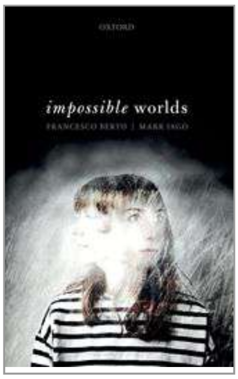

Impossible Worlds
Francesco Berto and Mark Jago

Print publication date: 2019
Print ISBN-13: 9780198812791
Published to Oxford Scholarship Online: August 2019
DOI: 10.1093/oso/9780198812791.001.0001

Relevant Logics

Francesco Berto
Mark Jago

DOI:10.1093/oso/9780198812791.003.0006

Abstract and Keywords

Relevant logics aim to avoid the 'paradoxes' of the material and strict conditionals. Their most natural semantics, the *Routley-Meyer semantics*, is given in terms of impossible worlds. By placing certain further conditions on those worlds, we can obtain stronger relevant logics. One of the main philosophical issues surrounding the general approach concerns how to interpret the Routley-Meyer ternary relation on worlds and the Routley star. The information-theoretic interpretation has proved popular but, it is argued, it faces philosophical issues. An alternative interpretation takes its cue from ways of thinking about conditionality in general. The three options are considered, but issues are found with each of them. A final option is the truthmaker interpretation of relevant logics, which is promising but under-developed.

Keywords: relevant logics, Routley-Meyer semantics, Routley star, information semantics, truthmaking

6.1 Basic Relevant Logic

Relevant logic (or, as they call it in the US, *relevance* logic) aims at developing a notion of conditionality free from the 'fallacies of relevance': the 'paradoxes' of the material and strict conditional (§1.3), including:

(6.1) $A \to (B \to B)$

(6.2) $A \to (B \lor \neg B)$

(6.3) $(A \land \neg A) \to B$

Whether ‘→’ is understood as material or strict, such conditionals come out logically valid in ordinary modal logic with possible worlds semantics. However, in each case, there need be no real connection between antecedent and consequent. ‘If Amsterdam is in the Netherlands, then if snow is white, then snow is white’ is an instance of (6.1), yet what has Amsterdam’s being in the Netherlands to do with (trivial consequences of) snow’s being white?

We can debate whether (6.1)–(6.3) are *really* invalid. ‘Be relevant’ was one of Grice’s conversational maxims, the *Maxim of Relation*. Perhaps what’s wrong with (6.1)–(6.3) is that they violate this pragmatic rule. Perhaps that’s all that’s wrong with them. They might be logically fine after all, but pragmatically difficult to assert. For relevant logicians, however, relevance is not merely pragmatics: it is **(p.126)** an integral feature of logic and formal semantics. They argue that this approach is more plausible than starting with classical logic, and then claiming that irrelevancies should not be asserted purely on the basis of Gricean maxims.

Anderson and Belnap (1975) pioneered relevant logics with the aim of avoiding irrelevance in logic. They held that a formula of the form $A \rightarrow B$ should be a theorem only if A and B shared a sentential atom or parameter. This was called the *Variable Sharing Property* (Dunn and Restall 2002, 27), and was meant to capture the idea of a real connection between antecedent and consequent. They intended the Variable Sharing Property to be necessary, but not sufficient, for a conditional to count as being relevantly acceptable.

Anderson and Belnap initially proceeded proof-theoretically, writing down lists of axioms and rules of inference which gave to the conditionals of their systems the required features. Soon, however, the issue of providing a semantics for such systems came to the top of the agenda:

> Yea, every year or so Anderson & Belnap turned out a new logic, and they did call it E, or R, or E_I, or $P - W$, and they beheld such logic, and they were called relevant. And these logics were looked upon with favor by many, for they captureth the intuitions, but by many they were scorned, in that they hadeth no semantics. Word that Anderson & Belnap had made a logic without semantics leaked out. Some thought it wondrous and rejoiced, that the One True Logic should make its appearance among us in the Form of Pure Syntax, unencumbered by all that set-theoretical garbage. Others said that relevant logics were Mere Syntax.
>
> (Routley and Meyer 1973, 194)

Anderson and Belnap (1975, §28.2) later found algebraic semantics, with soundness and completeness proofs, for these logics, or fragments of them. Usually, these made use of algebraic structures called *De Morgan lattices*. But in such approaches, the syntax and the semantics seem to copy each other. Lacking an independent understanding of the latter, these results often leave philosophically inclined logicians unsatisfied.

(p.127) In other words, Anderson and Belnap's approach looks like *pure*, rather than *applied*, semantics. (The terminology may be due to Plantinga (1974), although Carnap (1948) and Dummett (1973) make similar distinctions.) Pure formal semantics consists in mathematical structures which interpret the language but which have a merely mathematical meaning. In applied formal semantics, by contrast, we have a clear understanding of the connection between the mathematics and meaning. For Dummett, the former are of 'purely technical' interest, whereas the latter 'are taken to have a direct relation to the use which is made of the sentences of a language' (Dummett 1973, 6–7).

Relevant logicians sought to develop frame semantics for relevant logics which promised to move beyond syntax or pure algebra. But this seemed difficult. A logical truth is true everywhere; so how could a conditional with it as a consequent fail to be true everywhere? This requires a semantics that can, at the same time, (a) account for failures of logical truths, but (b) not relinquish their status as logical truths. The 'Routley-Meyer semantics' of Routley and Meyer 1973 was a breakthrough. They distinguished two kinds of points, *normal* and *non-normal*, and this allowed them to invalidate conditionals like (6.1)–(6.3). Non-normal points are naturally interpreted as impossible worlds (Priest 2008, 171–4).

Let us introduce a simple frame semantics of this kind. (We now follow Restall 1993, a simplified version of the original Routley-Meyer semantics presented also in Priest 2008, chapter 10.) $\mathscr{L}$ is as before but with a conditional →. A *Routley-Meyer frame* $\mathscr{F}$ for $\mathscr{L}$ is a quadruple ⟪$W, N, R, *$⟫, with W the set of worlds, $N \subseteq W$ the subset of normal worlds, and $W - N$ the non-normal worlds. $R \subseteq W \times W \times W$ is a ternary relation on worlds satisfying a *Normality Condition*:

(NC) If $w \in N$, then Rww_1w_2 iff $w_1 = w_2$.

Finally, * is the *Routley Star*: a *period two* operation on W ($w^{**} = w$ for each $w \in W$). Given w, let us call w^* the *twin* of w. We will come to the issue of what R and * may mean later on.

(p.128) A frame becomes a model $\mathscr{M} = ⟪ W, N, R, *, v ⟫$ when endowed with a valuation function v, assigning truth values to AT at worlds. We extend v to the whole language as follows:

(S¬) $v_w(\neg A) = 1$ if $v_{w^*}(A) = 0$, and 0 otherwise.

(S∧) $v_w(A \wedge B) = 1$ if $v_w(A) = v_w(B) = 1$, and 0 otherwise.

(S∨) $v_w(A \vee B) = 1$ if $v_w(A) = 1$ or $v_w(B) = 1$, and 0 otherwise.

(S→) $v_w(A \to B) = 1$ if for all $w_1, w_2 \in W$ such that Rww_1w_2, if $v_{w_1}(A) = 1$ then $v_{w_2}(B) = 1$, and 0 otherwise.

Logical validity and consequence are truth/truth preservation at all normal worlds in all models. The logic which is sound and complete with respect to this semantics is called **B**, for *Basic relevant logic*. (Don't confuse this with the modal logic from §4.1, also called **B**.)

The effect of (NC) is that $A \to A$ holds at all normal worlds. For $A \to A$ to hold at w, we require that, if Rww_1w_2 and A holds at w_1, then A holds at w_2 also. But if w is a normal world, then $w_1 = w_2$, and hence A holds at w_2 by assumption. Since validity is defined as truth at all normal worlds, it follows that $A \to A$ is valid in **B**. But it does not follow that each instance of $A \to A$ holds at all worlds. For now suppose w_1 is a non-normal world, and that $Rw_1w_2w_3$, where $w_2 \neq w_3$ and q holds at w_2 but not at w_3. Then $q \to q$ does not hold at w_1.

Now let's see how this allows us to deal with (6.1), $A \to (B \to B)$. We need a world at which A holds but $B \to B$ fails. We can use the model from above, in which $q \to q$ fails at w_1. Now add to the model a normal world w such that Rww_1w_1, and add that p holds at w_1. Then by definition, $p \to (q \to q)$ fails at w. Since w is a normal world, what holds there determines what's valid. Since an instance of $A \to (B \to B)$ fails at w, it isn't valid.

To understand how (6.2) and (6.3) are handled, we need to say something about the Routley Star. The clause (S¬) has it that $\neg A$ is true at a world w if and only if A is false at its twin, w^*. When w and w^* are distinct, we cannot read the value of $\neg A$ at w from the value of **(p.129)** A at w (as we could if ¬ were classical negation). Relevant negation is a modal operator: in order to evaluate negated formulas at w, one has to look at the goings on of a world that may be distinct from w. This negation is often called *De Morgan negation* in the literature, for all of De Morgan's Laws hold for it:

$$\neg\neg A \;\dashv\models\; A \qquad \neg(A \wedge B) \;\dashv\models\; \neg A \vee \neg B \qquad \neg(A \vee B) \;\dashv\models\; \neg A \wedge \neg B$$

This set-up delivers worlds which are classically impossible, for Excluded Middle fails at them; and contradiction-realizers where A and $\neg A$ are both true. This is what we need to have (6.2), $A \to (B \vee \neg B)$, and (6.3), $(A \wedge \neg A) \to B$, fail. For (6.2), a counterexample is given by a normal world w such that Rww_1w_1, with w_1 a world (which may be normal, or not) where p is true but neither q nor $\neg q$ are. This happens, by (S¬), when w_1 is such that q is false at it but true at its twin, w_1^*. For (6.3), a counterexample is given by a normal world w such that Rww_1w_1, with w_1 a world (normal or not) where both p and $\neg p$ are true but q is not. This happens, by (S¬) again, when w_1 is such that q is true at it but false at w_1^*.

One may not like the idea of normal worlds – which we think of as possible worlds – where contradictions are true, or where Excluded Middle fails. To alleviate such worries, one can add the *Classicality Condition* as a constraint on the semantics:

(CC) If $w \in N$, then $w = w^*$.

When a world w is its own twin, then by (S¬) the behaviour of negation at it is just the classical one: $\neg A$ is true at w just in case A is false at the very same world. That world is maximally consistent, making true exactly one of A and its negation. (CC) guarantees that all normal worlds are like that. This gives a logic stronger than the basic relevant **B** but, with the proviso that w_1 is a non-normal world, the counterexamples to (6.1)–(6.3) still go through.

There is a certain translatability between the Routley Star semantics for negation and the relational semantics for negation we met in §5.4. Suppose we define $\rho_w p 1$ if and only if $v_w p = 1$, and $\rho_w p 0$ if and only **(p.130)** if $v_{w^*} p = 0$. Then ρ will work just like a pair consisting of a w and its twin w^* (Priest 2008, 153). In fact, these are equivalent ways of presenting negation in **FDE** (§5.4). One reason for resorting to the Routley Star in this chapter is that adding a relevant conditional using the ternary R to the relational semantics does not give, in any but the simplest cases, the usual family of relevant logics.

On the Routley-Meyer semantics, the corresponding logics are, in an important way, *automatically* relevant. The Variable Sharing Property falls straight out of the Routley-Meyer semantics. As Priest says of this approach,

> relevance is not some extra constraint imposed on top of classical validity. Rather, relevance, in the form of parameter sharing, falls out of something more fundamental, namely the taking into account of a suitably wide range of situations.
>
> (Priest 2008, 174)

Contrast this situation with *filter logics* (Smiley 1959, Tennant 1984), in which relevant validity for conditionals is classical validity plus a constraint that filters out irrelevancies. There's something *ad hoc* about that approach. The Routley-Meyer semantics, by contrast, seems to be a natural extension of frame semantics with a binary relation R, from which the Variable Sharing Property naturally falls. What's crucial to this approach is that Priest's 'range of situations' is wider-than-classical, in that it now comprises non-normal, impossible worlds. These are essential to the frame semantics of mainstream relevant logic.

6.2 Stronger Relevant Logics

We can impose further constraints on the ternary relation R to obtain stronger relevant logics, which validate more principles than those of **B**. (This is similar to the way we obtained modal logics stronger than **K** in §4.1 by imposing conditions on the binary relation R.) The following conditions:

(p.131)

(6.4) If Rww_1w_2, then $Rww_2^*w_1^*$

(6.5) If there is a $x \in W$ such that Rw_1w_2x and Rxw_3w_4, then there is a $y \in W$ such that Rw_1w_3y and Rw_2yw_4

(6.6) If there is an $x \in W$ such that Rw_1w_2x and Rxw_3w_4, then there is a $y \in W$ such that Rw_2w_3y and Rw_1yw_4

validate, respectively, the following principles:

(Contraposition) $(A \rightarrow \neg B) \rightarrow (B \rightarrow \neg A)$

(Suffixing) $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$

(Prefixing) $(A \rightarrow B) \rightarrow ((C \rightarrow A) \rightarrow (C \rightarrow B))$

These are all desirable principles. With all three accepted, the corresponding logic is often called **TW**.

To add further desirable principles, we can add extra information to the Routley-Meyer frames, in the form a binary relation $\preccurlyeq$ between worlds. An *expanded Routley-Meyer frame* $\mathscr{E}$ is now a quintuple $\langle\langle W, N, R, *, \preccurlyeq \rangle\rangle$, which becomes an expanded model when the interpretation function v is added. All the familiar components are as before, and $\preccurlyeq \subseteq W \times W$ is a binary relation between worlds satisfying the following conditions. If $w \preccurlyeq w_1$, then:

(6.7) If $v_w(p) = 1$, then $v_{w_1}(p) = 1$

(6.8) $w_1^* \preccurlyeq w^*$

(6.9) If $Rw_1w_2w_3$, then ($w \in N$ and $w_2 \preccurlyeq w_3$) or ($w \notin N$ and Rww_2w_3)

Think of '$w \preccurlyeq w_1$' as saying that w_1 inherits the truths of w. (6.7) says that this is so for atomic truths, and (6.8) and (6.9) generalize to all formulas. Together, these guarantee that, if $w \preccurlyeq w_1$, then $v_w(A) = 1$ only if $v_{w_1}(A) = 1$, for all $A \in \mathscr{L}$.

There are two further important conditions, the second crucially involving $\preccurlyeq$:

(p.132)

(6.10) If $Rw_1w_2w_3$ then there is an $x \in W$ such that Rw_1w_2x and Rxw_2w_3

(6.11) If $Rw_1w_2w_3$ then there is an $x \in W$ such that $w_1 \leqslant x$ and Rw_2xw_3

These validate, respectively:

(Contraction) $(A \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow B)$

(Assertion) $(A \rightarrow ((A \rightarrow B) \rightarrow B)$

Adding all of these principles give us the relevant logic **R**, possibly the best-known relevant system. It can be shown that the Variable Sharing Property holds in **R**, and hence in the weaker systems as well (see Priest 2008, 205–6).

Let us now move on to the main topic of this chapter: how to make sense of the mainstream Routley-Meyer semantics for relevant logics and, in particular, of its non-normal worlds.

6.3 Relevant Worlds as Information States

Giving an intuitively plausible reading of the ternary relation and of the Routley Star has proved difficult. Copeland (1979) and Van Benthem (1979) claimed that the Routley-Meyer frame semantics are pure, not applied, semantics, for no independent understanding had been offered of what the ternary R and Routley Star * mean. Relevant logicians came to the rescue in various ways. We will now go through three main strategies, paying special attention to how the worlds of relevant frames are understood in them.

One well-known approach interprets worlds in relevant frames as states of information, and R as an informational relation on those states. Urquhart (1972) proposed that 'Rww_1w_2' be read as claiming that the information in w_2 is obtained by merging together the information in w and that in w_1.

(p.133) If this information merging is understood as entailing that w_1-information is included in w_2-information, then the following would seem to hold:

(6.12) If Rww_1w_2, then $w_1 \leqslant w_2$

But this principle makes $B \rightarrow B$ true at all worlds, non-normal as well as normal, which in its turn validates the irrelevant principle (6.1), $A \rightarrow (B \rightarrow B)$. So some other notion of information merging is required.

Systematic information-theoretic readings of the semantics have been proposed by Mares (2004) and Restall (1995). These rely on interpreting the worlds in the frames as representing *situations* in the sense of Barwise and Perry's *situation theory* (Barwise and Perry 1983).

In situation theory, situations are information-supporting structures, allowing the fine-grained distinctions unavailable within possible world semantics. Situations need not be maximal: they can fail to support information about certain topics. The situation consisting of Mark's office in Nottingham does not support the information that it is raining in Amsterdam, nor the information that it is not raining there. Situations may be abstract as well as concrete, and may represent logical impossibilities. Barwise and Seligman (1997) develop situation theory into a general theory of information flow in distributed systems.

Barwise and Parry's original approach did not rule out the possibility that situations act, not only as sites of information, but also as information channels or conduits between other situations. Restall (1995) adopts this view. The points of the relevant frame semantics are taken here as playing both roles. So the situation consisting of a living room with a TV turned on in Nottingham can support the information that it is raining in Amsterdam. It does so by connecting the rainy Dutch situation to the living room via the channel consisting of the cameras, wires, signals, and so on, connecting the two sites. We should read 'Rww_1w_2', then, as 'w is a conduit of information **(p.134)** from site w_1 to site w_2', or as 'situation w allows information to flow from situation w_1 to situation w_2'.

This helps to understand the semantic clause (S→) from §6.1. When w allows the information that $A \to B$ to flow from w_1 to w_2, and w_1 supports the information that A, then w_2 supports the information that B. Vice versa, if w does not allow the information that $A \to B$ to flow, there must be situations w_1 and w_2 such that w_1 supports the information that A, but w_2 does not support the information that B.

Mares (2004, 2009, 2010) too understands the worlds in relevant frames as information-conveying situations, in the sense of situation theory:

> a situation w can be said to contain the information that $A \to B$ if on the hypothesis that there is a w_1 in the same world that contains A, we can derive that there is a situation w_2 in the same world in which B. … [This theory] is about making inferences from the perspective of situations about the situations in a world.
>
> (Mares 2010, 211, notation modified)

(In Mares's terminology, there are both situations and worlds, and situations are included in worlds.) This analysis interprets R in terms of what can be derived from what, so that Rww_1w_2 says 'all the information that we can derive really using the information in both w and w_1 is all contained in w_2' (Mares 2010, 211, notation modified).

Mares thinks of this in terms of *situated inference*, facilitated by 'informational links' Mares (2004). An informational link is a 'perfectly reliable connection, such as a law of nature or a convention' (Mares 2009, 426). Similarly, Devlin (1991, 12) characterizes these constraints as 'natural laws, conventions, analytic rules, linguistic rules, empirical law-like correspondences, or whatever'. A sufficient condition for Rww_1w_2, for example, is that a law of nature of w relates w_1 to w_2. These informational links 'are themselves contained as information in situations' (Mares 2004, 44).

Mares (2004) also analyses *propositions* as sets of situations which satisfy some closure features. There is a relation of *situated implication,* Iww_1P, holding between situations w and w_1 and a proposition P. It holds when the information jointly supported by w **(p.135)** and w_1 allows us to infer the existence of a situation where P holds. We then read 'Rww_1w_2' as 'w_2 belongs to every proposition P such that Iww_1P'. When w supports the informational link $A \rightarrow B$, and w_1 supports the information that A, then w_2 represents a situation belonging to the proposition expressed by B. Vice versa, if w does not support $A \rightarrow B$ there must be some w_1 and w_2 such that w_1 supports the information that A, while w_2 does not belong to the proposition expressed by B.

Although these interpretations focus first of all on making sense of the ternary R, both Restall (1999) and Mares (2004) offer an interpretation of the Routley Star, too. They use a binary relation of *compatibility,* $C \subseteq W \times W$, between worlds (see Dunn 1993), whereby the negation of A holds at world w just in case, at all compatible worlds, A fails to hold. The clause for negation is then:

(SC¬) $v_w(\neg A) = 1$ if for all $w_1 \in W$ such that Cww_1, $v_{w_1}(A) = 0$, and 0 otherwise.

On this definition, ¬ is a 'negative modality': a quantifier over worlds, restricted by an accessibility relation interpreted as compatibility. Because we utter negations to express incompatibilities and exclusions (Berto 2015), a semantics for negation grounded in compatibility makes intuitive sense.

Restall (1999) then shows how to get the Routley Star negation (S¬) from §6.1 out of (SC¬), by imposing conditions on compatibility. It must be symmetric (if Cww_1 then Cw_1w) and serial (for all $x \in W$, there is a $y \in W$ such that Cxy). And each world w must have a maximal compatible world: some $x \in W$ such that Cwx and, for all y, if Cwy then $y \leqslant x$. Restall then claims that

> given that the compatibility semantics makes sense and is an applied semantics, it follows that its simple retelling, involving the Routley star, also makes sense, and it too is an applied semantics.
>
> (Restall 1999, 63)

These various ways of thinking about R in terms of information have been popular, and they allow for interplay between relevant logic **(p.136)** and other theoretical frameworks for reasoning about information, such as situation semantics. But let us briefly mention a worry for such interpretations. (We now draw on Jago 2013d.)

Information, as the term is used by Mares at least, has to be cognitively accessible, for 'what counts as a situation depends on the discriminatory capacities of human beings' (Mares 2009, 350). So, if w carries the information that A, then it should be possible for someone in w to get the information that A, and hence to come to know that A. But this idea is in tension with Excluded Middle, $A \vee \neg A$, which is valid in **R**. (It's also valid in other strong relevant logics, including Anderson and Belnap's favourite system **E**, which adds necessity to the relevant conditional.) But there's no reason to think that there is a situation in which, for every A, either the information that A or the information that $\neg A$ is available.

One can reply, correctly, that the semantics for **R** or **E** doesn't require *every* situation to support Excluded Middle. In fact, as we have seen in §6.1, to avoid irrelevancies it is vital that some points be inconsistent (i.e., both A and $\neg A$ hold there, for some A), and some be incomplete (i.e., neither A nor $\neg A$ holds there). But the objection is that (for logics like **R** and **E**) the semantics requires all *normal* points to support Excluded Middle. And yet, on the current interpretation, it is unlikely that there exist such situations. (The objection does not threaten weaker relevant logics in which Excluded Middle is not a theorem.)

6.4 Conditionality Interpretations

Beall et al. (2012) take a different approach to interpretations of R in the ternary semantics. (We follow the presentation in Jago 2013d in this section.) Beall et al. argue that, whichever way we think of conditionality in general, we get a suitable interpretation of R. They consider three general ways of thinking about conditionality:

(6.13) as the exclusion of counterexamples;

(p.137)

(6.14) as an operator or function; and

(6.15) as the kind of notion supported by conditional logic.

We'll consider options (6.13) and (6.14) only here, for the relevant arrows considered in this chapter are not connected to the kind of *ceteris paribus* conditionals studied in conditional logic.

Suppose, along with reading (6.13), we think of conditionality as the ruling out of certain situations: $A \to B$ says that there are no A-situations which are not also B-situations. That's a very classical way of thinking about the conditional, where a counterexample to $A \to B$ is any situation where A is true but B is false. We can also think of a strict conditional $A \dashv B$ (§1.2), understood as the necessitation of a material conditional, $\Box(A \supset B)$, in these terms: w_1 is a counterexample to $A \dashv B$ at w when w_1 is accessible from w and A but not B is true at w_1.

The difficulty with running this interpretation in the case of the ternary relation semantics is that the points of evaluation of antecedent and consequent may differ. To check whether $A \to B$ holds at w, we need to check for A at w_1 and B at w_2 whenever Rww_1w_2. A counterexample to $A \to B$ at w, therefore, depends on what goes on at some pair of points w_1 and w_2. In just this way, Beall et al. (2012) propose to treat 'split points' $\langle\langle w_1, w_2\rangle\rangle$ as potential counterexamples. Truth and falsity at a split point $\langle\langle w_1, w_2\rangle\rangle$ are fixed by truth at w_1 and falsity at w_2, respectively. Rww_1w_2 then says that $\langle\langle w_1, w_2\rangle\rangle$ is accessible from (or possible relative to) w. So, just as in the modal strict conditional case, $\langle\langle w_1, w_2\rangle\rangle$ is a counterexample to $A \to B$ at w when $\langle\langle w_1, w_2\rangle\rangle$ is accessible from w, A is true at $\langle\langle w_1, w_2\rangle\rangle$ but B is false there. We then get the relevant clause (S→) from §6.1 above.

If this approach is to provide a philosophical interpretation of R, as opposed to a useful bit of pure semantics, then the notion of a split point must be well understood. Notice that $\langle\langle w_1, w_2\rangle\rangle$ cannot in general be thought of as the pair set of points $\{w_1, w_2\}$, for this is identical to the pair set $\{w_2, w_1\}$. But not so for split points, in which the order of the points is essential, as the definitions of truth and **(p.138)** falsity at a split point make clear. For the same reason, we can't think of $\langle\langle w_1, w_2\rangle\rangle$ as the mereological composition or fusion of w_1 and w_2, for the fusion of thing a and thing b is the same as the fusion of thing b and thing a. We might think of $\langle\langle w_1, w_2\rangle\rangle$ as some sort of list or sequence of w_1 and then w_2. But what is a list or sequence of two situations, and in what sense are sentences true or false relative to such lists or sequences? In general, one needs a story on which w_1 and w_2, *taken in that order*, constitute a counterexample to $A \to B$, which does not assume that they constitute a counterexample when taken the other way around. This problem may not be insuperable, but more explanation is needed if this is to be a philosophically satisfying applied semantics.

Now let's turn to Beall et al.'s option (6.14) for interpreting R. This involves thinking of the conditional $A \to B$ in terms of an operator or function, taking us from A to B. Intuitionists might think of this in terms of a function from a proof of A to a proof of B, for example. More generally, suppose it makes sense to *apply* one situation w to another w_1, as if w were a function and w_1 an argument. And suppose the result of this application, $w(w_1)$, is in some sense contained in some other situation, w_2. Then we can set Rww_1w_2 whenever $w(w_1)$ is contained in w_2.

This gives us an understanding of R in terms of functional application and containment relations between situations. It's easy to make sense of these notions when the points in the frame semantics are proofs, programs, sets of evidence, or other syntactic constructions. This provides good reason to think that intuitionists and other constructivists can make sense of the ternary relation in this way.

This way of interpreting R resurrects the worry we raised in §6.3 for information-theoretic interpretations, however. Suppose points w and w_1 are understood as the kinds of entities which can be applied functionally to one another, such as proofs or sets of evidence. What justification do we then have for thinking that there's some such w at which, for every A, either A or $\neg A$ holds? If there are no such points, then Excluded Middle cannot be valid and we will be unable to give semantics along these lines for strong relevant logics such as **R**.

(p.139) 6.5 The Truthmaking Interpretation

We follow Jago 2013d in this section. An important feature of the points w, w_1, w_2 in the ternary semantics is that they may be partial or incomplete: it may be that neither A nor $\neg A$ holds at some w. Such points have fairly natural interpretations in epistemic terms, as we have seen, that is, as information states, evidence, or proofs. But such interpretations lead to problems in justifying Excluded Middle (§6.3). This suggests that a non-epistemic interpretation of partial points would be preferable, at least when considering strong relevant logics.

Restall (1996) suggests one such reading: the points are *truthmakers* (facts, states of affairs, or whatever else does truthmaking work). Restall briefly describes a truthmaker semantics which gives the first-degree fragment (i.e., without embedded conditionals) of the logic **RM**, a semi-relevant logic. (Van Fraassen (1969) had already spotted that a facts-based approach can give semantics for **FDE**, which we met in Chapter 5.)

This approach seems to us to overcome the Excluded Middle worry we raised in §6.3 for the epistemic, information-theoretic interpretation of worlds. A truthmaker for A will typically not be a truthmaker for B or for $\neg B$, unless there is some close relationship between A and B. So many truthmakers satisfy the partiality requirement. Yet plausibly, there are 'big truthmakers', such as complete-maximal worlds, which do make everything of the form $A \vee \neg A$ true. These can serve as the normal (validity-determining) points in the semantics. (We mentioned how one can stipulate that all normal points be maximal in our discussion of the Classicality Condition (CC) in §6.1 above. We saw there that (CC) poses no threat to the relevant requirement of the Variable Sharing Property.)

This approach has not been investigated in much detail, but it promises a philosophical interpretation of relevant logic in terms of familiar truth-like notions. It also suggests that principles of relevant logic are pertinent to the metaphysical debate over truthmaking. (The debate between Jago (2009a) and Rodriguez-Pereyra (2009) over certain metaphysical principles of truthmaking can be reinterpreted **(p.140)** as a semantic debate about whether ∧ and ∨ are idempotent, so that $A \wedge A$ and $A \vee A$ are both equivalent to A, for example.) As Restall says, the approach is of interest 'to all those who seek to understand contemporary work on relevant logic, and for those who wish to form a robust theory of truthmaking' (Restall 1996, 339).

Chapter Summary

Relevant logics aim to avoid the 'paradoxes' of the material and strict conditionals. Their most natural semantics – the *Routley-Meyer semantics* – is given in terms of impossible worlds (§6.1). By placing certain further conditions on those worlds, we can obtain stronger relevant logics, including **TW** and **R** (§6.2). One of the main philosophical issues surrounding the general approach concerns how to interpret the Routley-Meyer ternary relation R on worlds and the Routley Star *. The information-theoretic interpretation has proved popular but, we argue, it faces philosophical issues (§6.3).

An alternative interpretation takes its cue from ways of thinking about conditionality in general. We considered the three options suggested by Beall et al. (2012), but found issues with each of them. A final option is the truthmaker interpretation of relevant logics, suggested by Restall (1996), which is promising but underdeveloped.