2022

Mathematics

[Honours]

(B.Sc. Second Semester End Examination-2022)

PAPER-MTMH C202

(Real Analysis I)

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

Group-A

1. Answer any ten questions: **10x2= 20**

a) Prove that there is no least positive real number.

b) Find the solution set of $\left|\frac{x+2}{2x-1}\right| \leq 3$

c) Give the definition of Interior point and limit point of set in $\mathbb{R}$.

d) State principle of Induction for set of natural number.

e) Show that $\lim_{n\to\infty} \frac{1}{n}\left(1+2^{1/2}+3^{1/3}+\ldots+n^{1/n}\right)=1$

f) Prove that, $\lim_{n\to\infty} 2^{-n}n^2=0$

g) Find supremum of A and infimum of A of the set

$$A=\left\{1+(-1)^n\frac{1}{n}|n\in N\right\}$$

h) State the properties of supremum and infimum of a set in $\mathbb{R}$

i) Prove that the every point of the set $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}.....\right\}$ is isolated point.

j) If a $a\geq 0$ and $a<\in$ for every $\in>0$ prove that $a=0$

k) Let G be an open set in $\mathbb{R}$ and S be a non-empty finite subset of G. Prove that G-S is an open set.

l) Prove that the closer of a set S is the smallest closed super set of S

m) Prove that the set of all limit points of a bounded sequence is bounded.

n) Give an example of a sequence which have unique limit point but not convergent.

o) Is the series $1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+...+\frac{1}{n^n}+..$ is convergent?

2. Answer any four questions: 4x5 = 20

a) Let A and B are non empty bounded subsets of R: Prove that

i) Sub $A\cup B=min\{Sup\,A,Sup\,B\}$

ii) $\inf A\cup B=\min\{\inf A,\inf B\}$

b) State and prove Bol zano – weierestrass theorem.

c) Test the convergence of the series

$$1+\frac{1!}{2!}x+\frac{(2!)^2}{4!}x^2+\frac{(3!)^2}{6!}x^3+...x>0$$

d) Prove that the series $\sum u_n v_n$ converges absolutely if the series $\sum u_n$ be absolutely converge and $\{v_n\}$ be a bounded sequence..

e) Prove that every sequence of real number has a monotone sub-sequence.

f) Prove that the sequence $\{u_n\}$ is convergent by showing that the subsequence $\{u_{2n}\}$ and $\{u_{2n-1}\}$ converges to the same limit, $0<u_1<u_2$ and $u_{n+2}=\frac{1}{3}(u_{n+1}+2un)\,for\,n\geq 1$

Group -C

3. Answer any one questions: 2x10 = 20

a) Discuss the convergence of the series

i) $\sum\frac{(-1)^{n+1}}{\log(n+1)}$

ii) $\sum\frac{(-1)^n 3^n}{n!}$

b) Let $S=\left\{\frac{(-1)^m}{m}+\frac{1}{n}:m.n\in\mathbb{N}\right\}$

i) Show that 0 is a limit point of S

ii) If $K \in N$, show that $\frac{1}{K}$ is a limit point of S

iii) If $K \in N$, then show that $\frac{-1}{2K-1}$ is a limit point of S

c) i) Prove that the sequence $\{x_n\}$ and $\{y_n\}$ defined by

$$x_{n+1} = \frac{1}{2}(x_n + y_n), \quad \frac{2}{y_{n+1}} = \frac{1}{x_n} + \frac{1}{y_n} \quad \text{for} \quad n \geq 1, x_1 > 0, y_1 > 0$$

converges to a common limit l where $l^2 = x_1 y_1$

ii) Let S be a subset of $\mathbb{R}$. Then S is a closed set if and only if $S = S \cup S'$
