

# 9

## Superplurals

### 9.1 Introduction

Superplurals have come up time and again in the preceding chapters. We have observed on several occasions that the expressive resources afforded by superplurals would have useful applications. Superplurals would permit the formulation of a higher-order version of Cantor's theorem; they would help with the elimination of mereology and second-order logic in favor of plural logic; and they could be extensively used in semantics, for example by serving as values of plural predicates, coding models of PFO+, and restricting the range of plural quantifiers in Henkin interpretations.

However, the legitimacy of superplural resources cannot be taken for granted. This is why our appeals to superplurals have always been conditional so far. It is time to examine the matter more closely.

Can the step from the singular to the plural be iterated? Is there such a thing as the superplural that stands to the plural the way the plural stands to the singular? Some examples from natural language suggest an affirmative answer. We have already mentioned expressions that denote things articulated into distinct subpluralities, namely 'Russell and Whitehead, and Hilbert and Bernays' and 'these things, those things, and these other things'. These and other examples will be discussed shortly. First we need to clarify and sharpen the questions about superplurals that we seek to answer.

### 9.2 What superplural reference would be

The morphological operation of pluralization cannot be iterated—at least not in English or any other natural language with which we are familiar. For example, while 'cat' and 'cats' are permissible, 'catses' is not. So, as far as common nouns are concerned, the singular and the plural exhaust the options in English. This means that the quest for English superplurals cannot be based on morphology alone. Rather, it must be based on some

semantic feature. The most natural candidate is reference. Focusing on terms, we may thus attempt to single out superplural terms on the basis of what kind of reference they effect.

Terms may be semantically classified according to the number of things they are capable of referring to. Singular terms can refer to at most *one* thing, while plural terms are capable of referring to *a plurality* of things, that is, *many* things at once. What is semantically distinctive of superplural terms?

In earlier chapters, we often described the reference of a superplural term as a “plurality of pluralities”. But we noted that this expression does not have a clear meaning, given our stipulation that ‘plurality’ be used as a shorthand for a plural construction. In this sense, a plurality of things just is many things. So a plurality of pluralities would have to be, in Russell’s terms, *many many’s* (1903, 516). However, one may reasonably doubt that this gloss is meaningful. As a result, one may worry that the very the notion of a superplural term is semantically unintelligible (see, for example, Ben-Yami 2013).

On a set-based formulation of the semantics, there is a perfectly acceptable way to characterize superplural reference. Let our domain of discourse be a set  $d$ . Then a set-based interpretation assigns an individual in  $d$  to a singular term and a non-empty set of individuals in  $d$  to a plural term. So a superplural term would be one to which the interpretation assigns a *higher-level set*, namely a *set of sets* of individuals in  $d$  (subject to the conditions that all the relevant sets be non-empty). This characterization makes precise both the idea of iteration and the analogy used above, namely that the superplural stands to the plural the way the plural stands to the singular.

Linguistic semantics bears witness to the fact that the notion of superplural reference thus characterized is of substantial theoretical interest. For there is a rich and subtle debate over whether certain expressions of natural language should be interpreted by means of higher-level sets.<sup>1</sup> The debate concerns, in particular, the interpretation of plural noun phrases obtained by conjoining other plural noun phrases. Let  $\alpha$  be a noun phrase of the form

$$\beta_1 \text{ and } \dots \text{ and } \beta_n$$

where  $\beta_1, \dots, \beta_n$  are themselves plural noun phrases. The question is whether  $\alpha$  should be interpreted as the union of the semantic values of  $\beta_1, \dots, \beta_n$  or as the higher-level set whose elements are those semantic values. So we must

<sup>1</sup> See, for example, Gillon 1987, Lasersohn 1989, Gillon 1990, Lasersohn 1995, Schwarzschild 1996, and Winter 2001.

choose between a rule of semantic composition that makes  $\alpha$  a plural term, namely

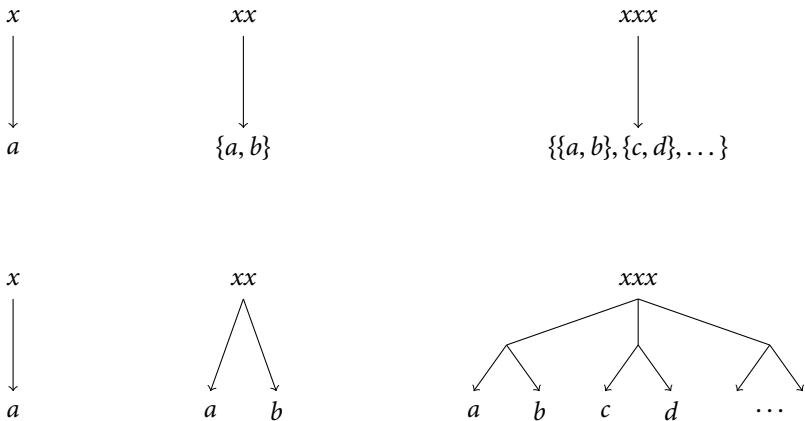
$$\llbracket \alpha \rrbracket = \llbracket \beta_1 \rrbracket \cup \dots \cup \llbracket \beta_n \rrbracket$$

and an alternative rule that makes  $\alpha$  a superplural term, namely

$$\llbracket \alpha \rrbracket = \{ \llbracket \beta_1 \rrbracket, \dots, \llbracket \beta_n \rrbracket \}$$

We will return to this debate below. For the moment, we simply want to emphasize that the notion of superplural reference has a clear and theoretically fruitful characterization within a set-based semantics. In this framework, there is no intelligibility problem.

A new problem arises, however, when we adopt a plurality-based semantics. If a plural term refers to many things, how are we to understand the reference of a superplural term? This question differs from the one discussed by linguists working within a set-theoretic framework (though the data they consider and many of their analyses remain relevant). The desired superplural reference would be a form of *articulated reference*: a superplural term refers not just to many objects but to *many objects articulated in a certain way*. The set-based semantics provides one way to make this articulation explicit. But the following visual analogy suggests another way to represent the articulation, consistent with a (generalized) plurality-based semantics.



Our aim, then, is to “articulate” some objects, just like a higher-level set “articulates” these objects through its members. If higher-level sets are required for a successful set-based semantics, something with the same functional role is required to carry out the semantics in a plurality-based setting.

If natural language contains no devices with this functional role, we may have to develop the needed devices ourselves. From a methodological point of view, the situation is not unusual. Sometimes theorizing requires the introduction of new expressive resources. Think of the huge expressive gains in the history of science afforded by the introduction of new mathematical resources. The justification for these resources derives from their theoretical fruits, not from their antecedent availability in natural language. The proponent of a plurality-based semantics may find herself in a similar situation. She might need to develop superplural resources to account for semantic phenomena that in the set-based semantics require higher-level sets. These new resources too are justified in part by their theoretical fruits.

An additional way to address the intelligibility problem is by adapting a lesson from plurals. It is customary to assume that denotation is a functional relation between a term and its semantic value. So, for any plural term  $tt$ , there are some things such that  $tt$  refers to them and to no other things. This requires the use of plurals in the specification of the semantic value of  $tt$ . But if we relax the functionality requirement and construe reference as a one-many relation, the use of plurals can be avoided. To characterize the reference of  $tt$ , we can say that  $tt$  refers to at least one thing, leaving open the possibility that it refer to some other thing. To express that  $tt$  achieves plural reference to  $a$  and  $b$ , we then say that  $t$  refers to  $a$  and  $t$  refers to  $b$ . The semantic characterization of a plural term is now given using only singular vocabulary. By making the right semantic adjustments, this notion of reference is entirely adequate, as shown by the model theory presented in Sections 7.3 and 7.5. Of course, as we learned there, plurals are not eliminated from the semantics. They are still needed, for example to state the satisfaction clause for plural predication. But it is possible to reduce the complexity of the definition of plural reference so that it only relies on singular resources.

An analogous strategy can be employed for superplural terms. We can say that a superplural term  $ttt$  refers to at least one plurality, leaving open the possibility that it refer to some other plurality. To express that  $ttt$  achieves superplural reference to  $aa$  and  $bb$ , we then say that  $ttt$  refers to  $aa$  and  $ttt$  refers to  $bb$ . This definition does not presuppose superplural notions. Relying only on our understanding of plurals, we now have access to a

semantic characterization of superplural reference. Of course, this does not mean that we can fully carry out the semantics without invoking superplural notions. But the strategy suffices at least to make intelligible what superplural reference would be in the context of a plurality-based semantics. So we submit that the notion of superplural reference is in good standing.

### 9.3 Grades of superplural involvement

Questions about superplural reference are part of a broader range of interesting questions concerning different ways in which superplurals may be said to be “available”. Affirmative answers to such questions correspond to different “grades of superplural involvement”, as we shall now explain.

The first question has to do with the availability of a certain logical system.

(Q1) Can we formulate a superplural logic?

This question receives a definitive answer in Rayo 2006. It is relatively straightforward to formulate an extension of plural logic that, in addition to singular and plural vocabulary, includes a third category of superplural expressions. These new expressions (terms, predicates, and quantifiers) are governed by logical rules similar to those governing singular and plural expressions. In the resulting system we can express, for instance, the plausible axiom that every superplurality is non-empty:

$$(9.1) \quad \forall xxx \exists yy \quad yy < xxx$$

Once a superplural logic has been formulated, an obvious semantic question arises:

(Q2) Can we provide a semantics, especially a model theory, for a superplural language?

A set-based model theory for such a language can easily be formulated as an extension of the set-based model theory for PFO+. If we are willing to countenance metalanguages with superplural resources, a plurality-based model theory is available as well (see Rayo 2006, Linnebo and Rayo 2012, and Section 11.A below).

The next two questions ask whether superplurals are available in a more demanding sense.

(Q3) Is superplural reference a legitimate expressive resource for beings like us?

This is a question in philosophical logic, unlike the fourth question, which is empirical and requires input from linguists.

(Q4) Is superplural reference realized in some (human) natural language?

Our primary target is the grade of superplural involvement associated with (Q3). This question involves the notion of “legitimate expressive resource”. How should this notion be understood? Some indications are provided in Lewis 1991, Hazen 1997, Linnebo 2003, and Rayo 2006. As Hazen and Lewis point out, we can show the legitimacy of certain expressive resources by showing that they can be taught and employed. Indeed, as discussed in Section 6.1, this process of training can include learning by what Williamson calls “the direct method”.

We will also be concerned with the last—and highest—grade of superplural involvement. While this is of obvious relevance to natural language semantics, our primary interest in the fourth question has to do with the fact that actuality entails possibility, or legitimacy, as we put it above. If some natural language truly realizes superplural reference, then such reference is certainly possible, which answers our primary question, (Q3).

## 9.4 Possible examples from natural language

Before reviewing a number of purported examples of superplurals in English, let us briefly consider two phenomena from other languages.<sup>2</sup> The first involves number words in Icelandic. Such words have plural forms which count not individual objects, but pluralities of objects that form natural groups. For instance,

<sup>2</sup> These examples are mentioned in Linnebo 2017. Grimau forthcoming offers an interesting discussion of a wider range of examples.

einn skór	means	one shoe
einir skó	means	one pair of shoes
tveir skór	means	two shoes
tvennir skó	means	two pairs of shoes

So we can talk about a “pluralities of pluralities” of shoes without the mediation of any singularizing device. This feature is lost in the translation, since ‘two pairs of shoes’ involves a singularization (via ‘pair of’) and thus amounts to regular plural talk of objects (the pairs of shoes).

For another example, consider these quantificational expressions in Lithuanian:

vienas batas	means	one shoe
vieneri batai	means	one group of shoes
du batai	means	two shoes
dveji batai	means	two groups of shoes
abu batai	means	both shoes
abieji batai	means	both groups of shoes
keleri batai	means	several groups of shoes
keli batai	means	both shoes

Here too we can talk about “pluralities of pluralities” of shoes, again without the mediation of any singularizing device.

Turning to English, there are many purported examples of superplural terms. Most of these examples involve *lists of plural terms*. Let us first reflect on the case of lists of singular terms. Some predicates applying to such lists appear to take a variable number of arguments, that is, they appear to be *multigrade*:

- (9.2) (a) Annie and Bonnie met/cooperated/helped each other.  
 (b) Annie, Bonnie, and Connie met/cooperated/helped each other.  
 (c) Annie, Bonnie, Connie, and Danny met/cooperated/helped each other.

These predicates are plausibly analyzed as *plural* predicates with a single argument filled by a plural term. So, on this analysis, the predicates have

fixed adicity after all, and each list of singular terms is in fact a plural term. By analogy, one might expect that predicates applying to lists of plural terms also have fixed adicity and should be analyzed as *superplural* predicates. If so, each list of plural terms would be a superplural term.

Here are some promising example involving lists.

- (9.3) The cards below seven and the cards from seven up have been separated. (Landman 1989a, 574)
- (9.4) The Beatles and the Rolling Stones gave a joint concert. (Inspired by Rosen and Dorr 2002, footnote 21, 172–3)
- (9.5) The joint authors of multivolume classics on logic are Whitehead and Russell, and Hilbert and Bernays. (Oliver and Smiley 2005, 1065)
- (9.6) These people, those people, and these other people played against each other. (Linnebo and Nicolas 2008, 193)
- (9.7) The square things, the blue things, and the wooden things overlap. (Linnebo and Nicolas 2008, 193)

It is worth noting that (9.5) also gives us a purported example of a superplural term other than a list, at least if one assumes that the sentence expresses a form of identity. On this assumption, the plural description has the same semantic status as the list.

Oliver and Smiley maintain that this is not an isolated case: there is a class of plural descriptions whose members are in fact uniformly superplural. These are descriptions of the form ‘the  $F$  that purport “to denote all the joint satisfiers of  $F$ ” rather than “the unique joint satisfiers of  $F$ ” (Oliver and Smiley 2016, 132). In addition to the description in (9.5), they mention various other descriptions as belonging to this class, including ‘the twin primes’ and ‘the creators of a great comic opera’. Let us therefore add another of their examples to our stock:

- (9.8) 3 and 5 are among the twin primes. (Oliver and Smiley 2016, 139)

On the superplural reading they propose, ‘are among’ is a higher-order counterpart of the relation of plural membership. It holds between a plural term (‘3 and 5’) and superplural one (‘the twin primes’) when, so to speak, the plurality denoted by the former is one of the pluralities denoted by the latter.



### 9.5 The possible examples scrutinized

To present a convincing case for a superplural interpretation of the examples just considered, we must rule out alternative analyses. The harder it is to find an alternative analysis, the stronger the case for superplurals. We will therefore examine a range of attempts to find an alternative analysis, without necessarily endorsing these attempts.

For some of the examples, alternative analyses are clearly available (see Linnebo and Nicolas 2008 for discussion). For instance, (9.3) can be analyzed as follows:

- (9.9) The cards below seven have been separated from the cards from seven up.

This sentence contains no *prima facie* superplurals. It features just ordinary plural descriptions. However, the possibility of this analysis depends on the fact that the original list is composed of *two* terms. So the analysis does not directly apply when a list has more terms:

- (9.10) The cards below four, the cards between four and six, and the cards from seven up have been separated.

To analyze (9.10), we can integrate the idea behind (9.9) with a *conjunctive analysis*:

- (9.11) The cards below four have been separated from the cards between six and four, the cards between four and six have been separated from the cards from seven up, and the cards from seven up have been separated from the cards below four.

The success of this conjunctive analysis requires the logical equivalence of (9.10) and (9.11). As is easy to see, this equivalence obtains, although one might question whether meaning is fully preserved.

However, the conjunctive analysis does not provide a general method for eliminating purported examples of superplural terms. Its limitations become apparent when we examine another example given above:

- (9.4) The Beatles and the Rolling Stones gave a joint concert.

As before, there is a simple analysis that avoids the list:

(9.12) The Beatles gave a concert with the Rolling Stones.

Still, the following variant of (9.4) is not amenable to a conjunctive analysis:

(9.13) The Beatles, the Rolling Stones, and Led Zeppelin gave a joint concert.

For it is clear that (9.13) is not equivalent to this conjunction:

(9.14) The Beatles gave a concert with the Rolling Stones, the Rolling Stones gave a concert with Led Zeppelin, and Led Zeppelin gave a concert with the Beatles.

While (9.13) implies that there is a single concert featuring all three bands, (9.14) does not.

Although it resists a conjunctive analysis, (9.13) does not yet provide a compelling example of superplurals. That is because it can plausibly be analyzed in a different way. Indeed, it has been suggested that terms like ‘the Beatles’ and ‘the Rolling Stones’, though syntactically plural, denote *groups* and are therefore semantically singular (see, e.g., Landman 1989a, Landman 1989b, and Landman 2000, Lectures Four, Five and Six). It is possible that the Beatles and the Rolling Stones gave a joint concert, though Ringo Starr was ill and was replaced by someone else. Assuming that pluralities have their members necessarily (see Chapter 10), this possibility shows that ‘the Beatles’ and ‘the Rolling Stones’ do not refer plurally and thus lends support to the group-based analysis. On this analysis, lists such as ‘the Beatles and the Rolling Stones’ amount to ordinary plural terms.

Is the conjunctive analysis applicable to the sentences suggested by Oliver and Smiley? It is not applicable directly to (9.5). For, clearly, these two sentences are not equivalent:

(9.5) The joint authors of multivolume classics on logic are Whitehead and Russell, and Hilbert and Bernays.

(9.15) Whitehead and Russell are joint authors of multivolume classics on logic, and Hilbert and Bernays joint authors of multivolume classics on logic.

To bridge the gap, one may apply the conjunctive analysis indirectly through a Russellian elimination of the plural description:

- (9.16) Whitehead and Russell are joint authors of multivolume classics on logic, Hilbert and Bernays are joint authors of multivolume classics on logic, and any joint authors of multivolume classics on logic are either Whitehead and Russell or Hilbert and Bernays.

So the force of (9.5) as an example of superplurals depends on the extent to which this elimination is objectionable.<sup>3</sup>

The sentence concerning twin primes is somewhat difficult to assess. In mathematics, it is common to treat ‘twin primes’ distributively, taking a twin prime to be a prime that differs by two from another prime. This makes ‘the twin primes’ an ordinary plural description. So a conjunctive paraphrase would be perfectly in order:

- (9.8) 3 and 5 are among the twin primes.  
 (9.17) 3 is a twin prime and 5 is a twin prime.

It is also common to use ‘twin prime’ as synonymous with ‘twin prime pair’. Then the semantic value of ‘the twin primes’ would be the collection of *pairs* of the form  $\langle p, p + 2 \rangle$ , where  $p$  and  $p + 2$  are prime. While this understanding is closer to the superplural interpretation, it will not persuade the skeptic who takes a pair to be an object and hence sees a collection of pairs as an ordinary plurality. This kind of skeptic may regard a true utterance of (9.8) as tantamount to:

- (9.18) The pair  $\langle 3, 5 \rangle$  is among the twin primes.

Linnebo and Nicolas (2008) argue that examples (9.6) and (9.7) strengthen the case for superplurals in English in that they are not amenable to either a conjunctive analysis or a group-based interpretation.

- (9.6) These people, those people, and these other people played against each other.  
 (9.7) The square things, the blue things, and the wooden things overlap.

<sup>3</sup> For objections, see Oliver and Smiley 2016, 135–6.

The group-based analysis is not available because, in each example, all the terms in the lists are paradigmatic instances of semantically plural terms. What about a conjunctive analysis? In its intended reading, (9.6) conveys that there is a three-way competition between these people, those people, and these other people. So a conjunctive analysis is not applicable (compare with ‘The Beatles, the Rolling Stones, and Led Zeppelin gave a joint concert’). Something similar may be said of (9.7). As noted by Linnebo and Nicolas (2008, 198), however, this sentence can be given an indirect conjunctive paraphrase:

(9.19) There is a thing such that it is one of the square things, it is one of the blue things, and it is one of the wooden things.

So the force of (9.6) as example of superplurals depends on whether this paraphrase is objectionable.<sup>4</sup>

Let us summarize our discussion so far. Some alleged examples of superplurals in English have limited force, since they can be given a conjunctive analysis or a group-based one. Other examples cannot easily be analyzed in those ways and are therefore more persuasive.

*How persuasive?* Again, the answer turns on the availability of alternative analyses. In the next section, we discuss an analysis according to which lists cannot give rise to superplural terms because they are not terms at all. If correct, this analysis undermines any example that essentially relies on lists. In Section 9.7, we consider an analysis purporting to undermine *all* alleged examples of superplural terms. It holds that lists, as well as plural descriptions of any kind, are ordinary plurals.

## 9.6 The multigrade analysis

If some lists are to count as superplural terms, they must be terms in the first place. The assumption that lists are terms is *prima facie* plausible and, as remarked in Section 9.4, it enables us to interpret seemingly multigrade predicates applying to lists of *singular* terms as plural predicates. By parity of reasoning, we can regard at least some seemingly multigrade predicates applying to lists of *plural* terms as superplural predicates.

<sup>4</sup> Linnebo and Nicolas suggest that the paraphrase could be resisted on grounds that ‘overlap’ is, plausibly, lexically primitive.

The assumption that lists are terms may, however, be disputed. One may instead interpret lists as strings of separate terms, taking predicates to be genuinely multigrade. There are several advocates of this multigrade analysis of predication, and Oliver and Smiley have concluded that the view that lists are string is just as plausible as the view that lists are terms (see Oliver and Smiley 2016, Chapter 10, for discussion and references).

To see how the multigrade analysis works, let us start with some basic examples.

- (9.20) (a) Annie and Bonnie met.  
 (b) Annie, Bonnie, and Connie met.  
 (c) Annie, Bonnie, Connie, and Danny met.

If we treat the predicate ‘met’ as having variable adicity, these sentences can be regimented as follows.

- (9.21) (a)  $M(a, b)$   
 (b)  $M(a, b, c)$   
 (c)  $M(a, b, c, d)$

Note that a general implementation of the multigrade analysis requires a distinction between argument places and argument positions.<sup>5</sup> Consider these sentences:

- (9.22) (a) Annie and Bonnie rescued Connie.  
 (b) Annie rescued Bonnie and Connie.

To avoid ambiguities, the regimentation must clearly separate the agents from the patients of the relation. So we must first have *argument places*, roughly corresponding to relevant thematic roles, such as agent and patient. Then, within each argument place, we must have *argument positions*. Using semicolons to separate argument places and commas to separate argument positions, we can regiment (9.22a) and (9.22b) as (9.23a) and (9.23b), respectively.

- (9.23) (a)  $R(a; b; c)$   
 (b)  $R(a; b, c)$

<sup>5</sup> For an elaboration of the distinction, see Oliver and Smiley 2016, 172–4.

The key point is that, on the multigrade analysis, ‘Annie’, ‘Bonnie’, and ‘Connie’ do not combine to form new terms. Rather, they occupy different argument positions, sometimes sharing the same argument place.

If we adopt this analysis, lists are strings of separate terms. As a result, we forsake all examples of superplurals based on lists. For instance, (9.6) has the form of (9.24), a formula containing only ordinary plural variables.

(9.6) These people, those people, and these other people played against each other.

(9.24)  $\varphi(xx, yy, zz)$

However, the multigrade analysis faces some difficulties. To begin with, there is the question of how to develop the approach systematically so that it can account for inferential relations involving sentences with lists. As observed in Florio and Nicolas 2015 (454–5), one would have to rely heavily on meaning postulates in order to account for the validity of inferences such as the following:

(9.25) (a) Annie and Bonnie are students and best friends.

(b) Therefore, some students are best friends.

(9.26) (a) Bonnie and Connie are students.

(b) Annie, Bonnie, and Connie met.

(c) Therefore, Annie and some students met.

(9.27) (a) Annie, Bonnie, and Connie visited Paris.

(b) Therefore, Annie and Bonnie visited Paris.

(9.28) (a) Annie and Bonnie met.

(b) Therefore, Bonnie and Annie met.

According to the multigrade analysis, these inferences have the following regimentations:

(9.29) (a)  $S(a, b) \wedge B(a, b)$

(b)  $\therefore \exists xx(S(xx) \wedge B(xx))$

(9.30) (a)  $S(b, c)$

(b)  $M(a, c, b)$

(c)  $\therefore \exists xx(S(xx) \wedge M(a, xx))$

- (9.31) (a)  $V(a, b, c; p)$   
 (b)  $\therefore V(a, b; p)$

- (9.32) (a)  $M(a, b)$   
 (b)  $\therefore M(b, a)$

To capture the validity of these inferences, we must introduce special rules governing the relevant classes of expressions. While there is no obvious obstacle, one may regard the need for special rules as a disadvantage over alternative accounts that can explain the validity of the inferences from simpler and more basic semantic principles.

Another difficulty has to do with the implementation of the analysis in a plurality-based setting. If the multigrade analysis is to serve as a genuine strategy for avoiding superplurals, it cannot rely on them in its implementation. Yet it is not clear that a plurality-based model theory for multigrade expressions can avoid superplurals. For example, to describe the interpretation of argument positions, one must refer to arbitrary long sequences of pluralities, which naturally calls for superplurals.

## 9.7 Covers

If successful, the multigrade analysis would undermine alleged examples of superplurals based on lists. However, there is an alternative analysis that is more far-reaching in its potential to weaken the case for superplurals. According to this analysis, lists as well as plural descriptions of any kind are *ordinary plurals*.

This idea might seem a non-starter. Aren't there obvious counterexamples to any ordinary plural analysis of alleged superplural terms? As we have seen above, there are clear cases in which the syntactic articulation of a plural term matters to the truth conditions. Other cases are not hard to produce. Consider these two sentences in a context where the participants in a competition are either students or teachers at a local school:

(9.33) The participants played against each other.

(9.34) The students and the teachers played against each other.

Given the context, these two sentences can differ in truth value. This suggests that the semantic values of their respective subjects are not the same,

contrary to the thesis that lists and plural descriptions are always ordinary plurals. If ‘the students and the teachers’ was an ordinary plural term, its semantic value would be same as that of ‘the participants’. So the two sentences would have to be equivalent, which they are not. By assuming, instead, that ‘the students and the teachers’ is not an ordinary plural term but a superplural one, we easily explain why the sentences are not equivalent.

Recall the dispute over the interpretation of a complex plural noun phrase  $\alpha$  of the form

$$\beta_1 \text{ and } \dots \text{ and } \beta_n$$

where  $\beta_1, \dots, \beta_n$  are themselves plural noun phrases (Section 9.2). We can choose between a rule of composition that renders  $\alpha$  a plural term and one that renders it a superplural term. That is, we have two options:

$$\llbracket \alpha \rrbracket = \llbracket \beta_1 \rrbracket \cup \dots \cup \llbracket \beta_n \rrbracket$$

and

$$\llbracket \alpha \rrbracket = \{ \llbracket \beta_1 \rrbracket, \dots, \llbracket \beta_n \rrbracket \}$$

Examples such as (9.33) and (9.34) put pressure on the plural interpretation. However, linguists such as Brendan Gillon (1987, 1990) and Roger Schwarzschild (1996) have argued that this interpretation can in fact be defended.

According to Gillon and Schwarzschild, articulated plural noun phrases (‘the students and the teachers’) have the same type of denotation as unarticulated ones (‘the participants’). The role of the articulation in the truth conditions is explained by appeal to *covers*.

To see how the analysis works, let us examine a sentence with multiple readings:

(9.35) Annie and her dogs weigh 50 kg.

There is, of course, a collective reading and a distributive one: Annie and her dogs may be said to weigh 50 kg collectively or individually. But another reading is easily available. This is an *intermediate* reading on which Annie is said to weigh 50 kg individually, and her dogs are said to weigh 50 kg collectively.

The three readings correspond to three *partitions* of the set containing Annie and each of her dogs. For concreteness, assume that the dogs are two,



$d_1$  and  $d_2$ , and hence the set can be represented as  $D = \{\text{Annie}, d_1, d_2\}$ . So the three readings correspond to partitions of  $D$  with one, three, and two cells, respectively:

$$\begin{aligned} P_1 &= \{\{\text{Annie}, d_1, d_2\}\} \\ P_2 &= \{\{\text{Annie}\}, \{d_1\}, \{d_2\}\} \\ P_3 &= \{\{\text{Annie}\}, \{d_1, d_2\}\} \end{aligned}$$

By appealing to these partitions, we can capture the multiple readings of (9.35) while retaining the assumption that

$$\llbracket \text{Annie and her dogs} \rrbracket = \llbracket \text{Annie} \rrbracket \cup \llbracket \text{her dogs} \rrbracket$$

This can be done by stating the truth conditions as follows:

‘Annie and her dogs weigh 50 kg’ is true in some context  $Q$  if and only if there is a partition  $P$  of  $\llbracket \text{Annie and her dogs} \rrbracket$  that is salient in  $Q$ , and every element of  $P$  is in  $\llbracket \text{weigh 50 kg} \rrbracket$ .

Thus, the three readings described above (and potentially more) can be seen to arise from contextually salient partitions. For instance, if the salient partition is  $P_3$ , the sentence is true if and only if  $\{\text{Annie}\} \in \llbracket \text{weigh 50 kg} \rrbracket$  and  $\{d_1, d_2\} \in \llbracket \text{weigh 50 kg} \rrbracket$ . That is, the sentence is true if and only if Annie weighs 50 kg individually and her dogs weigh 50 kg collectively.<sup>6</sup>

However, if this analysis is to succeed, we need something more general than partitions. This is shown by the next example. Suppose that Annie, Bonnie, and Connie variously cooperated to build sand castles; specifically, Annie cooperated with Bonnie, Bonnie with Connie, and Connie with Annie. None of them built any other sand castle. Then it seems that we can truly say:

(9.36) Annie, Bonnie, and Connie cooperated.

A moment’s reflection reveals that no partition of the denotation of ‘Annie, Bonnie, and Connie’ will capture the intended truth conditions. Indeed, the relevant articulation is represented by a set whose cells overlap:

<sup>6</sup> For convenience, here we follow Schwarzschild in assuming that a singleton set may be identified with its sole element. Since the sentence ‘Annie weighs 50 kg individually’ is true if and only if  $\text{Annie} \in \llbracket \text{weigh 50 kg} \rrbracket$ , it follows that the same sentence is true if and only if  $\{\text{Annie}\} \in \llbracket \text{weigh 50 kg} \rrbracket$ .

$$\{\{a, b\}, \{b, c\}, \{c, a\}\}$$

The notion of *cover* provides what is needed. A cover is just like a partition except that the cells are allowed to overlap. In the set-theoretic framework, the formal definition is as follows. A set  $C$  is a cover of a set  $D$  if and only if:

- (i)  $C$  is a set of non-empty subsets of  $D$   
(in symbols:  $\forall x(x \in C \rightarrow x \neq \emptyset \wedge x \subseteq D)$ );
- (ii) every element in  $D$  is in at least one set in  $C$   
(in symbols:  $\forall x(x \in D \rightarrow \exists y(y \in C \wedge x \in y))$ ).

Using covers, we can generalize the analysis of ‘Annie and her dogs weigh 50 kg’ presented above (see Schwarzschild 1996, 67):

$[_S \text{ NP}_{\text{plural}} \text{ VP}]$  is true in some context  $Q$  if and only if there is a cover  $C$  of the denotation of the NP that is salient in  $Q$ , and every element in  $C$  is in the extension of the VP.<sup>7</sup>

Let us call this the *cover analysis*.

It is now straightforward to account for the multiple readings of the sentences encountered in this section. The different truth conditions arise from contextual variations that select different covers. If a plural subject is syntactically structured, this structure might obviously influence the context and render salient a cover with the corresponding articulation. So the syntactic structure of a plural subject can affect the truth conditions even if one rejects the assumption that the subject’s semantic value is articulated.

The cover analysis is fully general and applies even when the plural subject is a definite description or a demonstrative noun phrase:

(9.37) The children cooperated.

(9.38) These children cooperated.

If the children in question are Annie, Bonnie, and Connie, these sentences are predicted to exhibit the same range of readings as:

<sup>7</sup> Gillon (1987) requires the additional condition that the cover be *minimal*, namely that no cell be properly contained in any other cell. For our purposes, we can safely leave out this complication. In so doing, however, we do not mean to rule out its correctness.

(9.36) Annie, Bonnie, and Connie cooperated.

This is because the noun phrases in the three sentences co-refer and are therefore associated with the same range of possible covers. Nevertheless, the difference in syntactic structure means that the sentences may make different covers salient.

We now wish to consider an objection to the cover analysis. Let us return to one of the sentences that we used to motivate superplurals in Section 9.4:

(9.7) The square things, the blue things, and the wooden things overlap.

Suppose that the denotations of the three definite descriptions are as follows:

$$\begin{aligned} \llbracket \text{the square things} \rrbracket &= \{a, b\} \\ \llbracket \text{the blue things} \rrbracket &= \{a, c\} \\ \llbracket \text{the wooden things} \rrbracket &= \{a, d\} \end{aligned}$$

Then, according to the cover analysis, we have:

$$\llbracket \text{the square things, the blue things, and the wooden things} \rrbracket = \{a, b, c, d\}$$

where, in the absence of further contextual cues, the salient cover is given by the syntactic structure of the plural subject:

$$C = \{\{a, b\}, \{a, c\}, \{a, d\}\}$$

It follows from the analysis that (9.7) is true if and only if every element in  $C$  is in the extension of ‘overlap’. This means that the truth of (9.7) requires:

$$\llbracket \text{overlap} \rrbracket = \{ \dots, \{a, b\}, \{a, c\}, \{a, d\}, \dots \}$$

But that seems incorrect. For this extension would also sanction the truth of:

(9.39) The square things overlap.

which is clearly false in the given context. Intuitively, the correct extension of ‘overlap’ is as follows:

$$\llbracket \text{overlap} \rrbracket = \{ \{ \{a, b\}, \{a, b\} \}, \dots, \{ \{a, b\}, \{a, c\}, \{a, d\} \}, \dots \}$$

That is,  $X$  is an element of the extension of ‘overlap’ just in case  $X$  is a set of non-empty sets that have some element in common. To make use of this extension, however, we would have to revise the cover analysis so that a

sentence is true if and only *the cover itself* is in the extension of the verb phrase. But this revision would not fit with the analysis of our previous examples where the extension of the verb phrase must contain *the elements of the cover*, not the cover itself.

Let us take a step back. The cover analysis is supported by evidence concerning intermediate readings. Still, at least as formulated above, it cannot handle some of the alleged examples of superplurals, as shown in the previous paragraph. Perhaps some alternative version of the analysis, coupled with a closer linguistic examination of the examples, can overcome this apparent difficulty. Even so, implementing the proposal in the plurality-based setting would have significant consequences for the debate over superplurals.

Indeed, superplurals seem required to formulate the cover analysis in the plurality-based setting. Consider the following examples:

(9.40) The ordinals and the cardinals overlap.

(9.41) The sets and the classes overlap.

If we assign to the descriptions their intended interpretations (the pluralities of ordinals, cardinals, and so on), we would have to rely on superplural terms to describe the salient covers. So appealing to covers in this setting concedes that some sentences express “superplural thoughts”. This suffices for the third grade of superplural involvement described in Section 9.3. It does not matter whether natural language is linked to superplural thoughts in a somewhat roundabout way.

The third grade of superplural involvement is secured even on a more instrumental conception of the semantic machinery associated with covers. The need to invoke superplurals to develop a semantics of natural language would still show that these resources are legitimate for theorizing and hence legitimate in the sense of the third grade.

## 9.8 Mixed-level predications

We now turn to an important challenge for the superplural analysis.<sup>8</sup> The challenge is based on a simple idea. Consider a purported example of a sentence with a superplural subject:

<sup>8</sup> See, e.g., Landman 1989a, 598–601; Schwarzschild 1996, 45–8; Ben-Yami 2013, 87.

(9.42) My children, your children, and her children played against each other.

Now extend the sentence as follows:

(9.43) My children, your children, and her children played against each other and then ate ice cream.

Notice that the added predicate, unlike the original one, does not call for an argument with a superplural articulation. Thus, the two predicates seem to belong to different levels, yet they are applied to the very same subject. How, then, can this subject involve superplural reference?<sup>9</sup>

One might try to handle such mixed-level predications by claiming that, when the semantic value of a noun phrase involves more articulation than is required by a predicate, the redundant articulation is simply “thrown away”, resulting in a less articulated semantic value that *is* appropriate for the predicate in question. To this end, one may postulate a type-shifting operation that maps a superplurality to the underlying plurality; for example:

$$a_1, a_2; b_1, b_2; c_1, c_2, c_3 \mapsto a_1, a_2, b_1, b_2, c_1, c_2, c_3$$

However, type-shifting should not be postulated beyond necessity. And in fact, it may be unnecessary to invoke this kind of mechanism in the present case. There is no need to shift the semantic value of a noun phrase by “throwing away” its redundant articulation. A predicate that does not *require* this extra articulation might nevertheless be *tolerant* of it—in the sense that the predicate simply ignores the articulation.<sup>10</sup> To ensure that the semantic values are tolerant of extra articulation in this way, we require that they be *upwards closed*: if  $X$  is an element of the semantic value, then so is any  $Y$  that is based on the same individuals but has a strictly richer articulation than  $X$ . As an example, consider the distributive plural predicate ‘ate ice cream’. For convenience, we represent its semantic value set-theoretically. Suppose we have:

$$\{a, b, c, d\} \subseteq \llbracket \text{ate ice cream} \rrbracket$$

<sup>9</sup> These examples also pose a challenge for the multigrade analysis. For this analysis has no obvious explanation of why (9.42) follows from (9.43). See the discussion in Section 9.6.

<sup>10</sup> This strategy is critically discussed in Schwarzschild 1996, Chapter 4. Schwarzschild favors a cover-based approach, as noted in Section 9.7.

Assuming that the semantic value of the predicate is upwards closed, it will contain all other sets based on the same individuals but ‘articulated’ in more complex ways. For example, we have:

$$\{\{a, b\}, \{c, d\}\} \subseteq \llbracket \text{ate ice cream} \rrbracket,$$

which corresponds to a superplural articulation of the four individuals. We give this idea a precise mathematical definition in Appendix 9.A.

To sum up, provided that the semantic values of predicates are required to be upwards closed, the challenge under discussion can be answered. Recall our guiding example, (9.43). Suppose the predication of ‘played against each other’ is analyzed as having a superplural subject, namely my children, your children, and her children. The upwards closure of  $\llbracket \text{ate ice cream} \rrbracket$  ensures that the associated predication is true of the mentioned superplurality just in case each of the children ate ice cream, precisely as desired. More generally, a highly articulated semantic value of some noun phrase is never a problem so long as predicates that do not require this degree of articulation simply ignore it.

### 9.9 Mixed-level terms, order, and repetition

The superplural analysis assumes that the semantic value of some plural noun phrases is articulated, namely structured into multiple sets or pluralities. Additional evidence for semantic articulation may come from the linguistic phenomena illustrated by the following sentences:

(9.44) Annie, Annie’s sisters, and Bonnie competed.

(9.45) ‘a’, ‘b’, ‘c’, and ‘d’ are consecutive letters.

(9.46) 1, 1, and 4 have mean 2.

The first example involves a mixed-level list, where singular terms are combined with a plural description. The other examples show sensitivity to order and to repetition. In (9.45), changing the order of the terms affects the truth conditions. So does removing the repetition in (9.46).

Those who accept the superplural analysis can simply take these examples to show the need for yet other forms of articulation. In particular, they can account for the data by countenancing these additional forms of articulated reference:

- (a) reference to *mixed-level pluralities*, including combinations of single objects and pluralities;
- (b) reference to *ordered pluralities*, respecting order;
- (c) reference to *multipluralities*, allowing for repetition.

Suggestions of this kind have been put forward by Hewitt (2012a), Ben-Yami (2013), and Hossack (2020).<sup>11</sup> Hewitt and Hossack claim that plural terms can refer to things in an order (serial reference). According to Ben-Yami, the reference of plural terms can be articulated in various ways (articulated reference): when the articulation represents information about order or repetition, this information can play a role in the truth conditions.

This appeal to additional forms of articulated reference can be challenged, however. Building on earlier work by Kay (1989), Oliver and Smiley (2004; 2016, Chapter 10), and Chaves (2012), Florio and Nicolas (2015) argue that order and repetition should be explained in a different way. They point out that assuming additional forms of articulated reference has limited scope and is unnecessary to give a unified account of the broad range of cases in which order and repetition are semantically relevant. On the account they propose, order and repetition enter the truth conditions through salient indexings introduced by context and by the meaning of special expressions (for example, ‘consecutive’, ‘in that order’, and ‘respectively’).

To see how their proposal works, consider this sentence:

- (9.47) Annie, Bonnie, and Connie arrived in the order they were called.  
(Florio and Nicolas 2015, 449)

Here the order of mention is irrelevant. There are two relevant orders: first, the one in which Annie, Bonnie, and Connie arrived; then, the one in which they were called. The sentence conveys that these two orders are, in some sense, the same. This can be explicated as follows: the sentence is true if and only if the indexing of the plurality of Annie, Bonnie, and Connie according to their time of arrival is isomorphic to the indexing of the same plurality according to the order of calling. There is no need to assume that the semantic

<sup>11</sup> Fine (2010) develops a more comprehensive approach, which can serve as a framework for these suggestions. At the heart of the approach is a general composition operation that can be specialized to obtain any form of articulated reference discussed here. In particular, this operation can be set to respect or ignore each of the following features: order, repetition, and articulation into higher-level pluralities. For a precise statement of these choices, see Fine 2010, 573.

contribution of ‘Annie, Bonnie, and Connie’ goes beyond that of supplying the mere plurality of objects on which the salient indexings operate.<sup>12</sup> So the proposal can be developed without postulating that the semantic value of a plural term is articulated.

Might the multigrade analysis provide an alternative way to account for the examples that seem to motivate the postulation of additional forms of articulated reference? This analysis, we recall, promises an alternative to superplurals. It is naturally extended to treat mixed-level terms, order, and repetition. Using multigrade predicates, one can provide the following regimentation of the examples discussed at the beginning of this section:

(9.44) Annie, Annie’s sisters, and Bonnie competed.

(9.48)  $C(a, tt, b)$

(9.45) ‘a’, ‘b’, ‘c’, and ‘d’ are consecutive letters.

(9.49)  $C(a, b, c, d)$

(9.46) 1, 1, and 4 have mean 2.

(9.50)  $M(1, 1, 4; 2)$

Thus, apparent mixed-level terms are simply cases in which the positions of an argument place are occupied by terms of different levels. Moreover, the semantic relevance of order and repetition is explained by an obvious fact about predication: changing the order of the arguments, as well as adding or removing an argument, does not in general preserve truth.

This extended use of the multigrade analysis has serious limitations, however. The problem is that order and repetition can matter even in the presence of a single, non-conjunctive argument:<sup>13</sup>

(9.51) There are some consecutive letters.

(9.52) These letters are consecutive.

(9.53) Some numbers have mean 2.

(9.54) These numbers have mean 2.

<sup>12</sup> For details, see Florio and Nicolas 2015, Section 5.

<sup>13</sup> See also Ben-Yami 2013, 96–7, and Florio and Nicolas 2015, 455.



The multigrade account of order and repetition just sketched does not apply to these cases, and it is unclear how it could explain them without borrowing from the other approaches discussed earlier in this section. In particular, articulated pluralities might still have to be invoked as covers or as possible values of plural variables. But if so, it becomes harder to justify the introduction of the multigrade apparatus.

It remains an open problem, then, how best to deal with the examples of mixed-level terms, order, and repetition. The case for additional forms of articulated reference is not as strong as that for superplural reference.

### 9.10 Conclusion

We have examined four questions concerning the availability of superplurals, corresponding to increasingly higher “grades of superplural involvement”. The first two grades are unproblematic: there is no obstacle to formulating a superplural logic and a model-theoretic semantics for a superplural language. It is not immediately obvious, however, whether superplural reference is a legitimate expressive resource for beings like us. Nor is it immediately obvious whether superplural reference is realized in natural language. So the third and fourth grades are less straightforward and require careful investigation.

Our investigation showed that a strong case can be made for the highest grade. Some sentences of natural language can plausibly be analyzed in superplural terms, and this analysis fares well compared to alternatives such as the multigrade analysis and the cover analysis. Let us review the pros and cons of each analysis.

The superplural analysis is supported by *prima facie* evidence from articulated noun phrases. However, it may require the assumption that the semantic values of predicates be upwards closed, which some might find problematic. Moreover, by avoiding superplural reference, the alternative analyses can claim better ideological economy.

The multigrade analysis offers a simple way to handle some cases of order and repetition, but it faces difficulties in accounting for all cases in which order and repetitions are semantically relevant. Moreover, it must rely heavily on meaning postulates to capture data about logical consequence.

The cover analysis is supported by evidence from intermediate readings. But it seems to falter on promising examples of superplurals, where the cover

itself, rather than its members, has to be included in the extension of the predicate.

What about the third grade of superplural involvement? If there is a good case for the highest grade, there is also a good case for the third grade. That is, if some natural language realizes superplural reference, it is hard to deny that this form of reference is a legitimate expressive resource for beings like us. In fact, the case for the third grade is probably even stronger than that for the highest grade. In particular, since we have accepted the plurality-based model theory as a new and valuable alternative to the traditional set-based model theory, it becomes difficult to avoid superplurals. Furthermore, in that style of model theory, superplurals are likely to be involved even in the cover analysis and in the multigrade one. The theoretical need to adopt superplural resources thus shows that these resources are legitimate for theorizing, which would also establish our third grade of superplural involvement.

## Appendix

### 9.A The notion of upwards closure

In this appendix, we make formally precise some of the ideas outlined in Section 9.8. Our first task is to define an analogue of the sets of finite rank based on a domain of individuals  $D$ , though modified so as to consider only sets of cardinality greater than or equal to 2. This modification is natural when modeling superplural phenomena. Sets are used to represent pluralities. But since a singleton plurality is identical with its single member, there is no need for a singleton set to represent this singleton plurality, since this plurality is already represented by whatever represents its single member. We therefore proceed as follows. First, we define our modified powerset operation by letting  $\wp^{\geq 2}(X)$  be the set of subsets of  $X$  of cardinality greater than or equal to 2. Next, we define the analogue of the sets of finite rank based on  $D$ :

- $V_0^{\geq 2}(D) = D$
- $V_{i+1}^{\geq 2}(D) = \wp^{\geq 2}(V_i^{\geq 2}(D))$
- $V_\omega^{\geq 2}(D) = \bigcup_{i < \omega} V_i^{\geq 2}(D)$

Finally, let  $D^\star = V_\omega^{\geq 2}(D)$ .

We are now ready to define the desired operation of upwards closure. Consider some  $X \in D^\star$ . What is it for some  $Y \in D^\star$  to be the result of imposing a richer articulation on  $X$ ? Let us proceed in steps. First, let us say that  $Y$  is a *simple articulation* of  $X$  just in case  $Y$  is the result of replacing zero or more members of  $X$  with subsets of  $X$ , ensuring that all of the members of  $X$  are “used” in the construction, that is, that all members of  $X$  are either in  $Y$  or in some member of  $Y$ . For example, let  $X$  be  $\{a, b, c\}$ . Then two simple articulations of  $X$  are  $\{a, \{b, c\}\}$  and  $\{a, \{a, b, c\}\}$ . Next, let us say that  $Y$  is an *articulation* of  $X$  just in case  $Y$  can be obtained by a finite chain of simple articulations starting with  $X$ .

Notice that this definition works for the representation of higher-order pluralities as well. For example, let  $X$  be  $\{\{a, a'\}, \{b, b'\}, \{c, c'\}\}$ . Then  $\{\{a, a'\}, \{\{b, b'\}, \{c, c'\}\}\}$  is a simple articulation of  $X$ .

These definitions can be made precise as follows.

**Definition 9.1** Let  $X \in D^\star$ . Then  $Y$  is a *simple articulation* of  $X$  if and only if there are  $U, V \in D^\star$  such that

- (i)  $Y = (X \setminus U) \cup V$
- (ii)  $x \in V \rightarrow x \subseteq X$
- (iii)  $U \subseteq \bigcup V$

Next,  $Y$  is an *articulation* of  $X$  if and only if there is a finite chain  $X_0, X_1, \dots, X_n$  such that

- (i)  $X = X_0$
- (ii)  $Y = X_n$
- (iii)  $X_{i+1}$  is a simple articulation of  $X_i$  for each  $i < n$ .

Finally, let  $A \in D^*$  be the semantic value of some predicate. Its *upwards closure* is defined as:

$$UC(A) = \{B \in D^* : B \text{ is an articulation of } A\}$$

Suppose we require that semantic values of predicates be upwards closed. Then there is no problem about predications of the sort discussed in Section 9.8. Consider our main example:

- (9.43) My children, your children, and her children played against each other and then ate ice cream.

Provided that the semantic value of 'ate ice cream' is upwards closed, it will apply to the articulated semantic value of 'my children, your children, and her children' just in case it applies to the individuals which are thus articulated.

In fact, the machinery just developed enables us to define what it is for a predicate to be plural (= plural of order 1), superplural (= plural of order 2), or plural of order  $n$ .

**Definition 9.2** Assume the semantic value of a predicate  $P$  is  $Y$ . Then  $P$  is *plural of order  $n$*  if and only if there is a  $X \subseteq V_n^{\geq 2}(D)$  such that  $Y = UC(X)$ .

That is, a predicate  $P$  on a domain  $D$  is plural of order  $n$  just in case its semantic value  $Y$  can be generated as the upwards closure of a set  $X$  whose members are of rank  $n$ .

