

End Semester Examination, 2022**Semester - IV****Physics****PAPER - CC8T**

Full Marks : 40

Time : 2 Hours

Answer any five questions :

1.a) Given the matrix $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Find the eigenvalues of it 2

b) Write down the necessary and sufficient conditions for analytic complex function. 2

c) Show that $(AB)^n = A^n B^n$ if $AB = BA$ 2

d) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1-x^2, & \text{for } |x| \leq 1 \\ 0, & \text{for } |x| > 1 \end{cases} \quad 2$$

e) The real part of an analytic complex function is $u = x^2 - 3y^2$. What is its imaginary part? 2

f) State Convolution and Parseval's theorem for Fourier's integral transforms. 2

g) Show that the real and imaginary parts of an analytical function are harmonic functions. 2

h) Define zeros and poles of complex functions. 2

(Turn Over)

Gr - B

2.a) Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ between the annular region $z=1$ and $z=2$.

b) Find the residue of $f(z) = \frac{ze^z}{(z-a)^3}$ at $z=a$ 3+2

3.a) Find the inverse of the matrix $A = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 2 & 1 \\ 3 & 4 & 0 \end{pmatrix}$

b) Find the rank of the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & -1 \end{bmatrix}$ 3+2

4. Show that if $f(z) = u + iV$ is an analytic function $\vec{F} = V\hat{i} + u\hat{j}$ is a vector, then $\text{div } \vec{F} = 0$ and $\text{curl } \vec{F} = 0$ are equivalent to Cauchy-Reimann equations. 5

5.a) Find the first three terms of Taylor expansion of $f(z) = \frac{1}{z^2 + 4}$ about $z = -i$ and give the region of convergence.

b) Test the analyticity of the functions given below
 $w = \ln z$ 3+2

6.a) Find the Fourier Cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$

b) Show that the matrix $[A]$, as given below is unitary

$$A = \begin{bmatrix} 1/\sqrt{2} & -i/\sqrt{2} \\ i/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \quad 3+2$$

7. Find the Fourier transform of the Gaussian distribution function $f(x) = Ne^{-\alpha x^2}$, where N and α are constant D Write down some comment on result. 4+1

Gr - C

8.a) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z by Milne-Thomson method.

b) State the Cauchy's integral formula. Evaluate

$$\int_c \frac{dz}{z^2 - 1} \quad \text{where } c \text{ is the circle } x^2 + y^2 = 4 \quad 5+5$$

9.a) For the matrix $[A]$, as given compute the following : the transpose $[A^T]$, the conjugate $[A^*]$ and the conjugate transpose $[A^\dagger]$

$$A = \begin{bmatrix} 5-3i & 3-5i \\ 3 & 6+2i \\ 4+i & 3-2i \end{bmatrix}$$

b) Solve with the help of matrices, the simultaneous equations

$$x + y + z = 3$$

$$x + 2y + 3z = 4$$

$$x + 4y + 9z = 6$$

5+5