2022

Mathematics

[Honours]

(B.Sc. Fourth Semester End Examination-2022)

PAPER-MTMH C403

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

[USE SEPARATE SHEET FOR EACH GROUP]

Group-A

(Vector Analysis – II)

1. Answer any six questions: **6x2= 12**

a) Show that a necessary and sufficient condition for a scalar point function ϕ is to be constant is that grad $\phi = 0$

b) If $\vec{a}$ is a constant vector and $\vec{r}$ is the radius vector then prove that $\operatorname{div}(\vec{r}\times\vec{a}) = 0$

c) Prove that curl of grad ϕ is a null vector.

d) If f and g are two scalar point function prove that $\operatorname{div}(f\nabla g) = f\nabla^2 g + \nabla f \nabla g$

e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is position vector then evaluate $\nabla(\log r)$ where $r = |\vec{r}|$

f) Derive the expression for maximum value of direational derivative of a scalar point function ϕ at the point (x, y, z) in the direction of unit vector $\hat{e}$.

g) What do you mean by conservative vector field. Show that the work done by a particular conservative field of force in a closed path is 0.

h) A vector field is given by $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$ is scalar potential of $\vec{A}$ exists?

i) If ϕ is differentiable scalar function of x, y, z, t, where x, y, z are differentiable scalar function of t prove that $\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + \Delta\phi . \frac{d\vec{r}}{dt}$

2. Answer any three questions: 3x5 = 15

a) Find the circulation of $\vec{F}$ around the curve C where $\vec{F} = (2x - y + 4z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z^3)\hat{k}$ and C is the circle $x^2 + y^2 = 9,\ z = 0$

b) Find the angle of intersection of the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1, 2)

c) Use stokes' theorem to evaluate $\int_c (2x - y)dx - yz^2 dy - y^2 z dz$ where c is the circle $x^2 + y^2 = 1$ corresponding to the surface of sphere of unit radius.

d) What is the physical significance of curl of a vector point function?

e) Show that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r = |\vec{r}|$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

3. Answer any one question: 1x10 = 10

a) Prove that a necessary and sufficient condition that the vector field $\vec{F}$ be conservative is that $\vec{\nabla} \times \vec{F} = \vec{0}$ where $\vec{F}$ has continuous derivatives in an open region R in which $\vec{F}$ is defined.

b) Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ where, $\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$. S is the surface of the region bounded by $x^2 + y^2 = 4,\ z = 0,\ z = 4$ in first octant.

Group-B

(Metric Space – I)

1. Answer any four questions: 4x2= 8

a) If d_1 and d_2 are two metrices on a non-empty set A. Prove that $ad_1 + bd_2$ is also a metric on A where $a, b \in \mathbb{R}^+$

b) If $x_1, x_2, \ldots\ldots x_m$ are arbitrary points of a metric space (A, d). Then prove that $d(x_1, x_m) \le d(x_1, x_2) + d(x_2, x_3) + \ldots + d(x_{m-1}, x_m)$

c) Let (x,d) be a metric space and $A, B \subseteq X$ then prove that $d(A,B) \le diam(A \cup B)$

d) Find all open and closed sets in a discrete metric space.

e) Define complete metric space (x,d). Give an example of a metric space which is not complete in a metric space but complete in another metric space. 1+1

f) Let $\mathbb{R}^2$ be the set of all order pairs of real numbers and define a metric $d : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ by $d(x,y) = |x_1 - y_1| + |x_2 - y_2|$ for all $x = (x_1, x_2)$ and $y = (y_1, y_2) \in \mathbb{R}^2$. Draw geometrically an ball $B(a,r)$ in $\mathbb{R}^2$

2. Answer any one questions: **1x5 = 5**

a) Let A denote the set of all Cauchy sequences of real numbers. . We define a distance function $d : A \times A \to \mathbb{R}$ by $d(x,y) = Sup|x_i - y_i|$ for all $x = \{x_n\}$ and $y = \{y_n\}$. Prove that (A,d) is metric space.

b) Let A be a subset of a metric space (X,d). Then prove that $x \in \overline{A}$ iff every open nbd of x intersect A

3. Answer any one questions: **1x10 = 10**

a) i) Prove that if A is closed iff each sequence in X which converges to a point A^c is eventually in A^c.

ii) Let (x,d) be a metric space and $A \subseteq X$ and F be a closed set containing A. Then prove that $\overline{A} \subseteq F$.

iii) Define separable metric space and give an example.

b) i) Let (x,d) be a metric space $U \subseteq X$ and $x \in X$. Then U is a nbd of x iff there is an open ball $B(y,r)$ for some $r > 0$ and some $y \in X$ (y is not necessarily x) such that $x \in B(y,r) \subseteq U$

ii) Define bounded set in a metric space (X,d) and give an of a bounded set.

iii) Let (X,d) be a metric space and $A, B \subseteq X$ prove that $A^0 \cup B^0 \subseteq (A \cup B)^0$. Is the converse true? Justify.