2022

MATHEMATICS

[HONOURS]

(B.Sc. Sixth Semester End Examination-2022)

PAPER-MTMH-C602

Full Marks: 60 ***Time: 03Hrs***

The figures in the right hand margin indicate marks
Candidates are required to give their answers in their own words as far as practicable
Illustrate the answers wherever necessary
[Use separate answer script for each group]

Group A
[Linear Algebra –II]

1. Answer any five questions: **5x2= 10**

a) Let, A be square matrix satisfy A^3=A. Then test the diagonalizability of A.

b) If α and β be two orthogonal vectors in a Euclidian space V, then prove that $\|\alpha+\beta\|=\|\alpha-\beta\|$.

c) Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be the linear transformation defined by $T(x_1,x_2,x_3)=(x_1+x_2,x_2+x_3,x_3+x_1)$ for all $(x_1,x_2,x_3)=\in\mathbb{R}^3$ then find Rank (T).

d) Find a non – null vector γ that is orthogonal to $\alpha=(1,2,1)$ and $\beta=(-1,1,2)$

e) Show that every orthogonal set of non null vectors in an inner product space is linearly independent.

f) Find the orthogonal complement of the row space of the matrix

$$A=\begin{pmatrix}1 & 1 & 2\\ 2 & 3 & 5\\ 3 & 4 & 7\end{pmatrix}$$

g) Let W be the subspace of $\mathbb{R}^3$ spanned by the vectors (1,1,0) and (0,1.1). Find basis of the annihilator of W.

h) Consider the following polynomials in polynomial space with the inner product $(f,g)=\int_0^1 f(t)g(t)dt, f(t)=t+2, \quad g(t)=3t-2.$ Then find $\|f\| and \|g\|$

2. Answer any two questions: 2x5 = 10

a) Suppose $T:P_2(\mathbb{R})\to P_2(\mathbb{R})$ be a linear operator defined by $T(f(x))=f(o)+f(1)(x+x^2)$ .Then obtain the eigen values of T. Also test T for diagonalizability.

b) Find the orthogonal basis of the row space of the matrix

$$A=\begin{pmatrix}1 & 2 & 0\\ 1 & 0 & -1\\ 2 & 2 & 1\end{pmatrix}$$

c) Let $\alpha=(1,-2,-1,3)$ be a vector in $\mathbb{R}^4$.Find the orthogonal and orthonormal basis for $\alpha^{\perp}$

3. Answer any one questions: 1x10 = 10

a) i) Using Gram-Schmidt orthogonalisation process find the orthonormal set of basis vector for the following set of vectors:

$V=\{(1\,1\,1\,1),(1\,2\,1\,0),(2\,1\,2\,3)\}$

ii) Determine all possible Jordan canonical forms for a linear operator $T:V\to V$ whose characteristic polynomial is $f(x)=(x-2)^3(x-5)^2$

b) i) Given $\{\alpha_1=1,\alpha_2=1+x,\alpha_3=x+x^2\}$ as a basis of $P_2(\mathbb{R})$.

Define the inner product as $(f,g)=\int_{-1}^{1} f(x)g(x)dx$

where f(x) , g(x) are elements of $P_2(\mathbb{R})$ .Construct an orthonormal basis of $P_2(\mathbb{R})$ from the given set.

ii) If $\{\alpha=(1,-1,3),\alpha_2=(0,1,-1),\alpha_3=(0,3,-2)\}$ is a basis of $\mathbb{R}^3$. Then find the dual basis of it.

Group B

[Complex Analysis]

1. Answer any five questions:: 5x2= 10

1.1) Show that the stereographic projection of y-axis is the circle.

1.2) Examine whether the function is continuous at $z=0$

$$f(z)=\begin{cases}|z|^2\sin\left(\dfrac{1}{|z|^2}\right), z\neq 0\\ 0,\ z=0\end{cases}$$

1.3) Find the radious of convergence of the series $1+\frac{a.b}{1.c}z+\frac{a(a+1)b(b+1)}{1.2.c(c+1}z^2+.......$

1.4) Prove that $\frac{\pi}{4}=\left(\frac{1}{2}+\frac{1}{3}\right)-\frac{1}{3}\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\frac{1}{5}\left(\frac{1}{2^5}+\frac{1}{3^5}\right).......$

1.5) Find the value of the integral $\oint_{|1-z|=1}\frac{e^z}{z^2-1}dz$

1.6) Let *f(z)* be an entire function such that for some constant $\alpha, |f(z)|\le\alpha|z|^3$ for $z\ge1$ and $f(z)=f(iz)$ for all $z\in\mathbb{C}$. Prove that *f(z)* is constant.

1.7) Show that when $0<|z|<4$ $\frac{1}{4z-z^2}=\sum_{n=0}^{\alpha}\frac{z^{n-1}}{4^{n+1}}$

2. Answer any two questions: 5x2 = 10

2.1) Let D be the rectangular region $\{(x,y):|x|\le4,|y|\le3\}$. suppose *f* is analytic in *D* and satisfies $|f(z)|\le1$ on *D* then show that $|f^1(0)|\le\frac{14}{9\pi}$

2.2) a) Let $f(z)=\frac{x^3-y^3}{x^2+y^2}+i\frac{x^3+y^3}{x^2+y^2}, z\ne0$
$=0,\ z=0$
Show that the function f satisfies *C-R* equations at the origin but f is not differentiable at *z=0*

b) Show that the function $u(x,y)=Cosx\,Coshy$ is harmonic and find its harmonic conjugate function v. 3+2

2.3) a) If $B(z,\bar{z})$ is continuous and has continuous partial derivatives in a region R and on its boundary C where $z=x+iy$ and $\bar{z}=x-iy$. Prove that $\oint_c B(z,\bar{z})dz=2i\iint_R\frac{\partial B}{\partial\bar{z}}dxdy.$

b) Find the upper bound of the integral (without evaluating the integral) $\left|\int_c\frac{(z-1)e^{2z}Logz}{z^2-7}dz\right|$ where $C=\{z:z=e^{i\theta},\pi/3\le\theta\le\pi/2\}$.

3. Answer any one questions: 1x10 = 10

3.1) a) If a function *f(z)* is analytic for all finite values of z and as $|z|\to\infty, |f(z)|=A\left(|z|^k\right)$ then prove that *f(z)* is a polynomial of degree $\le K$.

b) Find the value of the integral $\int_0^{1+i}(x-y+ix)dx$ along the straight line from *z=0 to z=1+i*.

c) Find the Taylor's series expansion of $f(z)=\frac{z^2-1}{(z+2)(z+3)}$ when $|z|<2$.

3.2) Find the image of the infinite strips

i) $\frac{1}{4}<y<\frac{1}{2}$ and

ii) $o<y<\frac{1}{2}$ under the transformation $W=\frac{1}{z}$.
