Total Pages-04 RNLKWC/B.Sc./CBCS/IVS/MTM-GE-401/22

2022

Mathematics

[Generic]

(B.Sc. Fourth SemesterEnd Examination-2022)

PAPER-MTM GE-401

Full Marks: 60 ***Time: 03 Hrs***

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer anytenquestions: **10x2= 20**

a) Determine the value of 'a', so that the function $f(x)$ is defined

by $f(x)=\begin{cases} \dfrac{a\cos x}{\lambda-2x}, x\neq \pi/2 \\ x=\pi/2 \end{cases}$

b) If $f(x)=\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ show that $f'(x)=\dfrac{1}{2\sqrt{1-x^2}}$

c) State Leibnitz's theorem on successive derivative.

d) Find the Integrating Factor of the differential equation

$\dfrac{dy}{dx}+\dfrac{2}{x}y=5e^{2x}$

e) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$$

f) Find the maximum value of the function $f(x)=3-|x-1|, x\in\mathbb{R}$

g) What do you mean by Random experiment.

h) Define the classical definition of probability.

i) Evaluate the limit $\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x^2\sin x}$

j) Evaluate $\int_0^{2\pi}|\cos x|\,dx$

k) When a differential equation is called a linear differential equation?

l) Evaluate $\frac{1}{D^2-2D+3}e^{2x}$

m) Write down the sample space for the experiment of tossing a coin thrice.

n) Define mutually exclusive events with example.

o) A coin is tossed three times successively. Find the probability of getting exactly one head or two heads.

2. Answer anyfourquestions: **4x5 = 20**

a) If $y=Sin\left(m\,Sin^{-1}x\right)$ then show that

i) $(1-x^2)y_2-xy_1+m^2y=0$

ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0$ 2+3

b) Solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+2y=\cos(\log x)$

c) For a die throwing experiment, define the following event

A : even of getting odd number.

B : even of getting even number.

C : even of getting prime number.

D : even of getting a number more than 3.

i) Write down the sample spare and the events A, B, C,. D.

ii) Obtain $A\cup C$, $B\cup D$, $D\cap A$, $B\cap C$

iii) Find A^C and C^C

d) Find the derivative of the function $e^{\sqrt{x}}$ by using first principle.

e) .Solve the differential equation $y=px+p^2x$

f) For a random variable *X*, $E(X-1)^2=10$ and $E(X-2)^2=6$. Find *E*(X) and var(X). If a variable X assume two values -2 and 1 such that $2P(X=-2)=P(X=1)$, then find var(X).

3. Answer any two questions: **2x10 = 20**

a) i) If A and B be any two events corresponding to a random experiment E, then prove that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ 5

ii) What is cumulative distributive function. Find the cumulative distributive function of the random variable X, the number of heads obtained in tossing of a fair coins three times 1+4

b) i) State Rolle's theorem

ii) Verify the Rolle's theorem for the function $f(x) = x\sqrt{4-x^2}$ in $0 \le x \le 2$.

iii) Find the value of a, b such that

$$\lim_{x \to 0} \frac{ae^x - b\cos x + e^{-x}}{x\sin x} = 2$$

2+3+5

c) i) Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$

ii) Solve $\left(D^2 - 9\right)y = \sin 2x$

iii) If $I_n = \int e^{-x}x^n dx$, show that $I_n = -e^{-x}x^n + +nI_{n-1}$

iv) Evaluate $\lim_{x \to \alpha} \frac{1 + 2^{10} + 3^{10} + \ldots\ldots\ldots + n^{10}}{n^{11}}$ 2+3+2+3