2022

Mathematics

[Honours]

(B.Sc. Second Semester End Examination-2022)

PAPER-MTMH C201

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group-A

(Group Theory - I)

F.M. - 37

1. Answer any six questions: **6x2= 12**

a) Let $(G,*)$ be a group and $a,b \in G$. If $a^2 = e$ and $a*b^2*a = b^3$ then prove that $b^5 = e$.

b) Find the number of generators of additive group $(Z_{36},+)$

c) Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 1 & 3 & 2 & 8 & 6 & 7 \end{pmatrix}$.

d) Define quasigroup and examine if $(Z,\cdot)$ is a quasigroup or not.

e) Let $S = \{a_1, a_2, ..., a_n\}$. How many binary operations can be defined on S?

f) If the set $\{1, x, y\}$ forms multiplicative group then show that $(xy)^{-1} = xy$ and $x^3 = y^3 = 1$

g) Prove that each element of a finite group is of finite order.

h) For what condition union of two subgroups of a group G is a subgroup of G? Justify.

i) Let G be a group and $a \in G$. Find the $o(a^8)$ if $o(a) = 17$

2. Answer any six questions: 3x5 = 15

a) Prove that the order of every subgroup of a finite group G is a divisor of the order of G. Is the converse true? Justify.

b) i) If (G,o) be a group in which $(aob)^3 = a^3ob^3$ and $(aob)^5 = a^5ob^5$ for all $a, b \in G$. Prove that the group (G,o) is an abelian.

ii) Let (G,o) be a finite abelian group with elements $a_1, a_2, ..., a_n$ and $x = a_1 = a_1oa_2o...........oa_n$ Show that $xox = e$

c) Show that A_3 the set of all even permutation of $\{1,2,3\}$ is a cycle group with respect of product of permutations. Is it Commutative? Answer with reason.

d) Let (G,o) be a group and $a \in G$. Prove that $Z(G)$ the centre of the group is subgroup of the $C(a)$, the eentraliser of a.

If there exists an element a in G such that $C(a) = Z(G)$, Prove that (G,o) is commutative group.

e) i) Prove that all proper subgroup of order 8 is commutative.

ii) Find all subgroup of A_3

3. Answer any one questions: 1x10 = 10

a) i) Let G be a group of order 15. A and B are two subgroups of order 3 and 5 respectively. Prove that $G=AB$

ii) Prove that set of all complex numbers of unit modulus forms a commutative group with respect to multiplication.

b) i) Let $S = \{1., w, w^2, -1, -w, -w^2\}$ where $w = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$, Prove that S is cyclic group under multiplication. Find the generators of the cyclic group.

ii) Let G be a infinite cyclic group generated by a. Prove that a and a^{-1} are the only generated of the group.

Group-B

(Vector Analysis - I)

F.M. - 23

1. Answer any four questions: 4x2= 8

a) If $\frac{d^2r}{dt^2} = 6t\hat{i} - 24t^2\hat{j} + 4\sin\hat{k}$ and if $\vec{r} = 2\hat{i} + \hat{j}$ and $\frac{d\vec{r}}{dt} = -\hat{i} - 3\hat{k}$ when $t = 0$ then show that $\vec{r} = (t^3 - t + 2)\hat{i} + (1 - 2t^4)\hat{j} + (t - 4\sin t)\hat{k}$

b) Find the vector $\vec{x}$ from the equation $\vec{x}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{x}.\vec{a}=0$ where $\vec{a}.\vec{b}\neq 0$. In particular, if $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=3\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+\hat{j}+3\hat{k}$ then find $\vec{x}$

c) Find the directional derivative of $\phi=x^2yz+4xz^2$ at $(1,-2,-1)$ in the direction of $2\hat{i}-\hat{j}-2\hat{k}$

d) Find the equation of the tangent plane to the surface $xyz=4$ at the point (1, 2, 2)

e) Find constants a, b, c so that the vector $\vec{v}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational.

f) Find the vector equation of the straight line passing through the point having position vector $\hat{i}-2\hat{j}+\hat{k}$ and perpendicular to the vector $2\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$

2. Answer any one question out of two: **1x5 = 5**

a) i) Find the shortest distance between the times

$$\vec{r}=(2-\lambda)\hat{i}+(\lambda-3)\hat{j}+(5-3\lambda)$$
$$\vec{r}=(\mu+2)\hat{i}+(3\mu-2)\hat{j}-(3\mu+2)\hat{k}$$

ii) If $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$ when $t=2$ and $\vec{r}=4\hat{i}-2\hat{j}+3\hat{k}$ when $t=3$ then show that $\int_2^3\left(\vec{r}.\frac{d\vec{r}}{dt}\right)dt=10$

b) If $\hat{e}_1$ and $\hat{e}_2$ be two unit vectors and θ be the angle between their directions, show that $2\sin\theta/2=|\hat{e}_1-\hat{e}_2|$

3. Answer any one question: **1x10 = 10**

a) i) Necessary and sufficient condition that a proper vector $\vec{u}$ always remains parallel to a fixed line i.e. to have a constant direction is $\vec{u}\times\frac{d\vec{u}}{dt}=0$

ii) Show that the lines $\vec{r}=\vec{a}+t(\vec{b}\times\vec{c})$ and $\vec{r}=\vec{b}+s(\vec{c}\times\vec{a})$ will intersect if $\vec{a}.\vec{c}=\vec{b}.\vec{c}$

iii) Show that the solution of the eqn^n $k\vec{r}+\vec{r}\times\vec{a}=\vec{b}$ where k is a non-zero scalar and $\vec{a}$ and $\vec{b}$ are two vectors, then the representation of $\vec{r}=\frac{1}{k^2+a^2}\left(\frac{\vec{a}.\vec{b}}{k}\vec{a}+k\vec{b}+\vec{a}\times\vec{b}\right)$ 4+3+3

b) i) Find the volume of the parallelepiped whose three concurrent edges are represented by $\vec{a}=3\hat{i}-5\hat{j}-4\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}-2\hat{k}$, $\vec{c}=3\hat{i}+\hat{j}-2\hat{k}$

ii) State Lami's theorem

iii) Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. 5+1+4