construction of

[4] If there is MFB in the BST_{92} structure under consideration, a certain modification is required; see Chapter 6.4.

[5] There is a tendency, which comes forcefully to the fore in the literature on free will, to keep insisting on contrastive explanations even after it has been acknowledged that the indeterminism in question is basic. See Ometto (2016) and Müller and Briegel (2018) for some discussion.

probability spaces, thus providing a close link between causation and probabilities and between indeterministic and probabilistic causation.

6.2 At least an inus condition

The idea behind the theory we develop in detail below is about tracing causality back to its beginnings in objectively indeterministic originating causes or *causae causantes* (we use these as synonyms).[6] The payoff will be a technical result: we will identify certain objects in a BST structure as *causae causantes* of a transition. Needless to say, these objects will be set-theoretical constructions. This raises the question of what reasons we have to believe that these objects are causes, or represent causes working in our indeterministic world. The question is reminiscent of Tarski's problem of how to ensure that a predicate T, as formally defined by him, is indeed a truth predicate that captures the meaning of the classical concept of truth, which is truth as correspondence. Tarski came up with his criterion of adequacy, namely, a definition of T adequately captures the classical concept of truth if every instance of the so-called truth schema is deducible from the definition of T.[7] Do we have any similar tool to ascertain that particular set-theoretical constructions, *causae causantes*, represent causes indeed? Well, in the BST framework, our constructions follow what we believe is a persuasive idea: in indeterministic settings, causes are those developments at risky junctions that keep the effect's occurrence possible. But surely this appeal to persuasiveness falls short of the rigor of Tarski's muster, as not everybody will be persuaded by our standards for persuasiveness. Fortunately, there is semi-formal support for the claim that our particular set-theoretic constructions represent causes, which comes from Mackie's (1974) analysis of causes as *inus* conditions for the occurrence of their effects: a *causa causans* of a transition is *at least an inus* condition for the occurrence of that transition.

[6] In this respect, our theory is indebted to the account of agency of Belnap, Perloff and Xu (2001), which shares an outline with the much earlier theory of von Kutschera (1986).

[7] Tarski discusses the adequacy issue in his famous paper on the concept of truth in formalized languages, which went through a number of versions in different languages: Polish (Tarski, 1933), German (Tarski, 1935), and English (Tarski, 1956). An instance of the truth schema, sometimes called "partial definition of truth", is a sentence like:

"Schnee ist weiß" is T in German iff snow is white.

Let us recall Mackie's idea:

> Quasi-definition of inus condition. An inus condition of an event type is 'an insufficient but non-redundant part of an unnecessary but sufficient condition'. (Mackie, 1974, p. 62)

The surrounding text makes it clear that Mackie has in mind a disjunction of conjunctions such as '*ABC* or *DEF* or *JKL*' (this is his example), such that the whole disjunction is a necessary condition of some *P* (this feature is implicit in Mackie's formula), and each disjunct is sufficient for *P*, and further each element, such as *A*, of a disjunct is a non-redundant part of 'its' conjunction. That is, if *A* is omitted from its conjunction *ABC*, then the remaining conjunction *BC* is no longer sufficient for *P*. Even so, since each *B* and *C* is non-redundant as well, *A* alone is insufficient to bring about *P* — to this end it needs to be combined with *BC*. Under these circumstances, *A* is an 'inus condition' of *P*, that is, an insufficient but non-redundant part of an unnecessary but sufficient condition for the occurrence of *P*. In an alternative mouthful, *A* is a non-redundant though insufficient conjunctive part of a sufficient condition for *P* that is a disjunctive part of a necessary condition for *P*. It is worth observing that all the concepts required for stating inus conditions are well-defined in terms of the relations between occurrence propositions in BST (see Definition 4.2), at least once we settle on a reading of 'non-redundant', which will be discussed in Chapter 6.3.2.1.

The central—and perhaps surprising—result of this chapter is that the *causae causantes* of BST have precisely this 'inus' structure. In cases similar to those considered by Mackie, that is, if an outcome part of a transition can occur in a number of alternative ways (i.e., it is a disjunctive outcome), the transition's *causae causantes* are typically inus conditions. In a less complex case *causae causantes* turn out to be *at least* an inus condition.[8] The notion of *at least* an inus condition is motivated by the observation that inus is a weak condition, which can be strengthened in the appropriate circumstances.

Causal circumstances can be stricter in a variety of ways. To explain this idea schematically, suppose that a necessary condition for *P* has the form '*A* or *DEF* or *JKL*' (that is, there is only *A* in the first disjunct). Since *A* has no companion, it is trivially a non-redundant part of itself; and it is *not* an insufficient part; in short, *A* is thus a nus and at least an inus condition for *P*.

[8] We owe this term to Paul Bartha.

A further case is if the condition has the form '*ABC*' (so *DEF* and *JKL* are missing). Since now the single disjunct *ABC* is both sufficient and necessary for *P*, in this case we say that *A* is an inns condition for *P*, and again we call *A* at least inus condition for *P*.

Yet another case is with '*BC*' and '*DEF* or *JKL*' missing (i.e., *A* stands alone). Then *A* is a necessary and sufficient, ns, and at least an inus, condition for *P*. As it is also non redundant, for trivial reasons, it serves as a case of an nns condition as well, and as that is more informative, we drop the abbreviation ns.

To sum up, by "at least an inus condition for *P*" we mean inus, inns, nus, or nns conditions for *P*.

One might object that by accommodating stricter causal settings and consequently moving from inus conditions to at least inus conditions, we betray Mackie's idea. But we believe that there are various settings for asking causal questions, and that strict causal settings are natural for singular event causation. For Mackie, both effects and inus conditions are types of events, types that may have instances (Mackie, 1974, p. 262). In the present development, however, at least inus conditions are concrete possible events. They are neither types, nor are they considered here as instances of types.[9] Now, a type event like a forest fire (Mackie's example) can be produced along different causal routes, but not so *that* particular forest fire, as the indexical phrase denotes a particular event with a fixed past history. Thus, 'u', abbreviating 'unnecessary' and occurring in 'inus', is to be expected only in a context of multiply realizable objects, which in BST are disjunctive outcomes. Turning to the 'i' in 'inus', its presence seems to be a contingent matter. In everyday examples it is natural to think that a single factor, say the lighting of a match, is *i*nsufficient for bringing about the fire (along a given causal route). On the other hand, in indeterministic contexts, where one focuses on risky junctions (i.e., cause-like loci) at which the occurrence of a putative effect can become jeopardized, a single risky junction is perfectly intelligible. Just think of a particle's radioactive decay that makes Schrödinger's cat well-fed.[10] The transition from the non-decayed to the decayed particle turns out, as it should, to be the single necessary and sufficient (ns) condition for that

[9] Of course every event is an instance of arbitrarily many types. The point is that we consider the concrete possible events themselves, not the types that they may instantiate.

[10] For our love of cats, we prefer the rendering of Schrödinger's cat paradox attributed to John Bell, with the cat well-fed or hungry, rather than the original story with the cat dead or alive.

particular event of feeding the cat, which is thereby also non-redundant (nns).

We note for fairness that a BST analysis of transitions to disjunctive outcomes $\check{\mathbf{O}}$ still falls short of delivering full-fledged generic, type-level causation. The reason is that in such a transition $I \rightarrowtail \check{\mathbf{O}}$, all the alternative component outcomes $\hat{O} \in \check{\mathbf{O}}$ still need to arise from a common initial I. That is, all these scattered outcomes are in the future of possibilities of the initial event I, which fully occurs in one history (see Def. 4.3(1)). So, to handle Mackie's example of the causes of a forest fire (type-event), we need to fix some initial event—which can be as large as the past of the world before the 20th century. This initial event then delimits the forest fires to those that occurred, or might really have occurred, in the 20th century. Clearly, that is not a fully generic notion of the type-event 'forest fire'. Ultimately, the difference between the type-event approach and our BST approach is due to the frugality of the causal structures represented in BST: they are specified merely by their spatio-temporal and modal aspects, without recourse to content. Such content would have to be added, for instance, via appropriate descriptions, implying a much richer formal machinery in the background.

Having explained what the main differences are between our approach and that of Mackie's, it is still worth being clear about three further similarities and dissimilarities, since our account is supposed to gain support from his analysis of causation. First, Mackie (1974, p. 265) distinguishes between 'explaining causes' (facts) and 'producing causes' (events). Given this dichotomy, we shall only concern ourselves here with ontology (i.e., with producing causes and with produced effects or results).

Second, Mackie notes that the 'occurrence' of an event makes no sense if an event is taken to be merely a chunk of space-time: "Causation is not something between events in a spatio-temporal sense" (Mackie, 1974, p. 296). In a crucially important shift, BST theory considers causation as a relation between concrete possible events. The causal relation between such events has a spatio-temporal component, but also a modal one, so that the occurrence of a BST cause or effect does not reduce to merely a spatio-temporal matter. The smallest BST objects that have these two components are non-trivial basic transitions.

Third and finally, Mackie's theory permits the possibility of backward causation. Various analytical shifts make it difficult to compare Mackie on this point directly with BST theory, but the following is true and may shed light on the issue. Central to our theory, as explained in Definition 6.1, is the notion of a 'cause-like locus' for a transition. It follows from the postulates of BST that no cause-like locus for a transition can lie in the future of any part of the outcome-part of that transition (see Fact 6.1(3) for outcome chains). This is perhaps a difference from Mackie. BST theory, however, leaves open whether or not every cause-like locus must occur in the past of the outcome-part. Some such cause-like loci might, as far as BST theory goes, be space-like related to the outcome part, and so neither past nor future. This seems to happen, for example, in the strange case of EPR-like quantum-mechanical correlations of Figure 5.1 (p. 107), which we discussed under the heading of "modal funny business" (MFB) in Chapter 5 and which we will consider in detail in Chapter 8. Our theory works both with and without MFB, but as it is more intuitive to develop it under the assumption of no MFB, we first assume this simplification. Later on, in Chapter 6.4, we then discuss the general case.

6.3 *Causae causantes* in BST_{92} in formal detail

We now turn to the formal theory of *causae causantes*, which we eventually show to be at least inus conditions. The leading idea is to identify *causae causantes* neither with initial events nor with outcome events, but instead with basic transitions. Since basic transitions are different in BST_{92} and BST_{NF}, we need to decide upon one of these frameworks. As before, we develop the theory in BST_{92} structures; we defer remarks about the corresponding BST_{NF} definitions to Chapter 6.5. As stated earlier, in this section we assume that there is no modal funny business.

6.3.1 Defining *causae causantes* in BST_{92}

A transition event is "where" something happens; it is "where" there is a transition from (to use Mackie's language) unfixity to fixity. (The shudder quotes remind us that for transition events there is no 'simple location'.) If Alice voluntarily sends a letter to Bob, there is in her personal life a decisive event after which (but not before which) it is settled that the letter is on its way. If Bob receives the letter that, by choice or chance, might or might not have been sent, then there is for him somewhere on his world line a transition from 'might not receive letter' to 'settled that he will have received letter', but that transition event is purely passive, a mere effect. The *causa causans* in this example is along the world line of Alice, not of Bob.

We have spoken of a single *causa causans*, but of course realistically, a great many *causae causantes* must cooperate in order to produce the receive-letter effect. Each *causa causans* of a given transition keeps the occurrence of that transition possible. We do not take deterministic transition events to be *causae causantes*. (Recall that a transition is deterministic if there is no transition which is locally alternative to it.)

As a first step in our formal construction, we return to the concept of a cause-like locus. In Def. 5.10 we introduced the set of cause-like loci for an outcome O, $cll(O)$. A cause-like locus e for O is a point event that marks a critical juncture for the occurrence of O: at e, the occurrence of O is still possible, but immediately after e, the occurrence of O may be impossible. Here we define cause-like loci not for outcomes, but for general transition events $I \rightarrowtail \mathcal{O}^*$. These may be from an initial I to an outcome chain O, to a scattered outcome $\hat{O}$, or to a disjunctive outcome $\check{\mathbf{O}}$. The basic idea is that a cause-like locus for a transition is a critical juncture for the outcome that is compatible with the occurrence of the initial.

There is one complication: For disjunctive outcomes, which are multiply realizable, it may turn out that some critical junctures for the disjuncts are not critical at all for the occurrence of the whole disjunction. Recall the example of rolling a die from the end of Chapter 4.1, and consider the transition from the die-rolling initial I to the disjunctive outcome $\check{\mathbf{O}} =_{\mathrm{df}} \{\boxed{1}, \ldots, \boxed{6}\}$ of *any* outcome occurring. In this example, each individual (scattered) outcome making up $\check{\mathbf{O}}$, such as $\boxed{3}$, certainly depends on what happens at some critical juncture, but the transition to the complete disjunctive outcome $\check{\mathbf{O}}$ is guaranteed to occur once the initial is finished—the transition $I \rightarrowtail \check{\mathbf{O}}$ is *deterministic* even though it is, in the relevant sense, a disjunction of individually *indeterministic* parts such as $I \rightarrowtail \boxed{3}$. Technically, this means that for a transition to a disjunctive outcome, we cannot simply identify the cause-like loci of that transition with the union of the cause-like loci for the individual transitions to the scattered outcomes that make up the disjunctive outcome. Rather, we need to remove all those cause-like loci that guarantee $\check{\mathbf{O}}$ to occur and whose occurrence therefore makes no difference for the occurrence of the disjunction. The following definition takes care of this complication.

Definition 6.1 (Cause-like loci of a transition). Let $I \rightarrowtail \mathscr{O}^*$ be a transition from an initial to an outcome chain or a scattered outcome. The set of cause-like loci for this transition is

$$cll(I \rightarrowtail \mathscr{O}^*) =_{\text{df}} \{e \in W \mid \exists h \in H_{[I]}[h \perp_e H_{\langle \mathscr{O}^* \rangle}]\}.$$

For $\mathscr{O}^*$ a disjunctive outcome $\check{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$, we take care of the mentioned complication by successively defining deterministic points for $\check{\mathbf{O}}$, reduced cause like loci ($cllr$) for $I \rightarrowtail \hat{O}_\gamma$, and finally $cll(I \rightarrowtail \check{\mathbf{O}})$. We rely on the definition of cll for scattered outcomes that we just gave.

$$\begin{aligned}
DET_{I \rightarrowtail \check{\mathbf{O}}} &=_{\text{df}} \{e \in W \mid H_e \cap H_{[I]} \subseteq H_{\langle \check{\mathbf{O}} \rangle}\}; \\
cllr(I \rightarrowtail \hat{O}_\gamma) &=_{\text{df}} cll(I \rightarrowtail \hat{O}_\gamma) \setminus DET_{I \rightarrowtail \check{\mathbf{O}}}; \\
cll(I \rightarrowtail \check{\mathbf{O}}) &=_{\text{df}} \bigcup_{\gamma \in \Gamma} cllr(I \rightarrowtail \hat{O}_\gamma).
\end{aligned}$$

To illustrate this definition, let us consider fully deterministic transitions to an exhaustive disjunctive outcome that occur in an indeterministic context. The simplest case is an exhaustive disjunctive transition $e \rightarrowtail \Omega_e$ from a single indeterministic event e to the set Ω_e of all its basic scattered outcomes (see Fact 4.8). In this case, $H_{\langle \Omega_e \rangle} = H_e$, $e \in DET_{e \rightarrowtail \Omega_e}$, and for any $\hat{O}_\gamma \in \Omega_e$, we have $cll(e \rightarrowtail \hat{O}_\gamma) = \{e\}$ and $cllr(e \rightarrowtail \hat{O}_\gamma) = \emptyset$. Accordingly, $cll(e \rightarrowtail \Omega_e) = \emptyset$.

More generally, we consider $I \rightarrowtail \mathbf{1}_I$, where $\mathbf{1}_I = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$ with $\bigcup H_{\langle \hat{O}_\gamma \rangle} = H_{[I]}$. (Our die rolling example can serve as an example: for the die-rolling-initial I, we have $\mathbf{1}_I = \{\boxed{1}, \ldots, \boxed{6}\}$.) By saying that the context is indeterministic, we mean that for an individual $\hat{O}_\gamma$, $cll(I \rightarrowtail \hat{O}_\gamma) \neq \emptyset$. However, for any $e \in cll(I \rightarrowtail \hat{O}_\gamma)$, if $h \in H_{[I]}$ splits from $H_{\langle \hat{O}_\gamma \rangle}$ at e, then $h \in H_{\langle \hat{O}_\delta \rangle}$ for some $\delta \in \Gamma$ distinct from γ. Thus, $h \in H_{\langle \check{\mathbf{O}} \rangle}$, so $e \in DET_{I \rightarrowtail \check{\mathbf{O}}}$ by the definition above. Accordingly, again for the reduced sets of cause-like loci, $cllr(I \rightarrowtail \hat{O}_\gamma) = \emptyset$, and hence $cll(I \rightarrowtail \check{\mathbf{O}}) = \emptyset$.

Before we proceed to define *causae causantes*, we state some useful general facts about cll. These are fairly simple in the no-MFB case. We will write them out anyway, with a view to the parallel, more complicated statement of Fact 6.4 for the MFB case to be discussed in Chapter 6.4.

Fact 6.1. *We assume no MFB. Consider a generic transition $I \rightarrowtail \mathscr{O}^*$, i.e., $\mathscr{O}^*$ is an outcome chain O, a scattered outcome $\hat{O}$, or a disjunctive outcome $\check{\mathbf{O}}$.*

(1) *Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$. Then e does not causally precede I, i.e., it is not the case that there is some $e' \in I$ for which $e < e'$.*
(2) *Let $\hat{O}$ be a scattered outcome. Then $cll(I \rightarrowtail \hat{O}) = \bigcup_{O \in \hat{O}} cll(I \rightarrowtail O)$.*
(3) *Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$ and $\mathcal{O}^*$ be an outcome chain O. Then $e < O$, i.e., for every $e' \in O$: $e \leqslant e'$.*
(4) *Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$ and $\mathcal{O}^*$ be a scattered outcome $\hat{O}$. Then there is $O \in \hat{O}$ such that $e < O$.*
(5) *Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$ and $\mathcal{O}^*$ be a disjunctive outcome $\check{\mathbf{O}}$. Then there is $\hat{O} \in \check{\mathbf{O}}$ with an $O \in \hat{O}$ such that $e < O$.*

Proof. (1) For reductio, assume that $e < e'$ for some $e' \in I$. Then for any $h \in H_{[I]}$: $e' \in h$; and by Fact 4.1, for any $h' \in H_{\langle \mathcal{O}^* \rangle}$: $e' \in h'$. Since $e < e'$ we have $h \equiv_e h'$ for any $h \in H_{[I]}$ and any $h' \in H_{\langle \mathcal{O}^* \rangle}$, so $e \notin cll(I \rightarrowtail \mathcal{O}^*)$.

(2) Left as Exercise 6.1.

(3, 4) By Fact 5.4 in Chapter 5 and Exercise 5.3 to that chapter and by noting that $cll(I \rightarrowtail \mathcal{O}^*) \subseteq cll(\mathcal{O}^*)$ for $\mathcal{O}^*$ an outcome chain or a scattered outcome.

(5) follows from the above as an immediate consequence. □

In what follows, we will appeal extensively to a useful consequence of items (3) and (4) of Fact 6.1: If a BST_{92} structure contains no MFB and $\mathcal{O}^*$ is an outcome chain or a scattered outcome for initial I, then all cause-like loci e of the transition $I \rightarrowtail \mathcal{O}^*$ are in the past of $\mathcal{O}^*$, and the outcome $\mathcal{O}^*$ determines a unique basic propositional outcome of e, written $\Pi_e\langle \mathcal{O}^* \rangle$.

Fact 6.2. *Assume no MFB, and let $I \rightarrowtail \mathcal{O}^*$ be a transition to an outcome chain or to a scattered outcome. Then for $e \in cll(I \rightarrowtail \mathcal{O}^*)$, (1) there is a unique basic outcome of e that is compatible with the occurrence of $\mathcal{O}^*$, which we denote by $\Pi_e\langle \mathcal{O}^* \rangle$, and (2) we have $H_{\langle \mathcal{O}^* \rangle} \subseteq \Pi_e\langle \mathcal{O}^* \rangle$.*

Proof. (1) By Fact 6.1(3) and (4), $e < \mathcal{O}^*$. The claim for outcome chains then follows from Fact 4.7, and the claim for scattered outcomes is the subject of Exercise 5.3(2) in Chapter 5 (for which a solution is given in Appendix B.5).

(2) We consider the case of $\mathcal{O}^* = \hat{O}$ a scattered outcome; the case for outcome chains is a straightforward simplification. Let $h \in H_{\langle \hat{O} \rangle}$, i.e., $h \in H_{\langle O \rangle}$ for all $O \in \hat{O}$, and let $e \in cll(I \rightarrowtail \hat{O})$. By Fact 6.1(4), $e < \hat{O}$, i.e., there is some $O \in \hat{O}$ for which $e < O$. As $h \in H_{\langle O \rangle}$, there is some $e' \in O \cap h$, and as $e < e'$, we have $h \in H_e$. Now $\Pi_e\langle \hat{O} \rangle$ is defined to be that unique outcome of e that is compatible with the occurrence of $\hat{O}$, i.e., $\Pi_e\langle \hat{O} \rangle \cap H_{\langle \hat{O} \rangle} \neq \emptyset$. Let $h' \in \Pi_e\langle \hat{O} \rangle \cap H_{\langle \hat{O} \rangle}$; by the above reasoning, there is $e'' \in O \cap h'$. Let $e^* =_{\mathrm{df}} \min\{e', e''\}$, we have $e^* \in O \cap h \cap h'$, and so e^* witnesses $h \equiv_e h'$, proving that $h \in \Pi_e\langle \hat{O} \rangle$. □

We arrive at the following definition of *causae causantes*. For disjunctive outcomes, we need to keep the disjuncts separate because the union of their *causae causantes* is generally inconsistent; we invoke the notion of a reduced set of cause-like loci, $cllr$, as in Def. 6.1.

Definition 6.2 (*Causae causantes* in no-MFB contexts)**.** Let $I \rightarrowtail \mathcal{O}^*$ be a transition from an initial I to an outcome chain or to a scattered outcome. The set of *causae causantes* for this transition is

$$CC(I \rightarrowtail \mathcal{O}^*) =_{\text{df}} \{e \rightarrowtail \Pi_e\langle \mathcal{O}^* \rangle \mid e \in cll(I \rightarrowtail \mathcal{O}^*)\}.$$

If $\mathcal{O}^*$ is a disjunctive outcome $\breve{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$ (where Γ is an index set), we define first the reduced set of *causae causantes* $CCr(I \rightarrowtail \hat{O}_\gamma)$ and then the set $\text{CC}(I \rightarrowtail \breve{\mathbf{O}})$ as the family of *causae causantes* of the disjuncts:

$$CCr(I \rightarrowtail \hat{O}_\gamma) =_{\text{df}} \{e \rightarrowtail \Pi_e\langle \hat{O}_\gamma \rangle \mid e \in cllr(I \rightarrowtail \hat{O}_\gamma)\};$$
$$CC(I \rightarrowtail \breve{\mathbf{O}}) =_{\text{df}} \{CCr(I \rightarrowtail \hat{O}_\gamma)\}_{\gamma \in \Gamma}.$$

Observe that for $\mathcal{O}^*$ an outcome chain or a scattered outcome, the set $CC(I \rightarrowtail \mathcal{O}^*)$ is consistent since $H_{\langle \mathcal{O}^* \rangle} \subseteq \Pi_e\langle \mathcal{O}^* \rangle$ for every $e \in cll(I \rightarrowtail \mathcal{O}^*)$ (Fact 6.2), and $H_{\langle \mathcal{O}^* \rangle} \neq \emptyset$. Accordingly, the set $CC(I \rightarrowtail \breve{\mathbf{O}})$, where $\breve{\mathbf{O}}$ is a disjunctive outcome, is a *family of consistent sets* of basic transitions, which keeps track of the individual causes of the disjuncts. Note that for a deterministic transition to a disjunctive outcome, $I \rightarrowtail \mathbf{1}_I$, the family $CC(I \rightarrowtail \mathbf{1}_I)$ just contains the empty set.

6.3.2 *Causae causantes* are at least inus conditions

Our goal now is to show that each *causa causans* for a transition is at least an inus condition for that transition. We must be clear on what form exactly these conditions take. Earlier, in Chapter 6.2, we noted that the multi-realizability of outcomes is essential for obtaining the "unnecessary" qualification in in*u*s, so we split our discussion into the case of uniquely realizable outcomes (outcome chains and scattered outcomes; Chapter 6.3.2.1), and the case of multiply realizable outcomes (disjunctive outcomes; Chapter 6.3.2.2). For the following discussion, we assume that there is no modal funny business.

6.3.2.1 Transitions to outcome chains or scattered outcomes

Our main Theorem 6.1 is stated for both outcome chain events and scattered outcomes alike. For these types of transitions, we first prove joint sufficiency: if every single *causa causans* of the transition $I \rightarrowtail \mathscr{O}^*$ occurs in a history h, then $I \rightarrowtail \mathscr{O}^*$ occurs in h as well. Joint necessity goes in the opposite direction: if the initial I and the transition $I \rightarrowtail \mathscr{O}^*$ occur, then all the *causae causantes* of $I \rightarrowtail \mathscr{O}^*$ occur as well. (Note that we have to require the occurrence of the initial, given the implication-like reading of the occurrence proposition for the transition.) Observe also that joint necessity entails necessity of each individual *causa causans* (i.e., it entails $H_{[I]} \cap H_{I \rightarrowtail \mathscr{O}^*} \subseteq \Pi_e \langle \mathscr{O}^* \rangle$ for each $e \in cll(I \rightarrowtail \mathscr{O}^*)$). In other words, each individual *causa causans* is necessary for the occurrence of $\mathscr{O}^*$.

It is the non-redundancy clause that raises some subtle issues. Is each individual $\tau \in CC(I \rightarrowtail \mathscr{O}^*)$ a non-redundant part of the set, in the set's capacity of bringing about $\mathscr{O}^*$? Note that if, for example, the set of *causae causantes* is finite and linearly ordered, then in some sense the last transition is sufficient for $\mathscr{O}^*$; in this sense all the remaining *causae causantes* are redundant. More generally, if there are $\tau_1, \tau_2 \in CC(I \rightarrowtail \mathscr{O}^*)$ with $\tau_1 \prec \tau_2$, then τ_1 is redundant—but only because its occurrence is already implied by τ_2 occurring. In concrete terms, that sense of redundancy is spurious, however, since if the concrete transition τ_2 occurs, all the preceding concrete transitions in the past of τ_2 must occur as well—so their specification is redundant, but their occurrence is not. There is another, more useful sense of non-redundancy, which comes from our implication-style reading of the occurrence proposition for a transition (Def. 4.5). Given that definition, we read the *non-occurrence* of a given basic transition $\tau = e_0 \rightarrowtail H$ as the occurrence of a basic transition that is a local alternative to it, $\tau' = e_0 \rightarrowtail H'$, with $H \neq H'$ and $H, H' \in \Pi_{e_0}$. To ask whether a given $\tau \in CC(I \rightarrowtail \mathscr{O}^*)$ is redundant, we thus produce a tweaked set S of basic transitions by taking $CC(I \rightarrowtail \mathscr{O}^*)$ and replacing its member τ by a local alternative τ', and we then ask if the tweaked set S is adequate for bringing about $\mathscr{O}^*$. This may fail to be so for two reasons: first, S may turn out to be inconsistent, or, second, it may happen to be consistent but insufficient for the occurrence of $\mathscr{O}^*$.[11]

[11] Given no MFB, the first case occurs if e_0 is a non-maximal element of $cll(I \rightarrowtail \mathscr{O}^*)$, and the second case occurs if e_0 is a maximal element of that set. In the presence of MFB, things are more complicated: we may still obtain an inconsistent set by replacing an outcome of a maximal element of $cll(I \rightarrowtail \mathscr{O}^*)$, which will thus fall under the first case.

With these explanations provided, we can state our Theorem:

Theorem 6.1 (nns for transitions to outcome chains or scattered outcomes in BST_{92} without MFB)**.** *Let $\langle W, < \rangle$ be a BST_{92} structure in which there is no MFB. Let $I \rightarrowtail \mathcal{O}^*$ be a transition to an outcome chain or a scattered outcome. Then the* causae causantes *of $I \rightarrowtail \mathcal{O}^*$ satisfy the following inus-related conditions:*

1. *joint sufficiency – nn$\underline{s}$: $\bigcap_{e \in cll(I \rightarrowtail \mathcal{O}^*)} H_{e \rightarrowtail \Pi_e \langle \mathcal{O}^* \rangle} \subseteq H_{I \rightarrowtail \mathcal{O}^*}$;*
2. *joint necessity – n$\underline{n}$s: $H_{\langle \mathcal{O}^* \rangle} = H_{[I]} \cap H_{I \rightarrowtail \mathcal{O}^*} \subseteq \bigcap_{e \in cll(I \rightarrowtail \mathcal{O}^*)} H_{e \rightarrowtail \Pi_e \langle \mathcal{O}^* \rangle}$;*
3. *non-redundancy – $\underline{n}$ns: for every $(e_0 \rightarrowtail H) \in CC(I \rightarrowtail \mathcal{O}^*)$ and every $H' \in \Pi_{e_0}$ such that $H' \cap H = \emptyset$:*

$$\textit{either} \quad H' \cap \bigcap_{e \in cll(I \rightarrowtail \mathcal{O}^*) \setminus \{e_0\}} \Pi_e \langle \mathcal{O}^* \rangle = \emptyset, \tag{6.1}$$

$$\textit{or} \quad H' \cap \bigcap_{e \in cll(I \rightarrowtail \mathcal{O}^*) \setminus \{e_0\}} \Pi_e \langle \mathcal{O}^* \rangle \not\subseteq H_{\langle \mathcal{O}^* \rangle} = H_{[I]} \cap H_{I \rightarrowtail \mathcal{O}^*}. \tag{6.2}$$

Proof. (1) If $H_{[I]} = H_{\langle \mathcal{O}^* \rangle}$, then $H_{I \rightarrowtail \mathcal{O}^*} = \text{Hist}$, so the inclusion is satisfied. Otherwise we argue for the contraposition of (1), considering a scattered outcome $\hat{O}$. Let us suppose that for some $h \in \text{Hist}$, $h \in H_{[I]}$, but $h \notin H_{\langle \hat{O} \rangle}$, so there is $O \in \hat{O}$ such that $h \notin H_{\langle O \rangle}$, i.e., $h \cap O = \emptyset$. On the other hand, there is some $h' \in H_{\langle \hat{O} \rangle}$, hence $h' \in H_{\langle O \rangle}$, so $h \cap O \neq \emptyset$. By Fact 4.1, $h' \in H_{[I]}$. Then $O' =_{\text{df}} O \cap h'$ is an initial segment of O for which we have $O' \subseteq h' \setminus h$. By PCP of BST_{92} and Fact 3.8, there is $c < O'$ such that $h \perp_c H_{\langle O' \rangle}$, which implies $c < \hat{O}$ and $h \perp_c H_{\langle O \rangle}$ (as O' is an initial segment of O). Since $H_{\langle \hat{O} \rangle} \subseteq H_{\langle O \rangle}$, we get $h \perp_c H_{\langle \hat{O} \rangle}$, which means that $c \in cll(I \rightarrowtail \hat{O})$. Since $c < \hat{O}$, $\Pi_c \langle \hat{O} \rangle$ is well-defined (see Fact 6.2(1)). Accordingly $h \perp_c \Pi_c \langle \hat{O} \rangle$. As $c \in h$, but $h \notin \Pi_c \langle \hat{O} \rangle$, we get $h \notin H_{c \rightarrowtail \Pi_c \langle \hat{O} \rangle}$, and hence $h \notin \bigcap_{e \in cll(I \rightarrowtail \hat{O})} H_{e \rightarrowtail \Pi_e \langle \hat{O} \rangle}$, which proves the contraposition of (1). By simplifying this argument, we can prove our claim for transitions to outcome chains.

(2) By no-MFB and by Fact 6.1(4), $e < \mathcal{O}^*$ for every $e \in cll(I \rightarrowtail \mathcal{O}^*)$, and so by Fact 6.2, $\Pi_e \langle \mathcal{O}^* \rangle$ is well-defined, and we have $H_{[I]} \cap H_{I \rightarrowtail \mathcal{O}^*} = H_{\langle \mathcal{O}^* \rangle} \subseteq \Pi_e \langle \mathcal{O}^* \rangle \subseteq (\text{Hist} \setminus H_e) \cup \Pi_e \langle \mathcal{O}^* \rangle = H_{e \rightarrowtail \Pi_e \langle \mathcal{O}^* \rangle}$. So $H_{[I]} \cap H_{I \rightarrowtail \mathcal{O}^*} \subseteq \bigcap_{e \in cll(I \rightarrowtail \mathcal{O}^*)} H_{e \rightarrowtail \Pi_e \langle \mathcal{O}^* \rangle}$.

(3) Pick an arbitrary $(e_0 \rightarrowtail H) \in CC(I \rightarrowtail \mathcal{O}^*)$. Recall that by no-MFB, $H =_{\text{df}} \Pi_{e_0} \langle \mathcal{O}^* \rangle$ is well-defined. Pick then an arbitrary $H' \in \Pi_{e_0}$ such that

$H' \cap H = \emptyset$. (Such H' exists as e_0 must be a choice point.) Since by Fact 6.2, $H_{\langle \mathscr{O}^* \rangle} \subseteq \Pi_{e_0}\langle \mathscr{O}^* \rangle = H$, $(*)$ $H' \cap H_{\langle \mathscr{O}^* \rangle} = \emptyset$. Let us abbreviate $H^- =_{\text{df}} \bigcap_{e \in cll(I \rightarrowtail \mathscr{O}^*) \setminus \{e_0\}} \Pi_e \langle \mathscr{O}^* \rangle$. Consider then two cases: (i) $H^- \cap H' = \emptyset$ and (ii) $H^- \cap H' \neq \emptyset$. Case (i) is identical to Eq. (6.1), and so we are done. In case (ii), by $(*)$ we have $H^- \cap H' \cap H_{\langle \mathscr{O}^* \rangle} = \emptyset$. Since $H_{\langle \mathscr{O}^* \rangle} \neq \emptyset$, it follows that $H^- \cap H' \nsubseteq H_{\langle \mathscr{O}^* \rangle}$, which is Eq. (6.2).[12] □

We thus obtain, for transitions to outcome chains or scattered outcomes, a result very much in line with Mackie's inus analysis, namely: each *causa causans* is an nns condition. The divergence from Mackie's account is due to our focus on singular causation rather than type causation. Since outcome chains and scattered outcomes are uniquely realizable, there is only one disjunctive part in the condition, and hence that part is necessary, not unnecessary, as Mackie has it. Also, we do not have insufficiency of a *causa causans* in general: if the set *cll* consists of exactly one element, there is also exactly one *causa causans*, and this one is then already sufficient for the occurrence of the transition effect in question.

We turn next to analyzing the causes of transitions to disjunctive outcomes.

6.3.2.2 Transitions to disjunctive outcomes

In what follows, we consider a transition $I \rightarrowtail \check{\mathbf{O}}$ to a non-trivial disjunctive outcome $\check{\mathbf{O}}$, by which we mean a disjunctive outcome consisting of two or more mutually incompatible scattered outcomes. (If there is only one disjunct, we are effectively considering a transition to a single scattered outcome, which has already been dealt with.) According to Def. 6.2, the set of *causae causantes* for a transition to a disjunctive outcome $I \rightarrowtail \check{\mathbf{O}}$ is the family of the *reduced* sets of *causae causantes* for the component transitions, $CCr(I \rightarrowtail \hat{O}_\gamma)$, $\hat{O}_\gamma \in \check{\mathbf{O}}$. There is, however, a subtle interplay between $CCr(I \rightarrowtail \hat{O}_\gamma)$ and $CC(I \rightarrowtail \hat{O}_\gamma)$ with respect to their role in our inus-related conditions. Beginning with sufficiency (the "s" in "nus"), we can show that each $CCr(I \rightarrowtail \hat{O}_\gamma)$ is sufficient for the occurrence of the transition

[12] One might be tempted to try and make the statements of the three parts of the Theorem more symmetrical, perhaps rewriting (2) as $H_{I \rightarrowtail \mathscr{O}^*} \subseteq \bigcap_{e \in cll(I \rightarrowtail \mathscr{O}^*)} H_{e \rightarrowtail \Pi_e \langle \mathscr{O}^* \rangle}$, or formulating Eq. (6.2) in condition (3) as $H_{e_0 \rightarrowtail H'} \cap \bigcap_{e \in cll(I \rightarrowtail \mathscr{O}^*) \setminus \{e_0\}} H_{e \rightarrowtail \Pi_e \langle \mathscr{O}^* \rangle} \nsubseteq H_{I \rightarrowtail \mathscr{O}^*}$. It can be shown, however, that both these variants are false, and philosophically dubious, which is the topic of Exercises 6.3 and 6.4.

to the disjunctive outcome. Clearly, $CC(I \rightarrowtail \hat{O}_\gamma)$, which is a superset of the former, is then sufficient for this occurrence as well. For non-redundancy (the "n" in "nus"), we can provide a reading that is fully in line with Mackie's idea of a "non-redundant part" (see the quote on p. 135): it turns out that each element of $CCr(I \rightarrowtail \hat{O}_\gamma)$ is non-redundant, in the sense that if it is removed from $CCr(I \rightarrowtail \hat{O}_\gamma)$, the resulting set is not sufficient for bringing about $I \rightarrowtail \breve{\mathbf{O}}$ via $I \rightarrowtail \hat{O}_\gamma$. On the other hand, there might be a redundant element in the possibly larger set $CC(I \rightarrowtail \hat{O}_\gamma)$. Finally, the unnecessary aspect (the "u" in "nus") comes from the fact that there is more than one $\hat{O}_\gamma$ in $\breve{\mathbf{O}}$, so a particular $I \rightarrowtail \hat{O}_\gamma$, and hence $CC(I \rightarrowtail \hat{O}_\gamma)$, is not necessary for the occurrence of $I \rightarrowtail \breve{\mathbf{O}}$.[13] Thus, the unnecessary aspect concerns $CC(I \rightarrowtail \hat{O}_\gamma)$, not the reduced set $CCr(I \rightarrowtail \hat{O}_\gamma)$. Having explained the various occurrences of CC vs. CCr, we state our Theorem:

Theorem 6.2. (nus for transitions to disjunctive outcomes in BST_{92} without MFB) *Let* $\breve{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma\in\Gamma}$ *be a disjunctive outcome consisting of more than one scattered outcome. The family* $CC(I \rightarrowtail \breve{\mathbf{O}}) = \{CCr(I \rightarrowtail \hat{O}_\gamma)\}_{\gamma\in\Gamma}$ *of* causae causantes *of* $I \rightarrowtail \breve{\mathbf{O}}$ *and the component* causae causantes $CC(I \rightarrowtail \hat{O}_\gamma)$ *satisfy the following inus-related conditions:*

1. *each* $CCr(I \rightarrowtail \hat{O}_\gamma)$ *is sufficient – nu̲s: for every* $\gamma \in \Gamma$:
   $\bigcap_{e\in cllr(I\rightarrowtail\hat{O}_\gamma)} H_{e\rightarrowtail\Pi_e\langle\hat{O}_\gamma\rangle} \subseteq H_{I\rightarrowtail\breve{\mathbf{O}}}$
2. *each* $CC(I \rightarrowtail \hat{O}_\gamma)$ *is unnecessary – nu̲s: for every* $\gamma \in \Gamma$:
   $H_{\langle\breve{\mathbf{O}}\rangle} = H_{[I]} \cap H_{I\rightarrowtail\breve{\mathbf{O}}} \not\subseteq \bigcap_{e\in cll(I\rightarrowtail\hat{O}_\gamma)} H_{e\rightarrowtail\Pi_e\langle\hat{O}_\gamma\rangle}$.
3. *for each* $\gamma \in \Gamma$, *each* $\tau_0 = (e_0 \rightarrowtail H) \in CCr(I \rightarrowtail \hat{O}_\gamma)$ *is non-redundant – n̲us. That is, for every* $H' \in \Pi_{e_0}$ *such that* $H \cap H' = \emptyset$:

$$\text{either} \quad H' \cap \bigcap_{e\in cllr(I\rightarrowtail\hat{O}_\gamma)\setminus\{e_0\}} \Pi_e\langle\hat{O}_\gamma\rangle = \emptyset, \tag{6.3}$$

$$\text{or} \quad H' \cap \bigcap_{e\in cllr(I\rightarrowtail\hat{O}_\gamma)\setminus\{e_0\}} \Pi_e\langle\hat{O}_\gamma\rangle \not\subseteq H_{\langle\hat{O}_\gamma\rangle} = H_{[I]} \cap H_{I\rightarrowtail\hat{O}_\gamma}. \tag{6.4}$$

[13] One might be tempted to think that, in contrast, $CCr(I \rightarrowtail \hat{O}_\gamma)$ is necessary for the occurrence of $I \rightarrowtail \breve{\mathbf{O}}$; but this is not the case. It is enough to consider a BST_{92} structure with one choice point c with three basic outcomes, such that $\hat{O}_1$ and $\hat{O}_2$ occur in the first and the second outcome, resp., whereas a history from the third outcome belongs to $H_{[I]}$. Then $c \in cllr(I \rightarrowtail \hat{O}_i)$, but $H_{\langle\breve{\mathbf{O}}\rangle} \not\subseteq \bigcap_{c\in cllr(I\rightarrowtail\hat{O}_i)} H_{c\rightarrowtail\Pi_c\langle\hat{O}_i\rangle}$ for each $i = 1, 2$.

Proof. (1) We consider two cases. If $I \rightarrowtail \breve{\mathbf{O}}$ is deterministic, i.e., if $H_{[I]} = H_{\langle \breve{\mathbf{O}} \rangle}$, then $H_{I \rightarrowtail \breve{\mathbf{O}}} =$ Hist, and the inclusion holds trivially. Otherwise we argue for the contraposition. Consider some $h \in H_{[I]} \setminus H_{\langle \breve{\mathbf{O}} \rangle}$, which must exist since $I \rightarrowtail \breve{\mathbf{O}}$ is not deterministic. As $H_{\langle \breve{\mathbf{O}} \rangle} = \cup_{\hat{O} \in \breve{\mathbf{O}}} H_{\langle \hat{O} \rangle}$ and $h \notin H_{\langle \breve{\mathbf{O}} \rangle}$, it follows that $h \notin H_{\langle \hat{O}_\gamma \rangle}$ for every $\gamma \in \Gamma$. Exactly like in the proof of Theorem 6.1(1), by PCP$_{92}$, we arrive at some $e \in cll(I \rightarrowtail \hat{O}_\gamma)$ with $h \perp_e \Pi_e \langle \hat{O}_\gamma \rangle$. Since $h \in H_e \cap (H_{[I]} \setminus H_{\langle \breve{\mathbf{O}} \rangle})$, we have that $e \notin DET_{I \rightarrowtail \breve{\mathbf{O}}}$, and hence $e \in cllr(I \rightarrowtail \hat{O}_\gamma)$. Accordingly, since $h \in H_e$ but $h \notin \Pi_e \langle \hat{O}_\gamma \rangle$, we get $h \notin H_{e \rightarrowtail \Pi_e \langle \hat{O}_\gamma \rangle}$, which proves the contraposition.

(2) Since $\breve{\mathbf{O}}$ consists of more than one scattered outcome, for every $\hat{O}_\gamma \in \breve{\mathbf{O}}$, there is $h \in H_{\langle \breve{\mathbf{O}} \rangle} \setminus H_{\langle \hat{O}_\gamma \rangle}$. Hence $h \notin H_{I \rightarrowtail \hat{O}_\gamma}$. By the contraposition of Theorem 6.1(1), $h \notin \bigcap_{e \in cll(I \rightarrowtail \hat{O}_\gamma)} H_{e \rightarrowtail \Pi_e \langle \hat{O}_\gamma \rangle}$, providing a witness for the non-inclusion claim (2).

(3) Pick some $\gamma \in \Gamma$. If $CCr(I \rightarrowtail \hat{O}_\gamma) = \emptyset$, the theorem trivially holds. Let us thus assume that $CCr(I \rightarrowtail \hat{O}_\gamma) \neq \emptyset$. Then $cllr(I \rightarrowtail \hat{O}_\gamma) \neq \emptyset$ as well. Pick an arbitrary $(e_0 \rightarrowtail H) \in CCr(I \rightarrowtail \hat{O}_\gamma)$. Recall that by no-MFB, $H =_{\text{df}} \Pi_{e_0} \langle \hat{O}_\gamma \rangle$ is well-defined. Pick then an arbitrary $H' \in \Pi_{e_0}$ such that $H' \cap H = \emptyset$. Since by Fact 6.2, $H_{\langle \hat{O}_\gamma \rangle} \subseteq \Pi_{e_0} \langle \hat{O}_\gamma \rangle$, $(*)$ $H' \cap H_{\langle \hat{O}_\gamma \rangle} = \emptyset$. Let us abbreviate $H^- =_{\text{df}} \bigcap_{e \in cllr(I \rightarrowtail \hat{O}_\gamma) \setminus \{e_0\}} \Pi_e \langle \hat{O}_\gamma \rangle$. Consider then two cases: (i) $H^- \cap H' = \emptyset$ and (ii) $H^- \cap H' \neq \emptyset$. Case (i) is identical to Eq. (6.3), and so we are done. In case (ii), by $(*)$ we have $H^- \cap H' \cap H_{\langle \hat{O}_\gamma \rangle} = \emptyset$. Since $H_{\langle \hat{O}_\gamma \rangle} \neq \emptyset$, it follows that $H^- \cap H' \nsubseteq H_{\langle \hat{O}_\gamma \rangle}$, which is Eq. (6.4). □

6.4 Causation in the presence of modal funny business

We have seen that the proofs of two clauses, (2) and (3), in our two theorems above go through on the assumption of no MFB. If we want to adapt our theory to allow for MFB, we need to reflect on what changes this brings. Observe that in the proofs we used Fact 6.1(3)–(5), which depends on no-MFB, and according to which every cause-like locus of a transition is below the outcome part of that transition (in the appropriate sense of 'below' from Def. 4.4). This further entails that each cause-like locus has exactly one basic outcome consistent with the outcome part of the transition, as noted in Fact 6.2(1).

In MFB contexts, however, it may happen that a cause-like locus e for a transition $I \rightarrowtail \mathscr{O}^*$ is *not* in the past of $\mathscr{O}^*$ and that e has *more than one* basic

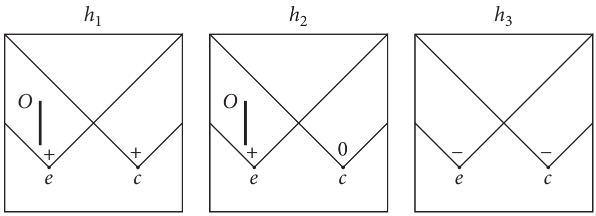

Figure 6.1 MFB with three histories, a binary choice point e (outcomes $+$ and $-$) and a ternary choice point c (outcomes $+$, 0, and $-$).

outcome that is consistent with $H_{\langle \mathscr{O}^* \rangle}$, the occurrence proposition for $\mathscr{O}^*$. In other words, there can be more than one basic outcome of a cause-like locus that keeps the possibility of $\mathscr{O}^*$ open. For illustration, consider Figure 6.1. That figure depicts a BST$_{92}$ structure with three histories, h^{++}, h^{+0}, and h^{--}. The outcome chain O occurs in h^{++} and h^{+0}, but not in h^{--}, whereas the initial I occurs in all these histories. The structure contains two choice points, e with double splitting $(+,-)$ and c with triple splitting $(+,-,0)$, and these choice points constitute the set of cause-like loci $cll(I \rightarrowtail O)$ (h^{--} is the witness). One of these events, e, is below O, but the other, c, is not.

The sets of basic outcomes of e and c are $\Pi_e = \{\{h^{++}, h^{+0}\}, \{h^{--}\}\}$ and $\Pi_c = \{\{h^{++}\}, \{h^{+0}\}, \{h^{--}\}\}$, respectively. Since $H_{\langle O \rangle} = \{h^{++}, h^{+0}\}$, there are two basic outcomes of c, $\{h^{++}\}$ and $\{h^{+0}\}$, that are consistent with $H_{\langle O \rangle}$. Note that e and c are *SLR* and the structure contains sets of transitions that are combinatorially consistent, but not consistent (see Exercise 6.5), thus exhibiting combinatorial funny business (Def. 5.6), for example, in the set $T = \{e \rightarrowtail \{h^{++}, h^{+0}\}, c \rightarrowtail \{h^{--}\}\}$.

This example suggests a natural generalization of our no-MFB definition of *causae causantes*. By this definition (Def. 6.2), a *causa causans* of $I \rightarrowtail \mathscr{O}^*$ (where $\mathscr{O}^*$ is either a scattered outcome or an outcome chain) is a *basic* transition from $e \in cll(I \rightarrowtail \mathscr{O}^*)$ to the *unique basic* outcome of e consistent with $H_{\langle \mathscr{O}^* \rangle}$, i.e., to $\Pi_e\langle \mathscr{O}^* \rangle \in \Pi_e$. However, as we have just seen, in the presence of MFB we lose the guarantee that there is always a unique basic outcome consistent with the outcome part $\mathscr{O}^*$ of $I \rightarrowtail \mathscr{O}^*$. Nevertheless, there is always a *non-empty set of basic outcomes* $H_1, H_2 \ldots \in \Pi_e$ each of which is consistent with the occurrence proposition $H_{\langle \mathscr{O}^* \rangle}$ for $\mathscr{O}^*$ (see Fact 6.3). Hence the set of those basic outcomes of e, which is a *basic disjunctive* propositional outcome of e (see Def. 4.9), is consistent with $H_{\langle \mathscr{O}^* \rangle}$ as well. Thus, if MFB is allowed, a *causa causans* of the transition

$I \rightarrowtail \mathcal{O}^*$ should be a transition from $e \in cll(I \rightarrowtail \mathcal{O}^*)$ to a basic *disjunctive* outcome of e.[14]

To make these observations more precise, we can use the following handy notation.

Definition 6.3. For $A \subseteq$ Hist, $\check{\mathbf{H}}_e\langle A\rangle$ is the basic disjunctive outcome of e that is consistent with A, i.e.,

$$\check{\mathbf{H}}_e\langle A\rangle = \{H \in \Pi_e \mid H \cap A \neq \emptyset\}.$$

Taking $H_{\langle\mathcal{O}^*\rangle}$ for A, we get $\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle$, which is just what we need: a basic disjunctive outcome of e that is the set of exactly those basic outcomes of e that are consistent with $H_{\langle\mathcal{O}^*\rangle}$. Of course, for the case that $e < \mathcal{O}^*$, the respective outcome of e is unique, so that in that case, we just acquire an extra layer of set-theoretic wrapping—that is the price we pay for a unified formal framework in the MFB case.

As the following fact shows, for $e \in cll(I \rightarrowtail \mathcal{O}^*)$, the basic disjunctive outcome $\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle$ captures the notion of "admitting $\mathcal{O}^*$" in the relevant sense.

Fact 6.3. *Let $\mathcal{O}^*$ be an outcome chain or a scattered outcome. For $e \in cll(I \rightarrowtail \mathcal{O}^*)$, $H_{\langle\mathcal{O}^*\rangle} \subseteq \bigcup \check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle$.*

Proof. Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$ and let $h \in H_{\langle\mathcal{O}^*\rangle}$. Then $h' \perp_e h$ for some $h' \in H_{[I]}$, so $e \in h$, and hence there is $H \in \Pi_e$ such that $h \in H$, hence $h \in H \cap H_{\langle\mathcal{O}^*\rangle}$. Accordingly, $h \in \bigcup\{H' \in \Pi_e \mid H' \cap H_{\langle\mathcal{O}^*\rangle} \neq \emptyset\}$, i.e., $h \in \bigcup \check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle$. □

We can now state an analogue of Fact 6.1 in the general case.

Fact 6.4. *We make no assumptions about MFB. Consider a generic transition $I \rightarrowtail \mathcal{O}^*$, i.e., $\mathcal{O}^*$ is an outcome chain O, a scattered outcome $\hat{O}$, or a disjunctive outcome $\check{\mathbf{O}}$.*

(1) Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$. Then e does not causally precede I, i.e., it is not the case that there is some $e' \in I$ for which $e < e'$.
(2) Let $\hat{O}$ be a scattered outcome. Then $\bigcup_{O \in \hat{O}} cll(I \rightarrowtail O) \subseteq cll(I \rightarrowtail \hat{O})$.
(3) Let $e \in cll(I \rightarrowtail \mathcal{O}^*)$ and $\mathcal{O}^*$ be an outcome chain O. Then for every $e' \in O$: $e \leqslant e'$ or $e\, SLR\, e'$.

[14] Note that a similar coarse-graining of the basic possibilities at a point lies behind the notion of an agent's choices in *stit* ('seeing to it that') theory. See Belnap et al. (2001) for details.

(4) *Let* $e \in cll(I \rightarrowtail \mathscr{O}^*)$ *and* $\mathscr{O}^*$ *be a scattered outcome* $\hat{O}$. *Then there is an initial segment* O' *of some* $O \in \hat{O}$ *such that for every* $e' \in O'$: $e \leqslant e'$ *or* $e\,SLR\,e'$.

(5) *Let* $e \in cll(I \rightarrowtail \mathscr{O}^*)$ *and* $\mathscr{O}^*$ *be a disjunctive outcome* $\breve{\mathbf{O}}$. *Then there is* $\hat{O} \in \breve{\mathbf{O}}$ *with an initial segment* O' *of some* $O \in \hat{O}$ *such that for every* $e' \in O'$: $e \leqslant e'$ *or* $e\,SLR\,e'$.

Proof. (1) The proof is exactly as for Fact 6.1(1).

(2) This direction follows as in the no-MFB case (see Exercise 6.1).

(3) Let $e \in cll(I \rightarrowtail \mathscr{O}^*)$, and let h be a history containing the whole chain O. This implies $h \in H_{\langle O \rangle}$. By the definition of cll, there must be some history $h_I \in H_{[I]}$ for which $h_I \perp_e H_{\langle O \rangle}$, implying $h_I \perp_e h$. Let now $e' \in O$. As $\{e, e'\} \subseteq h$, it must be that $e \leqslant e'$ or $e\,SLR\,e'$ or $e > e'$. But the latter case is ruled out: as $e \in h_I$, by the downward closure of histories we would have $e' \in h_I$ as well. Now $e' \in O$, so if $e' \in h_I$, then $h_I \in H_{\langle O \rangle}$. This, however, contradicts $h_I \perp_e H_{\langle O \rangle}$.

(4) and (5) are proved by an argument analogous to that for (3). We leave these proofs as Exercise 6.2. □

Having introduced the notion of a general (basic disjunctive) outcome $\breve{\mathbf{H}}_e\langle H_{\langle \mathscr{O}^* \rangle}\rangle$ of e consistent with $\mathscr{O}^*$, we generalize Definition 6.2 to both MFB and no-MFB contexts:

Definition 6.4 (*Causae causantes* generally)**.** Let $I \rightarrowtail \mathscr{O}^*$ be a transition from initial I to a scattered outcome or outcome chain $\mathscr{O}^*$. The set of *causae causantes* for this transition is

$$CC(I \rightarrowtail \mathscr{O}^*) = \{e \rightarrowtail \breve{\mathbf{H}}_e\langle H_{\langle \mathscr{O}^* \rangle}\rangle \mid e \in cll(I \rightarrowtail \mathscr{O}^*)\}.$$

If $\mathscr{O}^*$ is a disjunctive outcome $\breve{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$, where Γ is an index set, we define first the reduced set of *causae causantes* $CCr(I \rightarrowtail \hat{O}_\gamma)$ and then the set $\mathrm{CC}(I \rightarrowtail \breve{\mathbf{O}})$:

$$CCr(I \rightarrowtail \hat{O}_\gamma) =_{\text{df}} \{e \rightarrowtail \breve{\mathbf{H}}_e\langle H_{\langle \hat{O}_\gamma \rangle}\rangle \mid e \in cllr(I \rightarrowtail \hat{O}_\gamma)\};$$
$$CC(I \rightarrowtail \breve{\mathbf{O}}) =_{\text{df}} \{CCr(I \rightarrowtail \hat{O}_\gamma)\}_{\gamma \in \Gamma}.$$

Observe that if no-MFB is assumed, this definition almost agrees with Def. 6.2, since no-MFB implies that there is always a unique basic outcome

$\Pi_e\langle\mathcal{O}^*\rangle$ of e that is consistent with the occurrence of $\mathcal{O}^*$, so $\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle = \{\Pi_e\langle\mathcal{O}^*\rangle\}$. Thus in the no-MFB case, even according to our generalized definition, *causae causantes* turn out to be (singleton sets of) basic transitions whose initials are cause-like loci.

We can illustrate the generalized concept by returning to Figure 6.1. There the transition $I \rightarrowtail O$ has two cause-like loci: e and c. The *causa causans* associated with e is a basic transition $e \rightarrowtail \{\{h^{++}, h^{+0}\}\}$, as $\{h^{++}, h^{+0}\}$ is a basic outcome of e. However, c has two basic outcomes consistent with O, $\{h^{++}\}$ and $\{h^{+0}\}$, so the *causa causans* associated with c is the transition from c to the disjunctive basic outcome $\{\{h^{++}\}, \{h^{+0}\}\}$; that is, $c \rightarrowtail \{\{h^{++}\}, \{h^{+0}\}\}$.

Having given a general definition of *causae causantes*, we now provide analogues of Theorems 6.1 and 6.2 that prove that even in MFB cases, the *causae causantes* fulfill inus-like conditions. The two general Theorems read as follows:

Theorem 6.3. (nns for transitions to outcome chains or scattered outcomes in BST_{92} with MFB) *Let $\mathcal{O}^*$ be an outcome chain or a scattered outcome. Then the* causae causantes *of $I \rightarrowtail \mathcal{O}^*$ satisfy the following inus-related conditions:*

1. *joint sufficiency – nn<u>s</u>:* $\bigcap_{e\in cll(I\rightarrowtail\mathcal{O}^*)} H_{e\rightarrowtail\check{\mathrm{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle} \subseteq H_{I\rightarrowtail\mathcal{O}^*}$;
2. *joint necessity–n<u>n</u>s:* $H_{\langle\mathcal{O}^*\rangle} = H_{[I]} \cap H_{I\rightarrowtail\mathcal{O}^*} \subseteq \bigcap_{e\in cll(I\rightarrowtail\mathcal{O}^*)} H_{e\rightarrowtail\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle}$;
3. *non-redundancy – <u>n</u>ns: for every* $(e_0 \rightarrowtail \check{\mathbf{H}}) \in CC(I \rightarrowtail \mathcal{O}^*)$ *and every* $\check{\mathbf{H}}'$ *such that* $\check{\mathbf{H}} \cap \check{\mathbf{H}}' = \emptyset$*, where* $\check{\mathbf{H}}, \check{\mathbf{H}}' \subseteq \Pi_{e_0}$

$$\textit{either} \quad \bigcup \check{\mathbf{H}}' \cap \bigcap_{e\in cll(I\rightarrowtail\mathcal{O}^*)\setminus\{e_0\}} (H_e \cap H_{e\rightarrowtail\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle}) = \emptyset, \quad \textit{or} \tag{6.5}$$

$$\bigcup \check{\mathbf{H}}' \cap \bigcap_{e\in cll(I\rightarrowtail\mathcal{O}^*)\setminus\{e_0\}} (H_e \cap H_{e\rightarrowtail\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle}) \nsubseteq H_{[I]} \cap H_{I\rightarrowtail\mathcal{O}^*}. \tag{6.6}$$

Note the difference in the statement of the non-redundancy clause between this Theorem and Theorems 6.1–6.2. The difference is due to the extra set-theoretical level in the definition of (proposition-style) basic disjunctive outcomes (Def. 6.3). The two conditions of non-redundancy concern intersections of sets of histories, and both $H \in \Pi_e$ and $\Pi_e\langle\mathcal{O}^*\rangle$ are sets of histories. $\check{\mathbf{H}}' \subseteq \Pi_e$ and $\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle \subseteq \Pi_e$, however, are *sets* of sets of histories. We thus use the union $\bigcup \check{\mathbf{H}}'$ and the occurrence proposition $H_{e\rightarrowtail\check{\mathbf{H}}_e\langle H_{\langle\mathcal{O}^*\rangle}\rangle}$ (see Def. 4.9).

Theorem 6.4. (nus for transitions to disjunctive outcomes in BST_{92} with MFB) *Let* $\breve{\mathbf{O}} = \{\hat{O}_\gamma \mid \gamma \in \Gamma\}$ *be a disjunctive outcome consisting of more than one scattered outcome. The set of* causae causantes *of* $I \rightarrowtail \breve{\mathbf{O}}$*, i.e.,* $\{CCr(I \rightarrowtail \hat{O}_\gamma)\}_{\gamma\in\Gamma}$ *as well as each* $CC(I \rightarrowtail \hat{O}_\gamma)$*, satisfy the following inus-related conditions:*

1. *each* $CCr(I \rightarrowtail \hat{O}_\gamma)$ *is sufficient – nu*s̲*: for every* $\gamma \in \Gamma$*:* $\bigcap_{e\in cllr(I\rightarrowtail\hat{O}_\gamma)} H_{e\rightarrowtail\breve{\mathbf{H}}_e\langle H_{\langle\hat{O}_\gamma\rangle}\rangle} \subseteq H_{I\rightarrowtail\breve{\mathbf{O}}}$;
2. *each* $CC(I \rightarrowtail \hat{O}_\gamma)$ *is unnecessary – n*u̲*s: for every* $\gamma \in \Gamma$*:* $H_{\langle\breve{\mathbf{O}}\rangle} = H_{[I]} \cap H_{I\rightarrowtail\breve{\mathbf{O}}} \not\subseteq \bigcap_{e\in cll(I\rightarrowtail\hat{O}_\gamma)} H_{e\rightarrowtail\breve{\mathbf{H}}_e\langle H_{\langle\hat{O}_\gamma\rangle}\rangle}$.
3. *for each* $\gamma \in \Gamma$*, each* $\tau_0 = (e_0 \rightarrowtail \breve{\mathbf{H}}) \in CCr(I \rightarrowtail \hat{O}_\gamma)$ *is non-redundant –* n̲*us. That is, for every* $\breve{\mathbf{H}}' \subseteq \Pi_{e_0}$ *such that* $\breve{\mathbf{H}} \cap \breve{\mathbf{H}}' = \emptyset$*:*

$$either \ \bigcup \breve{\mathbf{H}}' \cap \bigcap_{e\in cllr(I\rightarrowtail\hat{O}_\gamma)\setminus\{e_0\}} (H_e \cap H_{e\rightarrowtail\breve{\mathbf{H}}_e\langle H_{\langle\hat{O}_\gamma\rangle}\rangle}) = \emptyset, \tag{6.7}$$

$$or \ \bigcup \breve{\mathbf{H}}' \cap \bigcap_{e\in cllr(I\rightarrowtail\hat{O}_\gamma)\setminus\{e_0\}} (H_e \cap H_{e\rightarrowtail\breve{\mathbf{H}}_e\langle H_{\langle\hat{O}_\gamma\rangle}\rangle}) \not\subseteq H_{[I]} \cap H_{I\rightarrowtail\hat{O}_\gamma}. \tag{6.8}$$

The proofs of these theorems, which parallel the proofs of Theorems 6.1 and 6.2, are left as Exercises 6.6 and 6.7.

Two comments about the BST analysis of causation in MFB contexts may be in order at this point. First, by PCP_{92}, there is always a *causa causans* in $cll(I \rightarrowtail \mathcal{O}^*)$ that is in the past of $\mathcal{O}^*$, in the relevant sense of "past", depending on the kind of $\mathcal{O}^*$ (see Def. 4.4). But as shown by Fact 6.4(3)–(5), a *causa causans* may also be space-like related to $\mathcal{O}^*$ (again, in the relevant sense). So the general situation is as depicted in Figure 6.1: there is a kosher *causa causans* for $I \rightarrowtail \mathcal{O}^*$, which is a basic transition whose initial lies below $\mathcal{O}^*$, but there is also a weird companion *causa causans*, which is a non-trivial basic disjunctive transition and whose initial is not in the past of $\mathcal{O}^*$. One might perhaps think that the weird companion is superfluous. But by the non-redundancy results of Theorems 6.3 and 6.4, the weird companion does some real work: if it were replaced by one of its alternatives, $I \rightarrowtail \mathcal{O}^*$ would not occur. Our second comment is related to the fact that in MFB contexts, a *causa causans* is a transition to a basic disjunctive outcome; the disjunctive outcome is a family of mutually inconsistent basic outcomes. One might worry whether such a transition can really occur. This worry is dispelled by our formal machinery, as the (non-trivial) occurrence proposition for such a transition is well-defined (see Def. 6.3 and Fact 6.3). We can dismiss this worry for non-technical reasons as well. Recall that the underlying idea of causation in BST is that causing X is making a difference by keeping the occurrence of X possible at a critical junction. Now, a transition from a cause-like locus e of $I \rightarrowtail \mathcal{O}^*$ to a basic disjunctive outcome $\breve{\mathbf{H}}_e\langle H_{\langle \mathcal{O}^* \rangle}\rangle$ keeps the occurrence of $I \rightarrowtail \mathcal{O}^*$ possible, and it also makes a difference, since an alternative transition from e precludes the occurrence of that transition. There is thus nothing objectionable to the notion of a transition to a basic disjunctive outcome keeping some outcome possible.

To finish this chapter, we provide some observations regarding how to reformulate our theory of causation so that it applies to $\mathrm{BST_{NF}}$ structures as well.

6.5 *Causae causantes* in $\mathrm{BST_{NF}}$ structures

Recall that a basic propositional transition in a $\mathrm{BST_{NF}}$ structure $\langle W, < \rangle$ is a pair $\langle \ddot{e}, H_e \rangle$, written as $\ddot{e} \rightarrowtail H_e$, where $e \in W$ and where $\ddot{e}$ is the choice set associated with e. The occurrence proposition for $\ddot{e} \rightarrowtail H_e$ is $H_{\ddot{e} \rightarrowtail H_e} = (\text{Hist} \setminus H_{\ddot{e}}) \cup H_e$, where $H_{\ddot{e}} = \bigcup_{e \in \ddot{e}} H_e$. The definition of cause-like loci for transitions generalize Definition 5.12 for outcomes:

Definition 6.5 (Cause-like loci in $\mathrm{BST_{NF}}$)**.** Let $\mathcal{O}^*$ be an outcome chain or a scattered outcome in a $\mathrm{BST_{NF}}$ structure $\langle W, < \rangle$. The set of cause-like loci for a transition $I \rightarrowtail \mathcal{O}^*$ is the following set:

$$cll(I \rightarrowtail \mathcal{O}^*) =_{\text{df}} \{\ddot{e} \subseteq W \mid \exists h \in H_{[I]} \; [h \perp_{\ddot{e}} H_{\langle \mathcal{O}^* \rangle}]\}.$$

If $\mathcal{O}^*$ is a disjunctive outcome $\breve{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$, where Γ is an index set,

$$DET_{I \rightarrowtail \breve{\mathbf{O}}} =_{\text{df}} \{\ddot{e} \subseteq W \mid H_{[I]} \cap H_{\ddot{e}} \subseteq H_{\langle \breve{\mathbf{O}} \rangle}\},$$
$$cllr(I \rightarrowtail \hat{O}_\gamma) =_{\text{df}} cll(I \rightarrowtail \hat{O}_\gamma) \setminus DET_{I \rightarrowtail \breve{\mathbf{O}}},$$
$$cll(I \rightarrowtail \breve{\mathbf{O}}) =_{\text{df}} \bigcup_{\gamma \in \Gamma} cllr(I \rightarrowtail \hat{O}_\gamma).$$

Assuming no MFB, it can be proved[15] that for every $\ddot{e} \in cll(I \rightarrowtail \mathcal{O}^*)$, where $\mathcal{O}^*$ is an outcome chain or a scattered outcome, there is a single

[15] See the Exercises to Chapter 5, esp. Exercise 5.5.

$e' \in \ddot{e}$ such that e' is appropriately below $\mathscr{O}^*$, where 'appropriately' indicates different senses of 'below', depending on the kind of outcome $\mathscr{O}^*$ (see Def. 4.4). It follows that there is a single outcome $H_{e'} \in \Pi_{\ddot{e}}$ consistent with $H_{\langle \mathscr{O}^* \rangle}$; we may refer to this outcome as $\Pi_{\ddot{e}}\langle \mathscr{O}^* \rangle$. In no-MFB contexts we thus define *causae causantes* as follows:

Definition 6.6 (*Causae causantes* with no MFB)**.** Let $I \rightarrowtail \mathscr{O}^*$ be a transition from an initial I to a scattered outcome or to an outcome chain. The set of *causae causantes* for this transition is

$$CC(I \rightarrowtail \mathscr{O}^*) = \{\ddot{e} \rightarrowtail \Pi_{\ddot{e}}\langle \mathscr{O}^* \rangle \mid \ddot{e} \in cll(I \rightarrowtail \mathscr{O}^*)\}.$$

If $\mathscr{O}^*$ is a disjunctive outcome $\check{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$, where Γ is an index set, we first define the reduced set of *causae causantes* $CCr(I \rightarrowtail \hat{O}_\gamma)$ and then the set $\mathrm{CC}(I \rightarrowtail \check{\mathbf{O}})$:

$$CCr(I \rightarrowtail \hat{O}_\gamma) =_{\mathrm{df}} \{\ddot{e} \rightarrowtail \Pi_{\ddot{e}}\langle \hat{O}_\gamma \rangle \mid \ddot{e} \in cllr(I \rightarrowtail \hat{O}_\gamma)\};$$
$$CC(I \rightarrowtail \check{\mathbf{O}}) =_{\mathrm{df}} \{CCr(I \rightarrowtail \hat{O}_\gamma) \mid \gamma \in \Gamma\}.$$

This definition can again be generalized to MFB contexts by identifying *causae causantes* with transitions from cause-like loci to basic disjunctive outcomes.

Definition 6.7 (General *causae causantes*)**.** Let $I \rightarrowtail \mathscr{O}^*$ be a transition from initial I to a scattered outcome or outcome chain $\mathscr{O}^*$. The set of *causae causantes* for this transition is

$$CC(I \rightarrowtail \mathscr{O}^*) = \{\ddot{e} \rightarrowtail \check{\mathbf{H}}_{\ddot{e}}\langle H_{\langle \mathscr{O}^* \rangle} \rangle \mid \ddot{e} \in cll(I \rightarrowtail \mathscr{O}^*)\}.$$

If $\mathscr{O}^*$ is a disjunctive outcome $\check{\mathbf{O}} = \{\hat{O}_\gamma\}_{\gamma \in \Gamma}$, where Γ is an index set, we define first the reduced set of *causae causantes* $CCr(I \rightarrowtail \hat{O}_\gamma)$ and then the set $\mathrm{CC}(I \rightarrowtail \check{\mathbf{O}})$:

$$CCr(I \rightarrowtail \hat{O}_\gamma) =_{\mathrm{df}} \{\ddot{e} \rightarrowtail \check{\mathbf{H}}_{\ddot{e}}\langle H_{\langle \hat{O}_\gamma \rangle} \rangle \mid \ddot{e} \in cllr(I \rightarrowtail \hat{O}_\gamma)\};$$
$$CC(I \rightarrowtail \check{\mathbf{O}}) =_{\mathrm{df}} \{CCr(I \rightarrowtail \hat{O}_\gamma) \mid \gamma \in \Gamma\}.$$

The challenge is then to prove, using these definitions, theorems analogous to Theorems 6.1, 6.2, 6.3, and 6.4. These tasks we also leave to the reader. As a hint, we provide here the statement of one of the theorems, and give its proof in the Appendix:

Theorem 6.5. *(nns for transitions to outcome chains or scattered outcomes in BST_{NF} with no MFB) Let $\mathscr{O}^*$ be an outcome chain or a scattered outcome. The* causae causantes *of $I \rightarrowtail \mathscr{O}^*$ satisfy the following inus-related conditions:*

1. *joint sufficiency – nn$\underline{s}$: $\bigcap_{\ddot{e} \in cll(I \rightarrowtail \mathscr{O}^*)} H_{\ddot{e} \rightarrowtail \Pi_{\ddot{e}}\langle \mathscr{O}^* \rangle} \subseteq H_{I \rightarrowtail \mathscr{O}^*}$;*
2. *joint necessity – n$\underline{n}$s: $H_{\langle \mathscr{O}^* \rangle} = H_{[I]} \cap H_{I \rightarrowtail \mathscr{O}^*} \subseteq \bigcap_{\ddot{e} \in cll(I \rightarrowtail \mathscr{O}^*)} H_{\ddot{e} \rightarrowtail \Pi_{\ddot{e}}\langle \mathscr{O}^* \rangle}$;*
3. *non-redundancy – $\underline{n}$ns: for every $(\ddot{e}_0 \rightarrowtail H) \in CC(I \rightarrowtail \mathscr{O}^*)$ and every $H' \in \Pi_{\ddot{e}_0}$ such that $H' \cap H = \emptyset$:*

$$\textit{either } H' \cap \bigcap_{\ddot{e} \in cll(I \rightarrowtail \mathscr{O}^*) \setminus \{\ddot{e}_0\}} \Pi_{\ddot{e}}\langle \mathscr{O}^* \rangle = \emptyset, \tag{6.9}$$

$$\textit{or } H' \cap \bigcap_{\ddot{e} \in cll(I \rightarrowtail \mathscr{O}^*) \setminus \{\ddot{e}_0\}} \Pi_{\ddot{e}}\langle \mathscr{O}^* \rangle \not\subseteq H_{[I]} \cap H_{I \rightarrowtail \mathscr{O}^*}. \tag{6.10}$$

6.6 Conclusions

We have proposed a non-probabilistic theory of indeterministic singular causation in which both causes and effects are given via transitions. A crucial definition is that of a cause-like locus for the effect. Such a cause-like locus should be thought of as the locus of a risky junction for the effect, such that the effect may cease to be possible right there. Our main idea is that an originating cause, or a *causa causans*, for an effect is a local indeterministic transition from a cause-like locus to that local outcome that keeps the possibility of the effect open (i.e., a local transition that does not prohibit the occurrence of the effect). The full cause of a given effect is then the set of its *causae causantes*.

We argued that our theory gains philosophical support from its agreement in spirit with the 'inus' analysis of type-level causation developed by Mackie. We proved that each *causa causans* of a given transition is at least an inus condition for its occurrence. We traced the differences between Mackie's 'inus' and our 'at least inus' conditions to Mackie's interest in types of events and our focus on particular, concrete events.

On a formal level, we defined cause-like loci for transitions of various types, $cll(I \rightarrowtail \mathscr{O}^*)$, and the resulting sets of *causae causantes*. Our theory can be made to work in contexts reminiscent of quantum non-locality as well, such as in cases in which there is (non-probabilistic) modal funny business as analyzed in Chapter 5. A *causa causans* $\tau \in CC(I \rightarrowtail \mathscr{O}^*)$ is formally defined as the transition $\tau = e \rightarrowtail H$ from a cause-like locus e for $I \rightarrowtail \mathscr{O}^*$ to that outcome of e that keeps $I \rightarrowtail \mathscr{O}^*$ possible. That outcome will be a basic one if no-MFB is assumed, and a basic disjunctive one in the presence of MFB.

6.7 Exercises to Chapter 6

Exercise 6.1. Prove clause (2) of Fact 6.1.

Hint: The argument from right to left is simple. For left to right, consider a witnessing chain O for $e < \hat{O}$ (guaranteed to exist by no-MFB), and extend the splitting-off claim from $H_{\langle \hat{O} \rangle}$ to $H_{\langle O \rangle}$. A full proof is given in Appendix B.6.

Exercise 6.2. Prove clauses (4) and (5) of Fact 6.4.

Hint: For the cases covered by clause (2), we can invoke clause (3). Otherwise, pick some O and find an additional cll, then use the fact that the cll are SLR in the appropriate way. A full proof is given in Appendix B.6.

Exercise 6.3. Exhibit a BST_{92} structure that falsifies a candidate for the joint necessity condition in Theorem 6.1, $H_{I \rightarrowtail \mathscr{O}^*} \subseteq \bigcap_{e \in cll(I \rightarrowtail \mathscr{O}^*)} H_{e \rightarrowtail \Pi_e \langle \mathscr{O}^* \rangle}$, which was mentioned in Footnote 12. Discuss why, philosophically speaking, this candidate is not to be expected to hold.

Hint: Consider the fact that a transition can occur vacuously, by its initial failing to occur.

Exercise 6.4. Exhibit a BST_{92} structure that falsifies an alternative formulation of Eq. (6.2) for the non-redundancy condition in Theorem 6.1, also mentioned in Footnote 12, namely:

$$H_{e_0 \rightarrowtail H'} \cap \bigcap_{e \in cll(I \rightarrowtail \mathscr{O}^*) \setminus \{e_0\}} H_{e \rightarrowtail \Pi_e \langle \mathscr{O}^* \rangle} \not\subseteq H_{I \rightarrowtail \mathscr{O}^*}.$$

Discuss why, philosophically speaking, this candidate is not to be expected to hold.

Hint: Again, consider the possibility of vacuous occurrence.

Exercise 6.5. Show explicitly that MFB is present in the BST_{92} structure depicted in Figure 6.1.

Exercise 6.6. Prove Theorem 6.3.

Exercise 6.7. Prove Theorem 6.4.

Exercise 6.8. Prove Theorem 6.5

Exercise 6.9. Formulate and prove the BST_{NF} versions of Theorems 6.2–6.4.